Application of Linear Programming Problem to Health Care

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Abstract— Operation research is a scientific approach to problem solving for executive decision-making which requires the formulation of mathematical, economic and statistical models for decision and control problems to deal with situations arising out of risk and uncertainty. In fact, decision and control problems in any organization are more often related to certain, production scheduling, manpower planning and distribution, and maintenance. In this content we are going to discuss about the application of Linear Programming Problem in medical centre. In this content we also discussed about the Diet Problem and to find the optimal state of nature.

Key words: Health Care, executive decision-making

I. INTRODUCTION

Operation Research is a science designed to provide quantitative tools to decision – making processes. Many researchers point out the development of linear programming method as one of the most important scientific advances of the second half of the 20th century.

Programming has demonstrated to be an alternative solution to plan Brach therapy, replacing the traditional solutions based on trial and error. Linear Programming has been used to formulate balanced diets at minimum cost and complying with a set of nutritional restrictions.

In order to attain optimized solutions of healthcare problems involving economics and nutrition. First, we present the formulation of a diet at minimum cost using only two nutritional restrictions aiming at making investigators in the field of healthcare more familiar with the terms and potentialities of the method reported.

Later, the describe an optimized solution for problems related to allocation of medical interventions that complies with a set of budget restrictions and medical visits. Decision tree have been applied to economic analysis, such a cost - effectiveness, to choose the best management strategies. The tree may indicate with interventions use less resource and represent better quality of life.

II. DESCRIBING A LINEAR PROGRAMMING METHOD

Optimization models are defined by an objective function composed of a set of decision-making variables, subject to a set of restrictions, and presented as mathematical equations. The objective of optimization is to find a set of decision-making variables that generates an optimal value for the objective function, a maximum or minimum value depending on the problem, and complies with a set of restrictions imposed by the model. Such restrictions are conditions that limit the decision-making variables and their relations to assume feasible values. In linear Programming models, the objective function is linear, that is, it is defined as a linear combination of decision-making variables and a set of constants, restricted to a set of linear equality or inequality equations. Therefore, the model composed of an objective function, restrictions, decision-making variables and parameters.

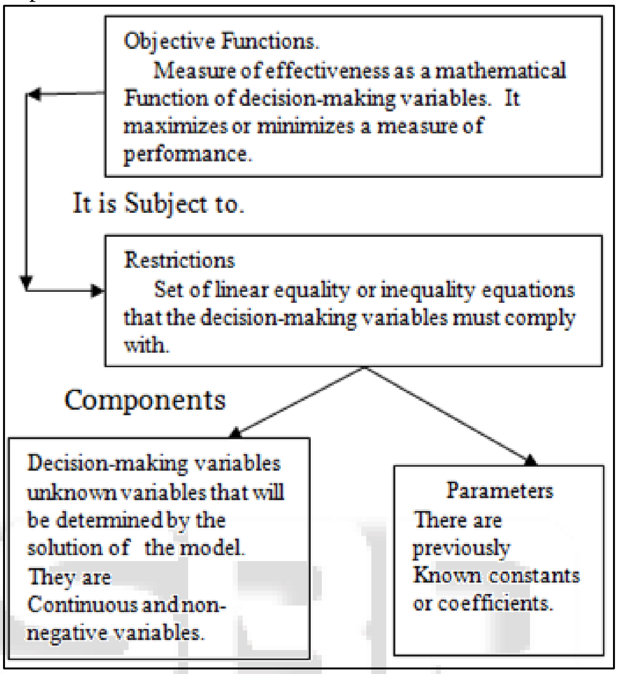

Fig. 1:

The chart represented in figure shows definitions and interactions among these components. A solution of a problem is called optimal when the decision-making variables assume value of the objective function and complies with all restrictions of the model.

An algebraic representation of a generic formulation of linear programming model could be presented as follows.

To maximize or minimize the objective function:

$$z = c_1 x_1 + c_2 x_2 + ------- + c_n x_n \quad (1.1)$$

It is subject to restrictions

$$a_{11} x_1 + a_{12} x_2 + ------- + a_{1n} x_n \le r_1 \quad (1.2)$$

$$a_{21} x_1 + a_{22} x_2 + ------- + a_{2n} x_n \le r_2 \quad (1.3)$$

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$$-------------------------$$

$$a_{m1} x_1 + a_{m2} x_2 + ------- + a_{mn} x_n \le r_m \quad (1.4)$$

$$X_j \ge 0 (j = 1, 2, ------, n) \quad (1.5)$$

Where

(a) Represents the mathematical function encoding the objective of the problem and is called objective function.

In linear Programming, this function must be linear.

(b)To (e) represents the linear mathematical function encoding the main restrictions identified.

(e) Non-negativity restrictions, i.e.; the decision-making variables may assume any positive value or zero.

"X_j" Corresponds to the decision-making variables that represent the quantities one wants to determine to optimize the global result.

"c_i" represents gain or cost coefficients that each variable is able to generate.

"r_j" represents the quantity available in each resource.

"a_{ij}" represents the quantity of resources each decision making variable consumes.

III. THE DIET PROBLEM

To illustrate an application of linear programming in diet formulation:

Let us assume that a diet for justifiable reasons is restricted to skimmed milk and a salad using well-known ingredients. We know that the nutritional requirements will be expressed as vitamin a and calcium controlled by their minimum quantities (in milligrams).

Table 1 summarizes the quality of each nutrient available in foods and their daily requirement for good health conditions of an individual, as well as the unitary cost of these foods. The objective is to minimize the total diet cost and comply with nutritional restrictions.

A. Cost, Nutrients and Predetermined Nutritional Restrictions

Nutrient	**Milk (glass)**	**Salad (500mg)**	Minimum Nutritional Requirement
Vitamin. A	2mg	50mg	11mg
Calcium	50mg	10mg	70mg
Cost/unit	Rs.1.50	Rs.3.00	

Table 1:

Decision – making variables:

X_1 = quantity of milk (in glasses)/day

x_2 = quantity of salad (in 500g portions)/day.

1) Objective Funtion (Z):

The function to be minimized - total diet cost - is the objective function of this problem. It is defined by the combination of food X_1 (milk) and X_2 (salad) and their unit costs, Rs.1.50 (one real and fifty cents) and Rs.3.00 (three real) respectively. This cost function is a linear function of X_1 and X_2.

i.e.; Z = $1.5x_1 + 3.0x_2$ (1.6)

B. Restrictions:

The total quantity of vitamin A in this diet should be equal or greater than 11mg. The food formulation should provide at least 70mg of calcium. We could not take into account negative quantities of food thus X_1 and X_2 should be non-negative quatities $(x_1 \geq 0)(x_2 \geq 0)$

C. Mathematical Model For The Diet Problem:

To minimize $z = 1.5x_1 + 3x_2$ (1.7)

It is subject to

$2x_1 + 50x_2 \geq 11$ (vitamin A restriction) (1.8)

$50x_1 + 10x_2 \geq 70$ (calcium restriction) (1.9)

$x_1 \geq 0$; $x_2 \geq 0$ (non-negative restriction) (1.10)

IV. DESCRIPTION OF THE PROBLEM OF HEALTHCARE RESOURCE ALLOCATION

Researchers want to allocate resources among five medical intervention program for a certain population aiming at maximizing the quality – adjusted life year (QALY).

QALY is a measure that considers quality and quality of life related to the medical intervention applied. It is estimated that a maximum of 300,000 monetary units be spent and the number of medical visits is expected to remain as 40,000, at most.

It is also assumed that each patient receives only one intervention. Moreover, fraction values of quality – adjusted life year QALY intervention costs and number of medical visits are allowed. The objective is to maximize total QALY accumulated by interventions in order to comply with the budget restrictions and the number of medical visits described in table 2.

Identification of Intervention programs	QALY	Intervention cost ($x10^3$ monetary units)	No. of medical visits ($x10^3$)
1	10	100	40
2	15	50	50
3	15	50	50
4	13	40	15
5	9	120	30
Maximum value		300	40

Table 2: QALY, intervention cost and medical

A. Formulation of The Mathematical Model

1) Decision - making variables

x_i = fraction of each intervention i to be designated by the model.

2) Objective function

To maximize the function z defined as total quality-adjusted life year due to interventions.

$10x_1 + 15x_2 + 15x_3 + 13x_4 + 9x_5$ (1.11)

the parameters of this equation represent the number of QALY obtained as a result of the intervention I.

3) Restrictions

Budget restrictions:

$100x_1 + 50x_2 + 50x_3 + 40x_4 + 120x_5 \leq 300$ (1.12)

The cost, including all interventions should not exceed 300,000 monetary units.

4) Visits:

$40x_1 + 50x_2 + 50x_3 + 15x_4 + 30x_5 \leq 40$ (1.13)

$x_i \geq 0; \quad x_i \leq 1$ i =1, 2, 3, 4, 5
the decision-making variables may assume any value between 0 and 1.

B. Mathematical Model

To maximize $z = 10x_1 + 15x_2 + 15x_3 + 13x_4 + 9x_5$ (1.14)
It is subject to a set of restrictions

$$100x_1 + 50x_2 + 50x_3 + 40x_4 + 120x_5 \leq 300 \qquad (1.15)$$

$$40x_1 + 50x_2 + 50x_3 + 15x_4 + 30x_5 \leq 40 \qquad (1.16)$$

$$0 \leq x_1 \leq 1; 0 \leq x_2 \leq 1; 0 \leq x_3 \leq 1; 0 \leq x_4 \leq 1; 0 \leq x_5 \leq 1$$

V. GRAPHIC SOLUTION FOR THE DIET PROBLEM

When we deal with decision–making variables (two foods), a geometric representation is possible and convenient for didactic purposes. In order to explore the geometry of the problem, the restrictions were first represented in a Cartesian plan, identifying the feasible region.

i.e.; the region of the Cartesian plan that complies with the set of restrictions of the model.
Second, we have to minimize cost, which is a constant for each combination of x_1 and x_2.

Hence, different costs generate parallel line where cost is a constant in each line. Then, the cost lines were drawn in the graphic to obtain the minimum cost that complies with nutritional restrictions.

Observe the cost (z) decreases as we move towards the intersection of lines that identify calcium and vitamin A restrictions. The exact Point in which cost is minimized corresponds to the intersection of these lines. This point is easily found by simultaneously determining the solution of the equations.

$$2x_1 + 50x_2 = 11 \text{ And } 50x_1 + 10x_2 = 70 \quad (1.17)$$

This solution x_1 =1.4 and x_2 =0.2 corresponds to the total minimum cost of z = Rs.2.55. In other words, the optimal solution corresponds to a diet of 1.4 glasses of skimmed milk/day and 100grams of salad/day, at a minimum cost of Rs.2.55

VI. GRAPH (THE FEASIBLE REGION)

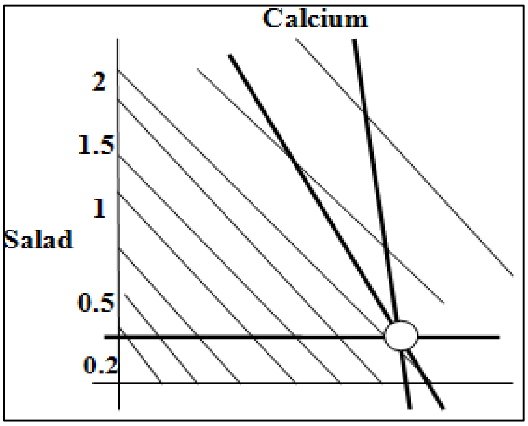

Fig. 2: Calcium

The feasible region is limited by the lines $2x_1 + 50x_2 = 11$ (vitamin A) and $50x_1 + 10x_2 = 70$ (calcium) and is identified by the arrows. The line is red (line 2) represents the 'objective function' to be minimized in order to have a minimum cost diet. The point in black determine the optimal solution x_1 =1.4; x_2 =0.2, corresponding to a minimum cost of Rs.2.55 (line in bold red).

A. Analytical Solution for the Diet Problem

The Simplex Method is an algorithm created to solve any Linear Programming problem. An algorithm is a set of rules that must be followed step by step, so that, in the end, the desired result is attained. The optimal solution obtained for this problem by Microsoft excel solver indicates a diet composed of 1.4 glasses of skimmed milk/day and 100 grams of salad/day (x_1 =1.4; x_2 =0.2 of a 500 g-portion), corresponding to total minimum cost of Rs.2.55. [10]

B. Solution for The Resource Allocation Problem:

The Optimized solution for the model of medical intervention program allocation calculated by Microsoft Excel Solver corresponds to the complete use (100%) of the intervention program number 4 and a fraction of 50% for the intervention program number 2, not using the interventions number 1,3 and 5(x_1 =1; x_2 =0.5; x_3 =0; x_4 =1; x_5 =0). This numerical solution provides a maximum value of 20.5 quality–adjusted life year QALY for the objective function of this model.

The graphic solution is easily obtained when there are two decision-making variables (as shown in the example of diet). When there are three decision-making variables it is still possible to have a graphic solution despite the difficulty in identifying the intersections between the planes defined in a three-dimensional space. In cases of four or more decision-making variables, graphic solution is impossible and the only alternative is an analytical solution by Simplex Method.

The type of algebraic modeling presented in this article known as linear programming could be considered a useful tool to support decision-making processes in healthcare. In a world with increasingly scarce resources and every day more competitive the search of optimized solutions to replace traditional methods based on common sense and trial and error may become an issue of survival for many organizations.

In dissertation, the structure of the linear programming problem is discussed in detail. Linear Programming Problem has many applications. In particular, the Fourier Variable Elimination Method of solving Linear Programming Problems are discussed in the following chapters with algorithms and examples. Linear Programming Problem and comparative study of the method of solving Linear Programming Problems are presented.

VII. CONCLUSION

Researchers have obtained, developed different algorithm to solve Linear Programming Problem. Some researcher has run the program to solve the problem. In this dissertation algorithm and C++ program are used to solve the problem.

This method takes less time when compared to all other methods researchers can come forward to generate different algorithm (Techniques) to solve Linear Programming Problem (in all types). Since this method takes less time, it can be applied to solve optimization problem in industries and other fields.

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