Design and Analysis of Reinforced Concrete Shells

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Abstract— A concrete shell, also commonly called thin shell concrete structure, is a structure composed of a relatively thin shell of concrete usually with no interior columns or exterior buttresses. The shells are most commonly flat plates and domes, but may also take the form of ellipsoids or cylindrical sections, or some combination thereof. A structural design of the thin shelled concrete structure will be computed using geometrical shape equations. Matrix methods offer an elegant and systematic means of formulating complex problems of structural analysis. Shells structures can be analysed using membrane theory and bending theory. The analysis and design of shell structures of circular cylinder shape have been done using D.K-J theory and Schorer theory D-K-J Theory has been used for design of short shell with edge beam and Schorer theory has been used for the analysis of long shell with edge beam. Reinforcement details have been provided based on calculations of stress resultants at different sections of the shell.

Key words: Reinforced Concrete Shells, Membrane

I. INTRODUCTION

A concrete shell, also commonly called thin shell concrete structure, is a structure composed of a relatively thin shell of concrete, usually with no interior columns or exterior buttresses. The shells are most commonly flat plates and domes, but may also take the form of ellipsoids or cylindrical sections, or some combination thereof. The America Concrete Institute (ACI) code defines a thin shell as a three dimensional spatial structure made-up of one or more curved slabs or folded plates whose thicknesses are small compared to their other dimensions. Thin shells are characterized by their three- dimensional load carrying behavior, which is determined by the geometry of their forms, by the manner in which they are supported, and by the nature of the applied load. Concrete shell structures are able to span large distances with a minimal amound of material. An arch spanning tens of feet, can be mere inches thick. In the maintaining this economy of material, these forms have a light, aesthetic, sculptural appeal.

Shells are spatially curved surface structures which support external applied loads. The exceptional behavior of shell structures can be referred to as “form resistant structures”. This implies a surface structure whose strength is derived from this shape, and which resists loads by developing stresses in its own plane. Thin shell concrete structures are pure compression structures formed from inverse catenary shapes. Caternary shapes are those taken by string or fabric when allowed to hang freely under their own weight. As string can bear no compression structure. Pure compression is ideal for concrete A concrete has high compressive the free hanging form is in pure tension. The inverse of this form is a pure compression structure. Pure compression is ideal for concrete as concrete has high compressive strength and very low tensile strength. These shapes maximize the effectiveness of concrete, allowing it to form thin light spans.

The effort in the design of shells is to make the shell as thin as the practical requirements will permit, so that the dead weight is reduced and the structure functions as a membrane free from the large bending stresses. By this means a minimum of materials is used to be the maximum structural advantage. Shells of eggs, nuts and the human skull are commonplace examples. These naturally occurring shells are hard to crack or break.

II. ADVANTAGES OF USING SHELL STRUCTURES

Like the arch, the curved shapes often used for concrete jshells are naturally strong structues, allowing wide areas to be spanned without the use of internal supports giving an open, unobstructed inferior.

1) The efficiency in its load carrying behavior is very high(being trated as a membrance)
2) Concrete shellstructures are able to span large distances with a minimal amount of material.
3) The use of concrete as a building material reduces both materials cost and construction cost in the case of shell construction, as concrete is relatively inexpensive and easily cast into compound curves.

The resulting structure may be immensely strong and safe.

A. Disadvantages

1) It is impossible to build a story that has a shell roof, thus shells are always used as a “ terminating roof”
2) Since concrete are porous material, concrete domes often having issues with sealing. If not treated, rainwater can seep through the roof and leak into the interior of the building.
3) On the other hand, the seamless construction of concrete domes prevents air from escaping and can lead to build up of condensation on the inside of the shell. Shingling or sealants are common soultions to the problem of exterior moisture, and dehumidiferes or ventilation can address condensation.

III. SHELL THEORIES

A. Membrane Theory:

Due to the initial curvature and low thickness to radius ratio, a thin shell has a much smaller flexural reigidity than extensional rigidity. When subjected to an applied load it mainly produces in plane actions a called membrane forces.

I the membrane theory, the shell is idealized as a membrane incapable of resisting bending stresses. For such a state of stress to exist it is essential that the shell be a closed surface if it is circular, elliptical, or cycloidal.

1) According to membrane theory, a thin shell acts partially as an arch and partially as a beam. The

arch action is responsible for the transfer of reactions to the edge beams, and the beam action for the transfer of reactions to the traverses through the medium of shear stresses that develop between adjacent rings of the shell. When the tangent is vertical at the ends, pure beam action results.

2) The external bending moment computed at any section treating the shell as a simply supported beam is resisted by
   – The resultant of the Nx forces.
   – The axial force P in the edge beams.
   – The vertical component of the bending moment to which the edge beams are subjected.
3) The value and variation of N is the same at all the cross sections, N is independent of the boundary conditions at the tranverse.
4) In the membrane state, equilibrium o fhteshell is maintained by the in plabne stresses N N and Nx .Bending moments and tranverse shears oif are absent.

IV. THE D.K. J THEORY

This theory was proposed by donnel in 1933-1934 who first used it in connection with his studies on the stability of thin walled circular cylinders. Karman and Tsein also employed the same theory in 1941 in their investigations on the buckling of cylindrical shells. The theory jis approximately known as the Donnel –Karman Jenkins theory..

V. EXPRESSIONS FOR STRESS RESULTANT AND DISPLACEMENTS

The expressions for the stress resultants and displacements are given below.

1) $Nx = -\frac{D\rho 4}{a3\lambda n}(e^{-\alpha 1\emptyset}(An2 sin\beta 1\emptyset - Bn2\cos\beta 1\emptyset) - e^{-\alpha 2\emptyset}(Cn2\sin\beta 2\emptyset - DN2\cos\beta 2\emptyset))\cos\frac{\lambda nx}{a}$
2) $Nx\phi = -D\rho 4/a3(e^{-\alpha 1\emptyset}(Anx^2 sin\beta 1\emptyset - Bn2\cos\beta 1\emptyset) - e^{-\alpha 2\emptyset}(Cn^2\sin\beta 2\emptyset - Dn2\cos\beta 2\emptyset))\sin\frac{\lambda nx}{a}$
3) $M\phi = -D\rho 4(e^{-\alpha 1\emptyset}(An2 sin\beta 1\emptyset - Bn2\cos\beta 1\emptyset) - e^{-\alpha 2\emptyset}(Cn2\sin\beta 2\emptyset - DN2\cos\beta 2\emptyset))\cos\frac{\lambda nx}{a}$

The constants, required for A_n^1, B_n^1, C_n^1 D_n^1this purpose are obtained as follows:

$$A_n^1 = -\alpha_1 A_n + \beta_1 B_n$$
$$B_n^1 = -\beta_1 A_n + \alpha_1 B_n$$
$$C_n^1 = -\alpha_1 C_n + \beta_2 D_n$$
$$D_n^1 = -\beta_2 C_n + \alpha_2 D_n$$

VI. DESIGN AND ANALYSIS OF CYLINDRICAL SHELL USING D-K-J THEORY

In order to organize the lengthy calculations systematically, the shell actions jof D-K-J theory may be represented in matrix form. Let 'H'represent any shell action, be it a stress resultant or displacement. Then we can write as

$$H = M\{\{e^{-\alpha 1\phi}[B1(An\cos\beta 1\phi + Bn\sin\beta 1\phi) + B2(Bn\cos\beta 1\phi - An\sin\beta 1\phi)] + e^{-\alpha 2\phi}[B3(Cn\cos\beta 2\phi - Dn\sin\beta 2\phi) + B4(Dn\cos\beta 2\beta\phi - Cn\sin\beta 2\phi]\}$$

Where M is a multiplier

Quantity	Whether odd or even	Multiplier
$\frac{\partial Nx\phi}{\partial x}$	odd	$\frac{-Dp4}{a4}\cos\frac{\lambda nx}{a}$
Qφ	odd	$\frac{-Dp4}{a3\sqrt{2}}\cos\frac{\lambda nx}{a}$
Nφ	Even	$\frac{-Dp4}{a3}\cos\frac{\lambda nx}{a}$
Nφ	Even	$\frac{-Dp4}{a3\sqrt{2}}\cos\frac{\lambda nx}{a}$
$\frac{\delta u}{\delta x}$	Even	$\frac{-\lambda_n^2}{\sqrt{2}\rho 2a}\cos\frac{\lambda_n^x}{a}$

Table 1: Multipliers M in the D-K-J theory

Quantity	B1	B2	B3	B4
$\frac{\partial Nx}{\partial x}$	+β1	+α1	-β2	-α2
Qφ	-{β1 + α(2k -1)	[(-α1 + β1k -1]	-{β2 + α(2k +1)	[(-α2 + β1k +1]
Nφ	0	-1		-1
Mφ	1+k2	-1	-(1-k2)	-1
$\frac{\delta u}{\delta x}$	-1	-(1+k2)	+1	0

Table 2: Coefficients B1, B2, B3 and B4 in the D-K-J theory

A. Design of a Single Short Cylinder Shell without Edge Beam

1) Sectional properties

Span length = 10 m
Radius (a) = 25 m
Thickness (d) = 0.075m
Semicentral angle (φ) = 40'

2) Loads:

Dead weight = 1.8 kN/m^2
Live load = 0.8 kN/sq.m
Total load = 2600N/sq.mm

3) Parameters:

$$\rho = \frac{12\pi r^4 a4}{L^4 A^4}$$
=16.366
$$k = \frac{\pi 4a4}{L^2\rho^2}$$
= 0.23113

Here ρ> 7, k> 0.12hence the shell is short
Hence D-K-J theory method can be used for the analysis

Φ	Nx	N φ	Nx φ
O'	-4307	-48599	8708
4'	-5224	-51290	7963
8'	-5387	-53765	7179
12'	-5702	-55799	6360
16'	-5899	-57971	5510
20'	-6068	-59575	4633
24'	-6028	-60942	3734
28'	-6317	-62071	2817
32'	-6395	-62781	1885
36	-6442	-63244	954
40	-6458	-63398	0

Table 3: Membrane Values of Nx, N φ, Nx φ

4) Final Values of Nx, N Φ, Nx Φ

Φ	Nx,	N φ	Nx φ	M φ
0	-628434	0	0	0
4	-52342	-29735	-97929	1232

8	-147858	-74040	-31367	-1260
12	-63729	-79497	-12843	-1272
16	-20864	-66765	-22586	-1306
20	-38379	-56796	-9737	-274
24	-23366	-55384	5193	262
28	-6739	-55623	-3180	287
32	-12683	-58293	2004	111
36	-11208	-62574	1567	-25
40	-7620	-64231	0	-60

B. Design of a Shell Reinforcement

1) Longitudinal Reinforcement

Since the Nx force is compressive throughout no reinforcement is required in the longitudinal direction. However a nominal reinforcement of 8mm diameter bars at 200 mm c/c in the longitudinal direction may be provided.

2) Transverse Reinforcement:

Since Nx is compressive throughout, no reinforcement isi provided. The maximum value of positive bending moment M φ = 1232 Nm/mm

Assuming a clear cover of 25 mm effective depth = 50mm

Area of steel requied will be

Ast = 1232 * 1000

= 0.87 * 150 * 50

Assuming safe stress in steel = 150MPa

Ast = 190sq.mm

Hence provide 10mm bars at 300mm c/c at bottom. Since the maximum value of negative bending moment is close to 1232 Nm/m the same reinforcement is provided at top. Hence to take care of both positive and negative moments provide two layers of steel of 10mm diameter at 300mm c/c

3) Diagnol Reinforcement

The principal tension is equal to the magnitude of Nxφ

The maximum value of Nxφ = 97929 N/m

Ast = 97929/150

= 650 sq.mm

Provide 12mm diameter bars @ 150mm c/c

For quarter span region provide 12mm diameter bars @ 300 mm c/c After the quarter span region the principal tension becomes small and hence theorectically no diagonal reinforcement is required beyond this region.

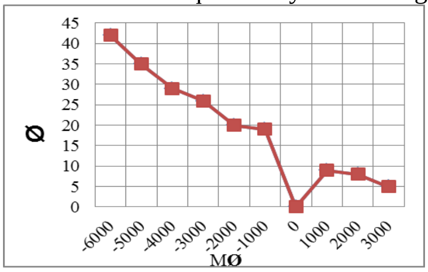

Fig. 1: GRAPH $M_Ø$ VS Ø

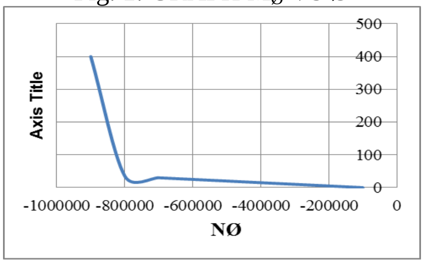

Fig. 2: GRAPH $N_Ø$ VS $_Ø$

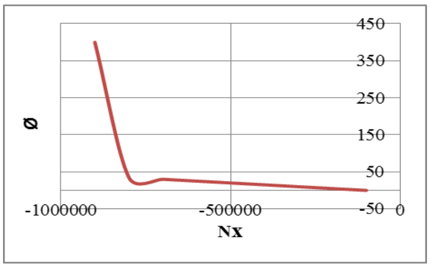

Fig. 3: Graph of Nx Final Value

VII. CONCLUSIONS

1) Concrete shell structures are able to span large distances with a minimal amount of material in Arch spanning tenses of feet mere inches thick
2) Keeping in view of various advantages of adopting shell structures different theories on the design of reignforced concrete shells were discussed. In particular, the design theory for the purpose of design cylindrical shells, viz,..
3) The D-K- J theory (for designing a short cylindrical shell without edge beam**.**

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