A New Method for Solving Deterministic Multi-Item Fuzzy Inventory Model with Three Constraints
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Abstract—The aim of this paper is to present a method in which fuzzy multi item inventory model together with the constraints is reduced to crisp inventory using ranking function and then the crisp multi item inventory problem (MIIP) with constraints is solved using NLPP technique.

Key words: Multi–Item Inventory Problem, Fuzzy Ware House Capacity, Fuzzy Maximum Investment, Fuzzy Maximum Average Number of Units, Fuzzy Multi-Item Inventory Problem, Trapezoidal Fuzzy Numbers, Fuzzy Ranking

I. INTRODUCTION

Most of the real world problems is inherently characterized by multiple, conflicting and incommensurate aspects of evaluation. These axes of evaluation are generally operationalized by objective functions to be optimized in framework of multiple objective linear programming models. Furthermore, when addressing real world problems, frequently the parameters are imprecise numerical quantities. In order to deal with such uncertain situations fuzzy model is used [2] in such cases, fuzzy set theory, introduced by Zadeh is acceptable. There are several studies on fuzzy EOQ model. Lin et al. [9] have developed a fuzzy model for production inventory problem. Katagiri and Ishii[7] have proposed an inventory problem with shortage cost as fuzzy quantity. This paper discusses a fuzzy multi-item inventory model subject to ware house capacity constraint, an investment constraint, maximum average number of units in all the items constraint. We introduced a method in which a fuzzy multi item inventory problem is reduced to crisp MIIP using ranking function GMR [11] and the resulting one is solved by crisp NLPP technique using LINGO software. The costs namely holding, setup of all items as well as the constraints is fuzzy in nature. A numerical example is given to illustrate the procedure.

II. REVIEW OF LITERATURE

In recent years inventory problems in fuzzy environment have received much attention Lin et al. [9] have developed a fuzzy model for production inventory. Katagiri and Ishii [7] have proposed an inventory problem with shortage cost as fuzzy quantity. Roy and Maiti (1995) considered a fuzzy inventory model with constraint. Srinivasan and Dhanam (2006) have considered cost analysis on a deterministic single item fuzzy inventory model with shortage. Also they have considered a multi-item EOQ inventory model with three constraints in a fuzzy environment. The model is solved by fuzzy non-linear programming method using Lagrange multiplies. Different kinds of inventory problems are solved in the papers[4] zimmermann[16] has introduced fuzzy programming approach to solve crisp multi objective linear programming problem. Recently H.M.Nehi et al.[10] used ranking function suggested to solve fuzzy MOLPP.

III. PRELIMINARIES

A. Definition 1:
The characteristic function μ a of a crisp set A ⊆ X assigns a value either 0 or 1 to each member in X. this function can be generalized to a function μ a such that the value assigned to the element of the universal set X fall within a specified range i.e. μ a : X→[0,1]. The assigned value indicates the membership grade of the element in the set A.

The function μ a is called the membership function and the set λ={(x, μ a(x)): x ∈ X} defined by μ a(x) for each x ∈ X is called a fuzzy set.

B. Definition 2:
A fuzzy set Š a defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

\[ \mu_a: X \rightarrow [0,1] \text{ is continuous.} \]
\[ \mu_a(x) = 0 \text{ for all } x \in (-\infty, a] \bigcup [d, \infty). \]
\[ \mu_a(x) \text{ Strictly increasing on } [a,b] \text{ and strictly decreasing on } [c,d]. \]
\[ \mu_a(x) = 1 \text{ for all } x \in [b,c], \text{ where } a < b < c < d. \]

C. Definition 3:
[4] A fuzzy number λ=(a1,a2,a3,a4) is said to be a triangular fuzzy number if its membership function is given by

\[ \mu_a(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1}, & a_1 \leq x < a_2 \\
\frac{a_3-x}{a_3-a_2}, & a_2 \leq x < a_3 \\
0, & \text{otherwise} 
\end{cases} \]

IV. MULTI-ITEM MULTI-OBJECTIVE INVENTORY PROBLEM

A. Assumptions:
A multi-item, multi-objective inventory model is developed under the following notations and assumptions.

- The inventory system pertains to multi-items.
- Demand rate is deterministic.
- The inventory is replenished in single delivery for each order.
- Replenishment is instantaneous.
- There is no lead time.
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B. Notations:
- n - total number of items being controlled simultaneously.
- Q_i - number of units ordered per order for i^{th} item (a decision variable).
- D_t - Annual demand for i^{th} item
- s_i - setup cost per order for i^{th} item.
- h_i - holding cost per unit quantity per unit time for i^{th} item.
- B - maximum available storage space to store all items.
- F - maximum investment.
- N - maximum average number of units for all items.
- A_i - Storage space required per unit of item i (i=1,2,3,.....n)
- c_i - Price per unit of item i (i=1,2,3,.....n)

\[ \tilde{h}_i - Fuzzy holding cost per unit quantity per unit time for i^{th} item. \]
\[ \tilde{B} - Fuzzy maximum average number of units for all items \]
\[ \tilde{N} - Fuzzy maximum average number of units for all items \]

C. Multi-item Crisp eqq Model with three Constraints

Minimize \( T(Q) = \sum_{i=1}^{n} \left[ \frac{D_t}{Q_i} + \frac{Q_i}{\tilde{h}_i} \right] \)

S.to:
\[ \sum_{i=1}^{n} (Q \geq \tilde{B}) \]
\[ \sum_{i=1}^{n} (Q \leq \tilde{F}) \]
\[ \frac{1}{2} \sum_{i=1}^{n} Q \leq \tilde{N} \]

D. Multi-item Fuzzy EOQ model with three constraints

\[ \min T(Q) = \sum_{i=1}^{n} \left[ \frac{D_t}{Q_i} + \frac{Q_i}{\tilde{h}_i} \right] \]

S.to:
\[ \sum_{i=1}^{n} A_i \leq \tilde{B} \]
\[ \sum_{i=1}^{n} C_i \leq \tilde{F} \]
\[ \frac{1}{2} \sum_{i=1}^{n} Q \leq \tilde{N} \]

Here \( \tilde{h}_i, \tilde{B}, \tilde{F}, \) and \( \tilde{N} \) are triangular fuzzy numbers represented
as \( \tilde{h}_i = (h_{il}, h_{ir}, h_i); \tilde{B} = (B_1, B_2, B_3); \tilde{F} = (F_1, F_2, F_3); \tilde{N} = (N_1, N_2, N_3) \)

V. Ranking of Triangular Fuzzy Numbers

A. The graded mean Integration Representation (GMIR)

A triangular fuzzy number \( \tilde{A} = (a, b, c) \) defined by its membership function as follows:

\[ L(x) = \frac{x-a}{b-a}; a \leq x \leq b \quad \text{And} \quad R(x) = \frac{c-x}{c-b}; b \leq x \leq c \]

The inverse functions \( L^{-1} \) and \( R^{-1} \) can be analytically expressed as given below:

\[ L^{-1}(h) = a + (b-a)h; R^{-1}(h) = c + (c-b)h \]

Then the graded mean integration representation (GMIR) of membership function of \( \tilde{A} \) is

\[ R(\tilde{A}) = \int_{0}^{\tilde{h}} \left[ aL^{-1}(h) + (1-a)R^{-1}(h) \right] dh \]

B. Numerical Example:

Let \( h_i = Rs.2100; h_2 = 1600; D_1 = 7; D_2 = 17; S_1 = 2100; S_2 = 1300; A_1 = 25 \) sq. units; \( A_2 = 25 \) sq. units; \( C_1 = 4500; C_2 = 2000; B = 250; F = 17,000; N = 5. \)
Suppose these are considered as triangular fuzzy numbers,
\( \tilde{h}_1 = (1950, 2075, 2150) \)
\( \tilde{B}_1 = (1400, 1550, 1700) \)
\( \tilde{N}_1 = (1800, 2100, 2175) \)
\( \tilde{S}_1 = (1125, 1200, 1300) \)
\( \tilde{B} = (175, 225, 300) \)
\( \tilde{F} = (15,000, 18,000, 20,000) \)
\( \tilde{N} = (3.5, 8) \)

Then using GMIR to these fuzzy numbers,
\[ R(\tilde{h}_1) = \frac{(\tilde{h}_1 - 200) + 6300}{3} \]
\[ R(\tilde{h}_2) = \frac{(\tilde{h}_2 - 300) + 4800}{3} \]
\[ R(\tilde{h}_3) = \frac{(\tilde{h}_3 - 375) + 6375}{3} \]
\[ R(\tilde{B}_1) = \frac{175 - 375 + 3700}{3} \]
So, \( R(\tilde{C}(Q)) = \min c(Q) \)

\[
\frac{R(\tilde{C})}{\tilde{C}} = \left(\frac{\alpha(125) + 750}{3}\right)
\]

\[
\frac{R(F)}{F} = \left(\frac{\alpha(-5000) + 56000}{3}\right)
\]

\[
R(\tilde{h}) = \left(\frac{\alpha(5) + 18}{3}\right)
\]

\( S.t. \quad 25Q + 25Q \leq \left(\frac{\alpha(-5000) + 56000}{3}\right) \)

\( \frac{Q}{2} + \frac{Q}{2} \leq 6 \) (1)

1) Case (i): when \( \alpha = 0 \)

\[
\min C(Q) = \frac{14875}{Q} + \frac{Q}{2} (2100) + \frac{Q}{2} (1600)
\]

S.t. \( 25Q + 25Q \leq 250 \)

\( 450Q + 2000Q \leq 18666.67 \)

\( \frac{Q}{2} + \frac{Q}{2} \leq 6 \)

2) Case (ii): when \( \alpha = 0.5 \)

\[
\min C(Q) = \frac{14437.5}{Q} + \frac{Q}{2} (2066.67) + \frac{Q}{2} (1550)
\]

\( S.t. \quad 25Q + 25Q \leq 229.17 \)

\( 450Q + 2000Q \leq 17833.33 \)

\( \frac{Q}{2} + \frac{Q}{2} \leq 5 \) (2)

3) Case (iii): when \( \alpha = 1 \)

\[
\min C(Q) = \frac{14000}{Q} + \frac{Q}{2} (2033.33) + \frac{19975}{Q} + \frac{Q}{2} (1500)
\]

S.t. \( 25Q + 25Q \leq 208.33 \)

\( 4500Q + 2000Q \leq 17000 \)

\( \frac{Q}{2} + \frac{Q}{2} \leq 4 \) (3)

VI. RESULTS AND DISCUSSION

The solutions obtained from (1),(2),(3) are given in table 1, 2, 3 respectively.

<table>
<thead>
<tr>
<th>Model</th>
<th>( h_1 )</th>
<th>( h_2 )</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( B )</th>
<th>( F )</th>
<th>( N )</th>
<th>( Q_1^* )</th>
<th>( Q_2^* )</th>
<th>( \min C(Q) )</th>
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<tr>
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<td>2100</td>
<td>1600</td>
<td>2100</td>
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<td>250</td>
<td>17000</td>
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<td>1000</td>
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VII. COMPARISON TABLE

Table: Comparison Table


