Mechanical Vibration of Visco-Elastic Circular Plate with Varying Thickness and Temperature in r-Direction
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Abstract— Mechanical vibration of circular plate with varying thickness and temperature in radial direction are investigated in the present study. Using the separation of variables method, the governing differential equation has been solved for vibration of visco-elastic isotropic circular plate. An approximate but quite convenient frequency equation is derived by using Rayleigh-Ritz technique with a two term deflection function. The frequencies corresponding to the first two modes of vibrations are obtained for a parallelogram plate for different values of taper constant and thermal gradient.

Key words: Visco-Elastic Circular Plate, Mechanical Vibration

I. INTRODUCTION

Plates of variable thickness are often encountered in engineering applications and their use in machine design, nuclear reactor technology, naval structures and acoustical components is quite common. The consideration of visco-elastic behavior of the plate material together with the variation in thickness of the structural components not only ensure the reduction in the rate and size but also meets the desirability of high strength in various technological situations of aerospace industry, ocean engineering and electronic and optical equipments. The research in the field of vibration is quite mesmerizing and continuously acquiring a great attention of scientists and design engineers because of its unbounded effect on human life. In the engineering we cannot move without considering the effect of vibration because almost all machines and engineering structures experiences vibrations. Study of effect of vibration can’t be restricted only in the field of science but, our day to day life is also affected by it.


In the aeronautical field, analysis of thermally induced vibrations in circular plates of variable thickness has a great interest due to their utility in aircraft wings. So, it is essentially required to have the knowledge of vibration for a designer. Here, present investigation is to study the vibrations of circular plate with varying thickness and temperature in radial direction. Rayleigh-Ritz’s method has been applied to derive the frequency equation of the plate. All results are illustrated with Graphs.

II. EQUATION OF TRANSVERSE MOTION

The equation of motion for a circular plate of radius a is governed by the equation [1]

\[ r \frac{\partial}{\partial r} \left[ \frac{1}{r} \left( \frac{\partial}{\partial r} (rM_r) - M_\theta \right) \right] = \rho h \frac{\partial^2 \omega}{\partial t^2} \]  (1.1)

The resultant moments of \( M_r \) and \( M_\theta \) for a polar visco-elastic material of plate are

\[ M_r = -\overline{BD} \left( \frac{\partial^2 \omega}{\partial r^2} + \frac{v \partial^2 \omega}{r \partial r} \right) \quad M_\theta = -\overline{BD} \left( \frac{\partial \omega}{\partial r} + \frac{v \partial^2 \omega}{r \partial r} \right) \]  (1.2)

where

\[ D = \frac{E h^3}{12(1-v^2)} \]  (1.3)

and \( \overline{BD} \) is the visco-elastic operator.

The deflection \( \omega \) can be sought in the form of product of two functions as follows

\[ \omega(r, \theta, t) = W(r, \theta)T(t) \]  (4)

where \( W(r, \theta) \) is the deflection function and \( T(t) \) is the time function.

Using equations (1.2) and (1.4) in (1.1) one gets

\[ r \frac{\partial^2 W}{\partial r^2} + \left( D + 2D \right) \frac{\partial W}{\partial r} \right] - \left( 2D - \frac{1}{r} \right) \frac{\partial^2 W}{\partial r^2} - \rho h \overline{BD} W = 0 \]  (1.5)
where \( p^2 \) is a constant.

These equations are expressions for transverse motion of a circular plate with variable thickness.

### III. ANALYSIS OF EQUATION OF MOTION

Assuming a steady temperature field in the radial direction for a circular plate as

\[
T = T_0 \left( 1 - \frac{r}{a} \right) \quad (1.6)
\]

where \( T \) denotes the temperature excess above the reference temperature at any point at the distance \( r/a \) from the centre of the circular plate of radius \( a \) and \( T_0 \) denotes the temperature excess above the reference temperature at \( r = 0 \).

The temperature dependence of the modulus of elasticity for most structural material is given as

\[
E(r) = E_0(1 - \nu T) \quad (7)
\]

where \( E_0 \) is the value of Young's modulus at the reference temperature, i.e., \( T = 0 \) and \( \nu \) is the slope of variation \( E \) of with \( T \). The module variation, in view of expressions (1.6) and (1.7), becomes

\[
E(r) = E_0 \left[ 1 - \alpha (1 - \frac{r}{a}) \right] \quad (1.8)
\]

where \( \alpha = \nu T_0 \) \((0 \leq \alpha < 1)\) is a parameter known as thermal gradient.

The expression for the maximum strain energy \( V \) and maximum kinetic energy \( T \) in the plate, when it vibrates with the mode shape \( W (r, \theta) \) are given as [1]

\[
V = \frac{1}{2} \int_0^{2\pi} \int_0^a \left[ \frac{\partial W}{\partial r} \left( \frac{1}{r} \frac{\partial W}{\partial r} + \frac{1}{r^2} \frac{\partial^2 W}{\partial \theta^2} \right) + \frac{2}{r} \frac{\partial W}{\partial \theta} \frac{\partial W}{\partial \theta} \right] r \, dr \, d\theta
\]

and

\[
T = \frac{1}{2} \rho \int_0^{2\pi} \int_0^a \left[ \frac{\partial W}{\partial r} \left( \frac{1}{r} \frac{\partial W}{\partial r} + \frac{1}{r^2} \frac{\partial^2 W}{\partial \theta^2} \right) - \frac{2}{r} \frac{\partial W}{\partial \theta} \frac{\partial W}{\partial \theta} \right] r \, dr \, d\theta
\]

### IV. SOLUTIONS AND FREQUENCY EQUATION

Rayleigh-Ritz technique requires that the maximum strain energy must be equal to the maximum kinetic energy. It is, therefore, necessary for the problem under consideration that

\[
\delta (V - T) = 0 \quad (1.14)
\]

For arbitrary variation \( W \) satisfying relevant geometric boundary conditions. Circular plate clamped at the edges \( r = a \) i.e., \( R = 1 \) the boundary conditions are

\[
\frac{dW}{dr} = 0 \quad \text{at} \quad r = 1 \quad (1.15)
\]

and the corresponding two terms of deflection function is taken as

\[
W (r) = C_1 (1 - r^2) + C_2 (1 - r^3) \quad (1.16)
\]

where \( C_1 \) and \( C_2 \) are undetermined coefficients. Now using equations (1.13) in equation (1.14), one has

\[
\delta (V - p^2 T) = 0 \quad (1.17)
\]

Where

\[
V = \int_0^1 [(1 - \alpha r) (1 - \beta r)] \left[ \frac{3}{2} \frac{dW}{dr} + R \left( \frac{dW}{dr} \right)^2 \right] R \, dR
\]

and

\[
T = \frac{1}{2} \int_0^1 (1 - \alpha r) (1 - \beta r) R^2 \frac{dW}{dr} \, dR \quad (1.18)
\]

where

\[
I = \frac{12 (1 - \nu^2)}{E_0} \frac{\rho a^4}{h_0} \quad (1.19)
\]

Equation (1.7) involves the unknowns \( C_1 \) and \( C_2 \) arising due to substituting of \( W (r) \) from (1.16). These unknowns are to be determined from equation (1.17), for which

\[
\frac{\partial}{\partial C_n} (V - p^2 T) = 0, \quad n = 1, 2 \quad (1.20)
\]

Equation (4.7) simplifies to the form

\[
\beta_{n1} C_1 + \beta_{n2} C_2 = 0, \quad n = 1, 2 \quad (1.21)
\]

Where \( \beta_{n1}, \beta_{n2} (n = 1, 2) \) involve the parametric constant and frequency parameter.

For a non-trivial solution, the determinant of the coefficient of equation (21) must be zero. Thus, one gets the frequency equation as

\[
\begin{vmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{vmatrix} = 0 \quad (22)
\]

On solving (22) one gets a quadratic equation in \( p^2 \), so it will give two roots.

### V. RESULT AND DISCUSSION

For calculations, the material of ‘Duralium’ which is an alloy of Aluminium, Copper, Magnesium and Manganese have been taken. Computations have been made for calculating the various values of frequency for different values thermal gradient (\( \alpha \)) and taper constant (\( \beta \)). In the present problem, latest software technology “MAT LAB” is used to get the numerical results with great accuracy and concentration.
Figure 1 illustrates the result of frequency for different values of thermal gradient $\alpha$ for first two modes of vibration. It can be seen from figures that as thermal gradient $\alpha$ increases then frequency decreases for both modes of vibration.

Figure 2 illustrates the result of frequency for different values of taper constant ($\beta$) for first two modes of vibration. It can be seen from figures that as taper constant ($\beta$) increases then frequency decreases for both modes of vibration quickly as compared with figure 1.

![Graph of frequency vs taper constant](image1)

**Fig. 1: Frequency of a visco-elastic circular plate vs Taper constant**

![Graph of frequency vs thermal gradient](image2)

**Fig. 2: Frequency of a visco-elastic circular plate vs Thermal Gradient**

VI. CONCLUSION

Since vibrations affect the efficiency and strength of machine as well as durability of the system, so the knowledge about the first few modes of vibration is essential & necessary to a mechanical engineers, before finalizing a design. Sometimes they had neither the analytical capability of solving the problem nor the money and time needed for an experimental program (setup) as a result; they are forced to drop the problem at this point. Therefore, it becomes the need of hour to present and develop more accurate and authentic mathematical models with practical approach and provide much better & reliable mechanical designs and structures for the welfare of human begins.

REFERENCES


