(τ_i,τ_j) – bĝ– Closed Sets in Bitopological Spaces

R.Subasree[1] Dr. M. Mariasingam[2]
[1]Assistant Professor [2]Associate Professor
[1,2]Department of Mathematics
[1]P.S.R.R College of Engineering for women, Sivakasi [2]V.O.Chidambaram College, Tuticorin

Abstract— In this paper we introduced the concept of (τ_i,τ_j) −bĝ –closed sets by using τ_i −ĝ – open set and τ_j −b–closure operator in bitopological space. Properties of these sets are investigated and we introduce two new bitopological spaces (i,j) −Tbĝ –space and (i,j) −T*bĝ – spaces as applications. Further we introduce and study (τ_i,τ_j) −bĝ–Neighbouhood,(τ_i,τ_j) − σ_K−bĝ–Continuous maps and Pair wise bĝ– irresolute maps.

Key words: Bitopological Spaces, Closed Sets

I. INTRODUCTION

In 1963, Kelly J.C[3] was the first who introduced the concept of bitopological spaces, where X is a non-empty set and τ_i,τ_j are two topologies on X. In 1985, Fukutake[2] introduced and studied the notions of generalized closed (g – closed) sets in bitopological spaces and after that several authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces. In this paper we find basic properties and characteristics of (τ_i,τ_j) − bĝ –closed sets. Also we investigate its relationships with certain types of closed sets with some new results and examples. We provide several properties and characterizations of (τ_i,τ_j) −bĝ–closed sets, (τ_i,τ_j) −bĝ–neighbourhoods, (τ_i,τ_j) − bĝ–continuous maps and Pair wise bĝ– irresolute maps.

Throughout this paper X and Y denoted the bitopological spaces (X,τ_1,τ_2) and (Y,σ_1,σ_2) respectively on which no separation axioms are assumed.

Notation: The following notation is used in this paper (i,j) denote the pair of topology (X,τ_1,τ_2).

II. PRELIMINARIES

A. *Definition 2.1:*

A subset A of a bitopological space (X, τ_i,τ_j) is called

1) (τ_i,τ_j) –semi open[4] if A ⊆ τ_j–cl[τ_i–int (A)]
2) (τ_i,τ_j) –α–open[3] if A ⊆ τ_i–int[τ_j–cl[τ_i–int (A)]]
3) (τ_i,τ_j) –b–open[8] if A ⊆ τ_i–int[τ_j–cl(A)] ∪ τ_j–cl[τ_i–int (A)]

The complement of (τ_i,τ_j)–semi open set (resp. (τ_i,τ_j)–α–open and (τ_i,τ_j)–b–open) is said to be (τ_i,τ_j)–semi closed set (resp. (τ_i,τ_j)– α– closed and (τ_i,τ_j)–b–closed).

B. *Definition 2.2:*

A subset A of a bitopological space (X, τ_i,τ_j) is called a

1) (τ_i,τ_j)−g-closed [2] if τ_j − cl(A) ⊆ U whenever A ⊆ U and U is τ_i−open in X.
2) (τ_i,τ_j)− gs-closed [1] if τ_j− scl(A) ⊆ U whenever A ⊆ U and U is τ_i−open in X.
3) (τ_i,τ_j) −sg-closed [6] if τ_j− scl(A) ⊆ U whenever A ⊆ U and U is τ_i−semi-open in X.
4) (τ_i,τ_j) −αg-closed [5] if τ_j−αcl(A)⊆U whenever A⊆U and U is τ_i−open in X.
5) (τ_i,τ_j)−gα-closed [5] if τ_j−αcl(A) ⊆ U whenever A ⊆ U and U isτ_i− α-open in X.
6) (τ_i,τ_j)−ĝ-closed [6] if τ_j−cl(A)⊆U whenever A ⊆ U and U is τ_i –semi-open in X.
7) (τ_i,τ_j)−gb-closed [8] if τ_j−bcl(A) ⊆ U whenever A ⊆ U and U is τ_i−open in X.
8) (τ_i,τ_j)−rgb-closed [7] if τ_j−bcl(A) ⊆ U whenever A ⊆ U and U is τ_i−regular-open in X.

III. (τ_i,τ_j) – BĜ–CLOSED SETS

A. *Definition 3.1:*

Let i,j ∈ {1,2} be fixed integers. A subset A of a bitopological space (X,τ_i,τ_j) is said to be (τ_i,τ_j) – bĝ–closed set if τ_j – bcl(A) ⊑ U whenever A⊑U and U is ĝ–open in (X,τ_i).

The family of all (τ_i,τ_j) – bĝ–Closed sets in a bitopological space (X,τ_i,τ_j) is denoted by (τ_i,τ_j)– bĝ– C(X).

B. *Remark 3.2:*

By setting $\tau_i = \tau_j$ in definition 3.1, (τ_i,τ_j) – bĝ–closed set is a bĝ–closed set.

C. *Proposition 3.3:*

1) If A is τ_j – closed subset of (X, τ_i, τ_j) then A is (τ_i,τ_j) – bĝ–closed set.
2) If A is τ_j – b – closed subset of (X, τ_i, τ_j) then A is (τ_i,τ_j) – bĝ–closed set.
3) If A is τ_j – α – closed subset of (X, τ_i, τ_j) then A is (τ_i,τ_j) – bĝ–closed set.
4) If A is τ_j –semi – closed subset of (X, τ_i,τ_j) then A is (τ_i,τ_j) – bĝ–closed set.
5) If A is τ_j – ĝ – closed subset of (X, τ_i, τ_j) then A is (τ_i,τ_j) – bĝ–closed set.
6) If A is τ_j – sg – closed subset of (X, τ_i, τ_j) then A is (τ_i,τ_j) – bĝ–closed set.

1) *Proof:*

1) Let A be any τ_j –closed subset of (X, τ_i, τ_j) and U be any τ_i–ĝ–open set containing A. Since τ_j–bcl(A) ⊑ τ_j – cl(A) ⊑ U, then τ_j – bcl(A) ⊑ U. Hence A is (τ_i,τ_j) – bĝ–closed set.
2) Let A be any τ_j –b–closed subset of (X, τ_i, τ_j) such that A ⊑ U where U is any τ_i– ĝ–open set. Since A is τ_j –b– closed which implies that τ_j – bcl(A) = A ⊑ U. Hence A is (τ_i,τ_j) – bĝ–closed set.
3) Let A be any τ_j – α – closed subset of (X, τ_i, τ_j) such that A ⊑ U where U is any τ_i– ĝ–open set. Since A is τ_j – α – closed set, which implies that τ_j

– bcl(A) ⊑ τ_j – α – cl(A) ⊑ τ_j – cl(A) ⊑ U, then τ_j – bcl(A) ⊑ U. Hence A is (τ_i, τ_j) – bĝ–closed set.

4) Let A be any τ_j – semi – closed subset of (X, τ_i, τ_j) such that A ⊑ U where U is any τ_i– ĝ–open set. Since A is τ_j–semi–closed set, which implies that τ_j–bcl(A) ⊑ τ_j– s –cl(A) ⊑ U, then τ_j – bcl(A) ⊑ U. Hence A is (τ_i, τ_j) – bĝ–closed set.
5) Let A be any τ_j –ĝ– closed subset of (X, τ_i, τ_j) such that A ⊑ U where U is any τ_i– ĝ–open set. Since A is τ_j – ĝ – closed which implies that τ_j – bcl(A) ⊑ τ_j – cl(A) ⊑ U, then τ_j – bcl(A) ⊑ U. Hence A is (τ_i, τ_j) – bĝ–closed set.
6) Let A be any τ_j – sg – closed subset of (X, τ_i, τ_j) such that A ⊑ U where U is any τ_i– ĝ–open set. Since A is τ_j – sg – closed set, which implies that τ_j – bcl(A) ⊑ τ_j–s–cl(A) ⊑ U, then τ_j – bcl(A) ⊑ U. Hence A is (τ_i, τ_j) – bĝ–closed set.

The converse of the above proposition need not be true in general as shown in the following example.

D. *Proposition 3.4:*

1) Every (τ_i, τ_j) – bĝ–closed set need not be τ_j –closed.
2) Every (τ_i, τ_j) – bĝ–closed set need not be τ_j –b– closed.
3) Every (τ_i, τ_j) – bĝ–closed set need not be τ_j – α – closed set.
4) Every (τ_i, τ_j) – bĝ–closed set need not be τ_j –*semi* – closed set.
5) Every (τ_i, τ_j) – bĝ–closed set need not be τ_j – ĝ – closed set.
6) Every (τ_i, τ_j) – bĝ–closed set need not be τ_j – sg – closed set.

1) *Proof:*

1) Let X = {a,b,c} with topologies
$\tau_i = \{ X, \Phi, \{a\}, \{b\}, \{a,b\}\}$ *and* $\tau_j = \{X, \Phi, \{a,c\}\}$
(τ_i, τ_j)– bĝ– C(X)) = {X,Φ, {a},{b},{c}, {a,b},{a,c},{b,c}}
Here {a},{c},{a,b},{a,c},{b,c} are (τ_i, τ_j) – bĝ–closed set but not τ_j – closed.
2) Let X = {a,b,c} with topologies $\tau_i = \{ X, \Phi, \{a\}, \{a,b\}\}$ and $\tau_j = \{X, \Phi, \{a\}\}$
(τ_i, τ_j)– bĝ– C(X) = { X, Φ,{b},{c},{a,c},{b,c}} τ_j – b –C(X) = { X, Φ, {b},{c},{b,c}}
Here the set{a,c} is (τ_i, τ_j) – bĝ–closed set but not τ_j – b – closed.
3) Let X = {a,b,c,d} with topologies
$\tau_i = \{ X, \Phi, \{b\}, \{a,b\}, \{b,c,d\}\}$ *and* $\tau_j =$ {X, Φ, {a}, {a, c}, {a, b, d}
(τ_i, τ_j)– bĝ– C(X) ={X, Φ,{b},{c},{d},{a,c},{a,d}, {b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}
τ_j – α –C(X) = { X, Φ, {b},{c},{d},{b,c},{b,d}, {c,d},{b,c,d}}
Here the sets{a,c},{a,d},{a,b,c},{a,b,d}and{a,c,d}are (τ_i, τ_j) – bĝ–closed sets but not τ_j –α– closed.
4) Let X = {a,b,c} with topologies $\tau_i = \{ X, \Phi, \{a\}, \{a,b\}\}$ and $\tau_j = \{X, \Phi, \{a\}\}$
(τ_i, τ_j)– bĝ– C(X) ={X, Φ,{b},{c},{a,c},{b,c}}
τ_j – *S*–C(X) = { X, Φ, {b},{c},{b,c}}
Here the set{a,c}is (τ_i, τ_j) – bĝ–closed set but not τ_j –*semi*– closed.
5) Let X = {a,b,c} with topologies $\tau_i = \{ X, \Phi, \{a\}, \{b\}, \{a,b\}\}$ *and* $\tau_j = \{X, \Phi, \{a,c\}\}$
(τ_i, τ_j)– bĝ– C(X) = { X, Φ,{a},{b},{c},{a,b}, {a,c},{b,c}}
τ_j – ĝ –C(X) = { X, Φ, {b},{a,b},{b,c}}
Here the sets {a},{c} and {a,c} are (τ_i, τ_j) – bĝ–closed sets but not τ_j –ĝ – closed.
6) Let X = {a,b,c,d} with topologies
$\tau_i = \{ X, \Phi, \{b\}, \{a,b\}, \{b,c,d\}\}$ *and* τ_j = {X, Φ, {a}, {a, c}, {a, b, d}}
(τ_i, τ_j)– bĝ– C(X ={X, Φ,{b},{c},{d},{a,c},{a,d}, {b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}
τ_j – sg –C(X) = { X, Φ, {b},{c},{d},{b,c},{b,d}, {c,d},{b,c,d}}
Here the sets{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}are (τ_i, τ_j)–bĝ–closed sets but not τ_j –sg– closed.

E. *Proposition 3.5:*

1) The concept of τ_j –g–closed sets and (τ_i, τ_j) – bĝ–closed sets are independent of each other as seen from the following example.
2) The concept of τ_j –gs–closed sets and (τ_i, τ_j) – bĝ–closed sets are independent of each other as seen from the following example.

1) *Proof:*

1) Let X = {a,b,c,d} with topologies
$\tau_i = \{ X, \Phi, \{a\}, \{a,c\}, \{a,b,d\}\}$ *and* τ_j = {X, Φ, {b}, {a, b}, {b, c, d}}
(τ_i, τ_j)– bĝ– C(X) ={X, Φ,{a},{c},{d},{a,c},{a,d}, {b,c},{b,d},{c,d},{a,b,c},{a,c,d},{b,c,d}}
τ_j – g –C(X) = { X, Φ, {a},{c},{d},{a,c},{a,d}, {c,d},{a,b,c},{a,b,d},{a,c,d}}
Here the set {a,b,d} is τ_j – g–closed but not (τ_i, τ_j) – bĝ–closed set and the sets{b,c} and {b,c,d} are (τ_i, τ_j) – bĝ–closed set but not τ_j –g– closed.
2) Let X = {a,b,c,d} with topologies
$\tau_i = \{ X, \Phi, \{a\}, \{a,c\}, \{a,b,d\}\}$ *and* $\tau_j =$ {X, Φ, {b}, {a, b}, {b, c, d}
(τ_i, τ_j)– bĝ– C(X) ={X, Φ,{a},{c},{d}, {a,c},{a,d},{b,c},{c,d},{a,b,c},{a,c,d},{b,c,d}}
τ_j – gs –C(X) = { X, Φ, {a},{c},{d},{a,c},{a,d}, {c,d},{a,b,c},{a,b,d},{a,c,d}}
Here the set{a,b,d}is τ_j –gs–closed but not (τ_i, τ_j)– bĝ–closed set and the sets{b,c}and {b,c,d} are (τ_i, τ_j) – bĝ–closed set but not τ_j –gs– closed.

F. *Proposition 3.6:*

1) The concept of τ_j – αg–closed sets and (τ_i, τ_j) – bĝ–closed sets are independent of each other as seen from the following example.
2) The concept of τ_j – $g\alpha$–closed sets and (τ_i, τ_j) – bĝ–closed sets are independent of each other as seen from the following example.

3) The concept of τ_j – gb–closed sets and (τ_i, τ_j) – bĝ–closed sets are independent of each other as seen from the following example.

1) Proof:

(i) (a) Let X = {a,b,c} with topologies

$\tau_i = \{X, \Phi, \{a\}, \{a, b\}\}$ *and* $\tau_j = \{X, \Phi, \{a\}\}$

(τ_i, τ_j)– bĝ– C(X) ={X, Φ,{b},{c},{a,c},{b,c}}

$\tau_j - \alpha g$ –C(X) = { X, Φ, {b},{c},{a,b},{a,c},{b,c}}

Here the set{a,b}is τ_j – αg–closed but not (τ_i, τ_j) – bĝ–closed set.

(b) Let X = {a,b,c}with topologies

$\tau_i = \{X, \Phi, \{a\}, \{b\}, \{a, b\}\}$ *and* $\tau_j = \{X, \Phi, \{a, c\}\}$

(τ_i, τ_j)– bĝ– C(X) ={X, Φ,{a},{b},{c},{a,b}, {a,c},{b,c}}

$\tau_j - \alpha g$ –C(X) = { X, Φ, {b},{a,b},{b,c}}

Here the sets {a},{c} and {a,c} are (τ_i, τ_j)– bĝ– closed set but not $\tau_j - \alpha$g–closed.

(ii) (a) Let X = {a,b,c} with topologies

$\tau_i = \{X, \Phi, \{a\}, \{a, b\}\}$ *and* $\tau_j = \{X, \Phi, \{a\}\}$

(τ_i, τ_j)– bĝ– C(X) ={X, Φ,{b},{c},{a,c},{b,c}}

$\tau_j - g\alpha$ –C(X) = { X, Φ, {b},{c},{a,b}, {a,c},{b,c}}

Here the set{a,b}is $\tau_j - g\alpha$–closed but not (τ_i, τ_j) – bĝ–closed set.

(b) Let X = {a,b,c} with topologies

$\tau_i = \{X, \Phi, \{a\}, \{b\}, \{a, b\}\}$*and* $\tau_j = \{X, \Phi, \{a, c\}\}$

(τ_i, τ_j)– bĝ– C(X) ={X, Φ,{a},{b},{c},{a,b}, {a,c},{b,c}}

$\tau_j - \alpha g$ –C(X) = { X, Φ, {b},{a,b},{b,c}}

Here the sets {a},{c} and {a,c}are (τ_i, τ_j)– bĝ– closed set but not τ_j–$g\alpha$–closed.

(iii) (a) Let X = {a,b,c} with topologies

$\tau_i = \{X, \Phi, \{c\}, \{a, b\}\}$ *and* $\tau_j = \{X, \Phi, \{b\}\}$

(τ_i, τ_j)– bĝ– C(X) ={X, Φ,{a},{c},{a,c}}

τ_j – gb –C(X) = { X, Φ, {a},{c},{a,b},{a,c},{b,c}}

Here the sets {a,b} and {b,c} are τ_j – gb–closed but not (τ_i, τ_j) – bĝ–closed set.

(b) Let X = {a,b,c,d} with topologies

$\tau_i = \{X, \Phi, \{b\}, \{a, b\}, \{b, c, d\}\}$ *and* $\tau_j = \{X, \Phi, \{a\}, \{a, c\}, \{a, b, d\}\}$

(τ_i, τ_j)– bĝ– C(X) ={X, Φ,{b},{c},{d},{a,c},{a,d}, {b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}

τ_j – gb –C(X) = {X, Φ,{b},{c},{d},{b,c},{b,d}, {c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}

Here the sets {a,c} and {a,d} are (τ_i, τ_j)– bĝ– closed set but not τ_j – gb–closed.

G. Proposition 3.7:

1) The concept of (τ_i, τ_j) –semi–closed sets and (τ_i, τ_j) – bĝ–closed sets are independent of each other as seen from the following example.
2) The concept of (τ_i, τ_j) –α–closed sets and (τ_i, τ_j) – bĝ–closed sets are independent of each other as seen from the following example.
3) The concept of (τ_i, τ_j) –g–closed sets and (τ_i, τ_j) – bĝ–closed sets are independent of each other as seen from the following example.

1) Proof:

(i) Let X = {a,b,c} with topologies

$\tau_i = \{X, \Phi, \{c\}, \{a, b\}\}$ *and* $\tau_j = \{X, \Phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$

(τ_i, τ_j)– bĝ– C(X) ={X, Φ,{b},{c},{a,c},{b,c}}

(τ_i, τ_j) – S–C(X) = { X, Φ, {c},{a,b}}

Here the set {a,b}is (τ_i, τ_j) –semi– closed but not (τ_i, τ_j) – bĝ–closed set. Also the sets {b},{a,c} and {b,c} are (τ_i, τ_j) – bĝ–closed but not (τ_i, τ_j) – semi–closed sets.

(ii) Let X = {a,b,c} with topologies

$\tau_i = \{X, \Phi, \{c\}, \{a, b\}\}$ *and* $\tau_j = \{X, \Phi, \{b\}\}$

(τ_i, τ_j)– bĝ– C(X) ={X, Φ,{a},{c},{a,c}}

(τ_i, τ_j) – α –C(X) = { X, Φ, {c},{a,b}}

Here the set {a,b} is (τ_i, τ_j) – α – closed but not (τ_i, τ_j) – bĝ–closed set. Also the sets {a}and {a,c} are (τ_i, τ_j) – bĝ–closed but not (τ_i, τ_j) –α –closed.

(iii) Let X = {a,b,c}with topologies

$\tau_i = \{X, \Phi, \{c\}, \{a, b\}\}$ *and* $\tau_j = \{X, \Phi, \{b\}\}$

(τ_i, τ_j)– bĝ– C(X) ={X, Φ,{a},{c},{a,c}}

(τ_i, τ_j) – g –C(X) = { X, Φ, {a,c},{b,c}}

Here the sets {b,c} is (τ_i, τ_j) – g– closed but not (τ_i, τ_j) – bĝ–closed set. Also the sets {a} and {c} are (τ_i, τ_j)– bĝ– closed but not (τ_i, τ_j)– g–closed.

H. Proposition 3.8:

1) The concept of (τ_i, τ_j) –b–closed sets and (τ_i, τ_j) – bĝ–closed sets are independent of each other as seen from the following example.
2) The concept of (τ_i, τ_j) –ĝ–closed sets and (τ_i, τ_j) – bĝ–closed sets are independent of each other as seen from the following example.
3) The concept of (τ_i, τ_j) –sg–closed sets and (τ_i, τ_j) – bĝ–closed sets are independent of each other as seen from the following example.
4) The concept of (τ_i, τ_j) –gs–closed sets and (τ_i, τ_j) – bĝ–closed sets are independent of each other as seen from the following example.
5) The concept of (τ_i, τ_j) – αg–closed sets and (τ_i, τ_j) – bĝ–closed sets are independent of each other as seen from the following example.
6) The concept of (τ_i, τ_j) – $g\alpha$–closed sets and (τ_i, τ_j) – bĝ–closed sets are independent of each other as seen from the following example.

1) Proof:

(i) (a) Let X = {a,b,c} with topologies

$\tau_i = \{X, \Phi, \{c\}, \{a, b\}\}$ *and* $\tau_j = \{X, \Phi, \{b\}\}$

(τ_i, τ_j)– bĝ– C(X) ={X, Φ,{a},{c},{a,c}}

(τ_i, τ_j) – b –C(X) = {X, Φ, {a},{b},{c}, {a,b},{a,c}}

Here the sets {b} and {a,b} are (τ_i, τ_j) – b– closed but not (τ_i, τ_j) – bĝ–closed.

(b) Let X = {a,b,c} with topologies

$\tau_i = \{X, \Phi, \{a\}, \{a, b\}\}$ *and* $\tau_j = \{X, \Phi, \{a\}\}$

(τ_i, τ_j)– bĝ– C(X) ={X, Φ,{b},{c},{a,c},{b,c}}

(τ_i, τ_j) – b –C(X) = { X, Φ, {b},{c},{b,c}}

Here the set {a,c} is (τ_i,τ_j)– bĝ– closed but not (τ_i,τ_j)– b–closed.

(ii) (a) Let X = {a,b,c} with topologies

$\tau_i = \{X, \Phi, \{a\}, \{b,c\}\}$ *and* $\tau_j = \{X, \Phi, \{a\}\}$

(τ_i,τ_j)– bĝ– C(X) ={X, Φ,{b},{c},{b,c}}

(τ_i,τ_j) – ĝ –C(X) = { X, Φ, {b},{c},{a,b}, {a,c},{b,c}}

Here the sets {a,b} and {a,c} are (τ_i,τ_j) – ĝ– closed but not (τ_i,τ_j) – bĝ–closed.

(b) Let X = {a,b,c} with topologies

$\tau_i = \{X, \Phi, \{a\}, \{b\}, \{a,b\}\}$ *and* $\tau_j = \{X, \Phi, \{a,c\}\}$

(τ_i,τ_j)– bĝ– C(X) ={X, Φ,{a},{b},{c},{a,b}, {a,c},{b,c}}

(τ_i,τ_j) – ĝ –C(X) = { X, Φ, {b}}

Here the sets {a},{c},{a,b},{a,c}and{b,c} are (τ_i,τ_j)– *b*ĝ– closed but not (τ_i,τ_j) –ĝ–closed set.

(iii) (a) Let X = {a,b,c} with topologies

$\tau_i = \{X, \Phi, \{a\}, \{b,c\}\}$ *and* $\tau_j = \{X, \Phi, \{a\}\}$

(τ_i,τ_j)– bĝ– C(X)={X, Φ,{b},{c},{b,c}}

(τ_i,τ_j) – sg–C(X) = {X, Φ, {b},{c},{a,b}, {a,c},{b,c}}

Here the sets {a,b} and {a,c}are(τ_i,τ_j) – sg– closed but not (τ_i,τ_j) – bĝ–closed.

(b) Let X = {a,b,c,d} with topologies

$\tau_i = \{X, \Phi, \{a\}, \{a,c\}, \{a,b,d\}\}$ *and* $\tau_j = \{X, \Phi, \{b\}, \{a,b\}, \{b,c,d\}\}$

(τ_i,τ_j)– bĝ– C(X)={X, Φ,{a},{c},{d},{a,c}, {a,d},{b,c},{c,d},{a,b,c},{a,c,d},{b,c,d}}

τ_j – *sg* –C(X) = { X, Φ, {a},{c},{d},{a,c}, {a,d},{c,d},{a,c,d},{b,c,d}}

Here the sets{b,c} and {a,b,c} are (τ_i,τ_j) – bĝ– closed set but not τ_j –*sg*– closed.

(iv) (a) Let X = {a,b,c} with topologies

$\tau_i = \{X, \Phi, \{a,b\}, \{c\}\}$ *and* $\tau_j = \{X, \Phi, \{b\}\}$

(τ_i,τ_j)– bĝ– C(X)={X, Φ,{a},{c},{a,c}}

(τ_i,τ_j) – gs–C(X) = { X, Φ, {a },{c},{a,c},{b,c}}

Here the sets{b,c} is (τ_i,τ_j) – gs– closed but not (τ_i,τ_j) – bĝ–closed.

(b) Let X = {a,b,c} with topologies

$\tau_i = \{X, \Phi, \{a\}, \{b\}, \{a,b\}\}$ *and* $\tau_j = \{X, \Phi, \{a,c\}\}$

(τ_i,τ_j)– bĝ– C(X)={X, Φ,{a},{b},{c},{a,b}, {a,c}{b,c}}

(τ_i,τ_j)- gs–C(X) = { X, Φ, {b},{c},{a,c},{b,c}}

Here the sets{a} and{a,b} are (τ_i,τ_j) – bĝ–closed but not (τ_i,τ_j) – gs– closed.

(v) (a) Let X = {a,b,c} with topologies

$\tau_i = \{X, \Phi, \{a\}, \{b,c\}\}$ *and* $\tau_j = \{X, \Phi, \{a\}\}$

(τ_i,τ_j)– bĝ– C(X)={X, Φ,{b},{c},{b,c}}

(τ_i,τ_j)-αg–C(X) = { X, Φ, {b},{c},{a,b},{a,c},{b,c}}

Here the sets {a,b} and {a,c} are (τ_i,τ_j) – αg– closed but not (τ_i,τ_j) – bĝ–closed.

(b) Let X = {a,b,c} with topologies

$\tau_i = \{X, \Phi, \{a\}, \{b\}, \{a,b\}\}$ *and* $\tau_j = \{X, \Phi, \{a,c\}\}$

(τ_i,τ_j)– bĝ – C(X)={X, Φ,{a},{b},{c},{a,b}, {a,c}{b,c}}

(τ_i,τ_j) – αg–C(X) = { X, Φ, {b},{c},{a,c},{b,c}}

Here the sets{a} and{a,b} are (τ_i,τ_j) – bĝ–closed but not (τ_i,τ_j) – αg– closed.

(vi)(a) Let X = {a,b,c} with topologies

$\tau_i = \{X, \Phi, \{a,b\}, \{c\}\}$ *and* $\tau_j = \{X, \Phi, \{b\}\}$

(τ_i,τ_j)– bĝ– C(X)={X, Φ,{a},{c},{a,c}}

(τ_i,τ_j) – gα–C(X) = { X, Φ, {a },{c},{a,c},{b,c}}

Here the sets{b,c} is (τ_i,τ_j) – gα– closed but not (τ_i,τ_j) – bĝ–closed.

(b) Let X = {a,b,c,d} with topologies

$\tau_i = \{X, \Phi, \{b\}, \{a,b\}, \{b,c,d\}\}$ *and* $\tau_j = \{X, \Phi, \{a\}, \{a,c\}, \{a,b,d\}\}$

(τ_i,τ_j)– bĝ– C(X)={X, Φ,{b},{c},{d},{a,c},{a,d}, {b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}

τ_j – *gα* –C(X) = { X, Φ, {b},{c},{d},{b,c},{b,d}, {c,d},{a,c,d},{b,c,d}}

Here the sets{a,c},{a,d},{a,b,c}and {a,b,d} are (τ_i,τ_j) – bĝ–closed but not τ_j –*gα*– closed.

I. *Proposition 3.9:*

1) If A is (τ_i,τ_j) – bĝ–closed subset of (X,τ_i,τ_j) then A is (τ_i,τ_j) – gb–closed set.
2) If A is (τ_i,τ_j) – bĝ–closed subset of (X,τ_i,τ_j) then A is (τ_i,τ_j) – rgb–closed set.

1) Proof:

1) Let A be any (τ_i,τ_j) – bĝ–closed subset of (X,τ_i,τ_j) such that A ⊑ U where U is any τ_i–open set. Since A is (τ_i,τ_j) – bĝ– closed set, which implies that τ_j – bcl(A) ⊑ U. Hence A is (τ_i,τ_j) – gb–closed set.
2) Let A be any (τ_i,τ_j) – bĝ–closed subset of (X,τ_i,τ_j) such that A ⊑ U where U is any τ_i–regular open set. Since A is (τ_i,τ_j) – bĝ– closed set, which implies that τ_j – bcl(A) ⊑ U. Hence A is (τ_i,τ_j) – rgb–closed.

J. *Proposition 3.10:*

1) Every (τ_i,τ_j) – gb–closed set need not be (τ_i,τ_j) – bĝ–closed set.
2) Every (τ_i,τ_j) – gb–closed set need not be (τ_i,τ_j) – bĝ–closed set.

1) Proof:

(i) Let X = {a,b,c} with topologies

$\tau_i = \{X, \Phi, \{a\}, \{b,c\}\}$ *and* $\tau_j = \{X, \Phi, \{a\}\}$

(τ_i,τ_j)– bĝ– C(X)={X, Φ,{b},{c},{b,c}}

(τ_i,τ_j)- gb–C(X) = { X, Φ, {b},{c},{a,b}, {a,c},{b,c}}

Here the sets {a,b} and {a,c} are (τ_i,τ_j) – gb– closed but not (τ_i,τ_j) – bĝ–closed.

(ii) Let X = {a,b,c,d} with topologies

$\tau_i = \{X, \Phi, \{a\}, \{a,b\}, \{a,d\}, \{a,b,d\}\}$ *and* $\tau_j = \{X, \Phi, \{a\}, \{a,b\}\}$

(τ_i,τ_j)– bĝ– C(X)={X, Φ,{b},{c},{d},{a,c},{b,c}, {b,d},{c,d},{a,b,c},{a,c,d},{b,c,d}}

(τ_i,τ_j)– rgb– C(X)={X, Φ,{a},{b},{c},{d},{a,b}, {a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d}, {a,c,d},{b,c,d}}

Here the sets {a},{a,b},{a,d}and{a,b,d} are (τ_i,τ_j) – rgb–closed but not (τ_i,τ_j) – bĝ–closed.

IV. (τ_i,τ_j) – BĜ–OPEN SETS AND (τ_i,τ_j) – BĜ–NEIGHBORHOODS

A. Definition 4.1:

A subset A of a bitopological space (X,τ_i,τ_j) is said to be (τ_i,τ_j) – bĝ–open set if its complement is (τ_i,τ_j) – bĝ–Closed in X.

The family of all (τ_i,τ_j) – bĝ–open sets of X are denoted by (τ_i,τ_j)– bĝ– O(X).

B. Lemma 4.2:

Let (X,τ_i,τ_j) be a bitopological space. Then

(i) The arbitrary union of (τ_i,τ_j) – bĝ–Closed sets is (τ_i,τ_j) – bĝ–Closed.

(ii) The arbitrary intersection of (τ_i,τ_j) – bĝ–open sets is (τ_i,τ_j) – bĝ–open.

C. Remark 4.3:

(i) Let A and B are two (τ_i,τ_j) – bĝ–Closed sets in (X,τ_i,τ_j). The intersection A∩B is not generally (τ_i,τ_j) – bĝ–Closed set.

(ii) Let A and B are two (τ_i,τ_j) – bĝ–Open sets in (X,τ_i,τ_j). The union A∪B is not generally (τ_i,τ_j) – bĝ–Open set.

1) Proof:

(i) Let X = {a,b,c,d} with topologies

$\tau_i = \{X, \Phi, \{b\}, \{a,b\}, \{b,c,d\}\}$ *and* $\tau_j = \{X, \Phi, \{a\}, \{a,c\}, \{a,b,d\}\}$

(τ_i,τ_j)– bĝ– C(X)={X, Φ,{b},{c},{d},{a,c},{a,d}, {b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}

Here the sets A= {a,c}and B={a,d} are two (τ_i,τ_j) – bĝ–closed sets. But A∩B={a} which is not (τ_i,τ_j)– bĝ– Closed set.

(ii) Let X = {a,b,c,d} with topologies

$\tau_i = \{X, \Phi, \{b\}, \{a,b\}, \{b,c,d\}\}$ *and* $\tau_j = \{X, \Phi, \{a\}, \{a,c\}, \{a,b,d\}\}$

(τ_i,τ_j)– bĝ– O(X)={X, Φ,{a},{b},{c},{d},{a,b},{a,c}, {a,d},{b,c},{b,d},{a,b,c},{a,b,d},{a,c,d}}

Here the sets A= {c}and B={d} are two (τ_i,τ_j) – bĝ–open sets. But A∪B={c,d} which is not (τ_i,τ_j)– bĝ– open set.

D. Definition 4.4:

Let (X,τ_i,τ_j) be a bitopological space and let g∈ X. A subset N of X is said to be (τ_i,τ_j) – bĝ–neighbourhood (briefly (τ_i,τ_j) – bĝ–nbd) of a point g if and only if there exists a (τ_i,τ_j) – bĝ–open set G such that g ∈ G ⊑ N.

The subset of all (τ_i,τ_j) – bĝ–nbd of a point g is denoted by (τ_i,τ_j) – bĝ–N(g).

E. Proposition 4.5:

(i) Every of τ_j – nbd of g ∈ X is a (τ_i,τ_j) – bĝ–nbd of g∈X.

(ii) If N a subset of a bitopological space (X,τ_i,τ_j) is (τ_i,τ_j) – bĝ–open set, then N is (τ_i,τ_j) – bĝ–nbd of each of its points.

1) Proof:

(i) Since N is a τ_j – nbd of g ∈ X, then there exists τ_j – open set G such that g ∈ G ⊑ N. Since from proposition 3.3 (i), "Every τ_j – open set is (τ_i,τ_j) – bĝ–open set, G is (τ_i,τ_j) – bĝ–open. By definition 4.4, N is (τ_i,τ_j) – bĝ–nbd of X.

(ii) Let N be a (τ_i,τ_j) – bĝ–open set. By definition 4.4, N is a (τ_i,τ_j) – bĝ–nbd of each of its points.

F. Proposition 4.6:

(i) Every (τ_i,τ_j) – bĝ–nbd of g ∈ X need not be τ_j – nbd of g.

(ii) Every (τ_i,τ_j) – bĝ–nbd of g∈X need not be (τ_i,τ_j) – bĝ–open set of X.

1) Proof:

(i) Let X = {a,b,c,d} with topologies

$\tau_i = \{X, \Phi, \{a\}, \{a,b\}, \{a,d\}, \{a,b,d\}\}$ *and* $\tau_j = \{X, \Phi, \{a\}, \{a,b\}\}$

(τ_i,τ_j)– bĝ– O(X)={X, Φ,{a},{b},{d},{a,b},{a,c}, {a,d},{b,d},{a,b,c},{a,b,d},{a,c,d}}

Here the set {b,d}is (τ_i,τ_j) – bĝ–nbd of b, since there exists a (τ_i,τ_j)– bĝ– open set G={b} such that b∈{b}⊑ {b,d}. However {b,d} is not τ_j – nbd of b, since there is no τ_j – open set G such that b ∈ G ⊑ {b,d}.

(ii) Let X = {a,b,c} with topologies $\tau_i = \{X, \Phi, \{a\}, \{a,b\}\}$ *and* $\tau_j = \{X, \Phi, \{a\}\}$

(τ_i,τ_j)– bĝ– O(X)={X, Φ,{a},{b},{a,b},{a,c}}

Here the set {b,c}is (τ_i,τ_j) – bĝ–nbd of b, since there exists a (τ_i,τ_j)– bĝ– open set G={b} such that b∈{b}⊑ {b,c}. However {b,d} is not (τ_i,τ_j) – bĝ–open set.

G. Remark 4.7:

In general (τ_i,τ_j) – bĝ–C(X) is not equal to (τ_j,τ_i) – bĝ–C(X)

Let X = {a,b,c,d} with topologies

$\tau_i = \{X, \Phi, \{b\}, \{a,b\}, \{b,c,d\}\}$ *and* $\tau_j = \{X, \Phi, \{a\}, \{a,c\}, \{a,b,d\}$

(τ_i,τ_j)– bĝ– C(X)={X, Φ,{b},{c},{d},{a,c},{a,d}, {b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}

(τ_j,τ_i) – bĝ–C(X)={ X, Φ,{a},{c},{d},{a,c},{a,d}, {b,c},{c,d},{a,b,c},{a,c,d},{b,c,d}

Hence they are not equal.

V. APPLICATIONS

A. Definition 5.1:

- A bitopological space (X,τ_i,τ_j) is said to be (i,j) – Tbĝ–space, if every (τ_i,τ_j)– bĝ–closed set in it is τ_j– b–closed.
- A bitopological space (X,τ_i,τ_j) is said to be (i,j) – T*bĝ–space, if every (τ_i,τ_j)– bĝ–closed set in it is τ_j– closed.

B. Example 5.2:

(i) Let X = {a,b,c} with topologies

$\tau_i = \{X, \Phi, \{a\}, \{b,c\}\}$ *and* $\tau_j = \{X, \Phi, \{a\}\}$

(τ_i,τ_j)– bĝ– C(X)={X, Φ,{b},{c},{b,c}}

τ_j – b–C(X) = { X, Φ, {b},{c},{b,c}}
Hence (X, τ_i, τ_j) is (i,j) – Tbĝ–space.

(ii) Let X = {a,b,c} with topologies
$\tau_i = \{X, \Phi, \{a, b\}, \{c\}\}$ and $\tau_j = \{X, \Phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$
(τ_i, τ_j)– bĝ– C(X)={X, Φ,{b},{c},{a,c},{b,c}}
τ_j – C(X) = { X, Φ, {b},{c},{a,c},{b,c}}
Hence (X, τ_i, τ_j) is (i,j) – T*bĝ–space.

C. Remark 5.3:

(i) If $\tau_i = \tau_j$, then (i,j) – Tbĝ–space becomes a Tbĝ–space.
(ii) If $\tau_i = \tau_j$, then (i,j) – T*bĝ–space becomes a T*bĝ–space.

D. Proposition 5.4:

Every (i,j) – T*bĝ–space is a (i,j) – Tbĝ–space.

1) *Proof:*

Since from [9], remark every τ_j –Closed set is τ_j – b – closed, the proposition is valid.

E. Example 5.5:

Every (i,j) – Tbĝ–space is not (i,j) – T*bĝ–space.
Let X = {a,b,c} with topologies $\tau_i = \{X, \Phi, \{a, b\}, \{c\}\}$ and $\tau_j = \{X, \Phi, \{b\}\}$
(τ_i, τ_j)– bĝ– C(X)={X, Φ,{a},{c},{a,b}}
τ_j – b–C(X) = { X, Φ, {a},{c},{a,c}}
Here (X, τ_i, τ_j) is (i,j) – Tbĝ–space but not (i,j)–T*bĝ–space, since {a} and {c} are τ_j – b–Closed but not τ_j – Closed .

VI. (τ_i, τ_j) – σ_K–BĜ–CONTINUOUS MAPS & (τ_i, τ_j) – BĜ–IRRESOLUTE MAPS

A. Definition 6.1:

A map f: (X, τ_i, τ_j) → (Y, σ_e, σ_k) is called (τ_i, τ_j) – σ_K–bĝ–Continuous map if the inverse image of every σ_K–closed set is (τ_i, τ_j) –bĝ–Closed.

B. Example 6.2:

Let X = {a,b,c} with topologies $\tau_1 = \{X, \Phi, \{a\}, \{b, c\}\}$ and $\tau_2 = \{X, \Phi, \{a\}\}$
Let Y = {p,q,r} with topologies $\sigma_1 = \{Y, \Phi, \{p\}, \{p, q\}\}$ and $\sigma_2 = \{Y, \Phi, \{p, r\}\}$
(τ_1, τ_2)– bĝ– C(X)={X, Φ,{b},{c},{b,c}}
Define f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a) = p, f(b) = r and f(c) = q, then f is (τ_1, τ_2) – σ_1–bĝ–Continuous Map as well as (τ_1, τ_2) – σ_2–bĝ–Continuous Map.

C. Proposition 6.3:

1) If a map f: (X, τ_i, τ_j) → (Y, σ_e, σ_k) is τ_j – σ_K–continuous then it is an (τ_i, τ_j) – σ_K–bĝ–Continuous map.
2) If a map f: (X, τ_i, τ_j) → (Y, σ_e, σ_k) is (τ_i, τ_j) – σ_K–bĝ–Continuous then it is (τ_i, τ_j) – σ_K–gb–Continuous map.
3) If a map f: (X, τ_i, τ_j) → (Y, σ_e, σ_k) is (τ_i, τ_j) – σ_K–bĝ–Continuous then it is (τ_i, τ_j) – σ_K– rgb–Continuous map.

1) *Proof:*

(i) Let V be any σ_K– closed set in Y, then f^{-1}(V) is τ_j – closed set, since f is τ_j –σ_K–Continuous. By proposition 3.3(i), f^{-1}(V) is (τ_i, τ_j) – bĝ – closed. Hence f is (τ_i, τ_j) – σ_K–bĝ–Continuous map.
(ii) Let V be any σ_K– closed set in Y, then f^{-1}(V) is (τ_i, τ_j) – bĝ – closed set in X, since f is (τ_i, τ_j) – σ_K–bĝ–Continuous. By proposition 3.9(i), f^{-1}(V) is (τ_i, τ_j) – gb – closed in X and hence f is (τ_i, τ_j) – σ_K–gb–Continuous map.
(iii) Let V be any σ_K– closed set in Y, then f^{-1}(V) is (τ_i, τ_j) – bĝ – closed set in X, since f is (τ_i, τ_j) – σ_K–bĝ–Continuous. By proposition 3.9(ii), f^{-1}(V) is (τ_i, τ_j) – rgb – closed in X and hence f is (τ_i, τ_j) – σ_K– rgb–Continuous map.

D. Proposition 6.4:

(i) Every (τ_i, τ_j) – σ_K–bĝ–Continuous map need not be τ_j – σ_K–continuous.
(ii) Every (τ_i, τ_j) – σ_K–gb–Continuous map need not be (τ_i, τ_j) – σ_K–bĝ–Continuous.
(iii) Every (τ_i, τ_j) – σ_K– rgb–Continuous map need not be (τ_i, τ_j)–σ_K–bĝ–Continuous.

1) *Proof:*

(i) Let X = {a,b,c,d} with topologies
$\tau_1 = \{X, \Phi, \{a\}, \{a, c\}, \{a, b, d\}\}$ and $\tau_2 = \{X, \Phi, \{b\}, \{a, b\}, \{b, c, d\}\}$ and
Let Y = {a,b,c,d} with topologies
$\sigma_1 = \{Y, \Phi, \{d\}, \{b, c\}, \{b, c, d\}\}$ and $\sigma_2 = \{Y, \Phi, \{a\}\{b\}, \{a, b\}\}$
(τ_1, τ_2)– bĝ– C(X)={X, Φ,{a},{c},{d},{a,c},{a,d},{b,c},{c,d},{a,b,c},{a,c,d},{b,c,d}}
Define f: (X, τ_1, τ_2)) → (Y, σ_1, σ_2) by f(a) = a, f(b) = b, f(c) = c and f(d) = d.
Hence f is (τ_1, τ_2) – σ_1–bĝ–Continuous Map but not τ_2 – σ_1–Continuous Map.
Since for the τ_2 – σ_1–closed set {a,d} in Y, f^{-1}({a,d})={a,d} which is not τ_2 – closed in X, but it is (τ_1, τ_2) –bĝ–Closed set.
(ii) Let X = {a,b,c} with topologies
$\tau_1 = \{X, \Phi, \{a\}, \{b, c\}\}$ and $\tau_2 = \{X, \Phi, \{a\}\}$ and
Let Y = {a,b,c} with topologies
$\sigma_1 = \{Y, \Phi, \{a, c\}\}$ and $\sigma_2 = \{Y, \Phi, \{a\}, \{b\}, \{a, b\}\}$
(τ_1, τ_2)– bĝ– C(X)={X, Φ,{b},{c},{b,c}}
(τ_1, τ_2)– gb– C(X)={X, Φ,{b},{c},{a,b},{a,c},{b,c}}
Define f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a) = a, f(b) = b, f(c) = c.
Hence f is (τ_1, τ_2) – σ_2–gb–Continuous Map but not (τ_1, τ_2)– σ_2– bĝ–Continuous Map. Since for the σ_2–closed set {a,c} in Y, f^{-1}({a,c})={a,c} which is not (τ_1, τ_2) –bĝ–Closed set, but it is (τ_1, τ_2) –gb–Closed set.
(iii) Let X = {a,b,c,d} with topologies
$\tau_1 = \{X, \Phi, \{a\}, \{a, b\}, \{a, b, c\}\}$ and $\tau_2 = \{X, \Phi, \{a\}, \{a, b\}, \{a, d\}, \{a, b, d\}\}$ and
Let Y = {a,b,c,d} with topologies

$\sigma_1 = \{ Y, \Phi, \{a\}, \{a,c\}, \{a,b,d\}\}$ *and* $\sigma_2 = \{Y, \Phi, \{b\}, \{a,b\}, \{b,c,d\}\}$

(τ_1, τ_2)– bĝ– C(X)={X, Φ,{b},{c},{d},{a,d},{b,c},{b,d},{c,d},{a,b,d},{a,c,d},{b,c,d}}

(τ_1, τ_2)– rgb– C(X)={X, Φ,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d}{a,c,d},{b,c,d}}

Define f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a) = a, f(b) = c, f(c) = d and f(d) = b.

Hence f is (τ_1, τ_2) – σ_2–rgb–Continuous Map but not (τ_1, τ_2)–σ_2– bĝ–Continuous Map. Since for the σ_2–closed set {a} and {a,c,d} in Y, f^{-1}({a})={a} and f^{-1}({a,c,d})={a,b,c} which is not (τ_1, τ_2) –bĝ–Closed set, but it is (τ_1, τ_2) –rgb–Closed set.

E. *Definition 6.5:*

A map f: (X, τ_i, τ_j) → (Y, σ_e, σ_k) is called Pair wise–bĝ–irresolute, if the inverse image of every (σ_e, σ_k)– bĝ–closed set in Y is (τ_i, τ_j) –bĝ–Closed set in X.

F. *Example 6.6:*

Let X = {a,b,c,d} with topologies
$\tau_1 = \{ X, \Phi, \{b\}, \{a,b\}, \{b,c,d\}\}$ *and* $\tau_2 = \{X, \Phi, \{a\}, \{a,c\}, \{a,b,d\}\}$ and
Let Y = {a,b,c,d} with topologies
$\sigma_1 = \{ Y, \Phi, \{a\}, \{a,c\}, \{a,b,d\}\}$ *and* $\sigma_2 = \{Y, \Phi, \{b\}, \{a,b\}, \{b,c,d\}\}$

(τ_1, τ_2)– bĝ– C(X)={X, Φ,{b},{c},{d},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}

(τ_1, τ_2)– bĝ– C(Y)={Y, Φ,{a},{c},{d},{a,c},{a,d},{b,c},{c,d},{a,b,c},{a,c,d},{b,c,d}}

Define f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a) = b, f(b) = a, f(c) = c and f(d) = d.

Clearly f is pair wise–bĝ–irresolute.

G. *Theorem 6.7:*

If a map f: (X, τ_i, τ_j) → (Y, σ_e, σ_k) is a pair wise–bĝ–irresolute map, then it is an (τ_i, τ_j) – σ_K–bĝ–Continuous.

1) *Proof:*

Assume that f: (X, τ_i, τ_j) → (Y, σ_e, σ_k) is a pair wise–bĝ–irresolute map. Let V be σ_k–closed set in Y. Then by proposition 3.3 (i), it is (σ_e, σ_k) – bĝ–closed in Y. By our assumption, $f^{-1}(V)$ is (τ_i, τ_j) –bĝ–Closed in X and hence f is (τ_i, τ_j) – σ_K–bĝ–Continuous map.

H. *Example 6.8:*

Every (τ_i, τ_j) – σ_K–bĝ–Continuous map need not be pair wise–bĝ–irresolute map.

Let X = {a,b,c} with topologies
$\tau_1 = \{ X, \Phi, \{a,b\}, \{c\}\}$ *and* $\tau_2 = \{X, \Phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$ and
Let Y = {a,b,c} with topologies
$\sigma_1 = \{ Y, \Phi, \{a\}, \{b\}, \{a,b\}\}$ *and* $\sigma_2 = \{Y, \Phi, \{a,c\}\}$

(τ_1, τ_2)– bĝ– C(X)={X, Φ,{b},{c},{a,c},{b,c}}

(σ_1, σ_2)– bĝ– C(Y)={Y, Φ,{a},{b},{c},{a,b},{a,c},{b,c}}

Define f: (X, τ_1, τ_2) → (Y, σ_1, σ_2) by f(a) = a, f(b) = b, f(c) = c.

Hence f is (τ_1, τ_2) – σ_1– bĝ –Continuous map but not pair wise bĝ– irresolute map, since the inverse image of (σ_1, σ_2) – bĝ–closed sets {a} and {a,b} in Y are {a} and {a,b} which are not (τ_1, τ_2)– bĝ– Closed sets in X.

I. *Theorem 6.9:*

Let f: (X, τ_i, τ_j) → (Y, σ_e, σ_k) be a map, then the following statements are equivalent

(i) f is (τ_i, τ_j) – σ_K–bĝ–Continuous map.

(ii) The inverse image of each σ_k–open set in Y is (τ_i, τ_j) – bĝ–open in X.

1) *Proof:*

(i) ⇒ (ii)

Assume that f is (τ_i, τ_j) – σ_K–bĝ–Continuous map. Let G be σ_k–open in Y. Then G^c is σ_k–closed in Y. Since f is (τ_i, τ_j) – σ_K–bĝ–Continuous map, $f^{-1}(G^c)$ is (τ_i, τ_j) –bĝ–Closed in X. But $f^{-1}(G^c)$ = X - $f^{-1}(G)$. Thus $f^{-1}(G)$ is (τ_i, τ_j) –bĝ–open in X.

(ii) ⇒ (i)

Conversely assume that the inverse image of each σ_k–open set in Y is (τ_i, τ_j) –bĝ–open in X. Let F be any σ_k–closed set in Y, then $f^{-1}(F^c)$ is (τ_i, τ_j) –bĝ–open. But $f^{-1}(F^c)$ = X – $f^{-1}(F)$. Thus $f^{-1}(F)$ is (τ_i, τ_j) –bĝ–closed in X. Therefore f is (τ_i, τ_j) – σ_K–bĝ–Continuous map.

J. *Theorem 6.10:*

Let (X, τ_i, τ_j) and (Z, μ_p, μ_q) be any bitopological spaces and Y be a (σ_e, σ_k) –T*bĝ–space, then the composition g°f: (X, τ_i, τ_j) → (Z, μ_p, μ_q) is (τ_i, τ_j) – μ_p–bĝ–Continuous map, if f: (X, τ_i, τ_j) → (Y, σ_e, σ_k) is (τ_i, τ_j) – σ_K–bĝ–Continuous map and g: (Y, σ_e, σ_k) →(Z, μ_p, μ_q) is (σ_e, σ_k)– μ_p–bĝ–Continuous map.

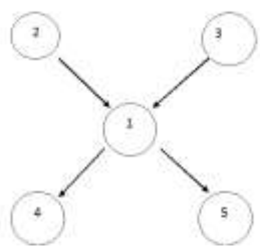
1) *Proof:*

To Prove: For every μ_p– closed set F in Z, $(g°f)^{-1}$(F) is (τ_i, τ_j) –bĝ–closed in X

Let F be any μ_p– closed set in Z. Since g is (σ_e, σ_k)–μ_p–bĝ–Continuous map, g^{-1}(F) is (σ_e, σ_k) –bĝ–closed in Y. But Y is (σ_e, σ_k) –T*bĝ–space and hence g^{-1}(F) is σ_K–closed in Y. Now since f is (τ_i, τ_j) – σ_K–bĝ–Continuous map, $f^{-1}(g^{-1}(F))$ is (τ_i, τ_j) –bĝ–closed in X. But $f^{-1}(g^{-1}(F))$= $(g°f)^{-1}$(F). Therefore $(g°f)^{-1}$(F) is (τ_i, τ_j) –bĝ–closed in X. Hence $g°f$ is (τ_i, τ_j) – μ_p–bĝ–Continuous map.

K. *Remark 6.11:*

The following diagram shows the relationship among (τ_i, τ_j) – σ_K–bĝ–Continuous map with other existing functions.



1) (τ_i, τ_j) – σ_K–bĝ–Continuous

2) $\tau_j - \sigma_K$–Continuous
3) Pairwise–bĝ–irresolut**e**
4) $(\tau_i, \tau_j) - \sigma_K$–rgb–Continuous
5) $(\tau_i, \tau_j) - \sigma_K$– gb–Continuous

REFERENCES

[1] El-Tantawy.O.A and Abu-Donia.H.M, Generalized Separation Axioms in Bitopological Spaces, The ArabianΠ for Science and Engg.Vol.30,N0.1A,117-129(2005)
[2] Fukutake, T., On generalized closed sets in bitopological spaces, Bull.Fukuoka Univ.Ed.Part III,35(1986),19-28.
[3] Kelly, J.C., Bitopological spaces, Proc.London Math.Soc.13(1963), 71-89.
[4] Maheswari, S.N., and Prasad, R., Semi open sets and semi continuous functions in bitopological spaces. Math Notae, 26, (1977/78),29-37.
[5] F.H.khber andH.S.Al-Saadi,On pairwise semi generalized closed sets ,JKAU ,Sci Vol.21,No.(2009)2,269-295.
[6] T.Indra ,τ1τ2 - #g Closed Sets in Bitopological spaces, Annals of Annals of Pure and Applied Mathematics Vol. 7, No. 2, 2014, 27-34.
[7] Bushra Jaralla Tawfeeq, Dunya MohamedHammed (**τi**, **τj**)− rgbClosed Sets in Bitopological Spaces , IOSR Journal of Mathematics (IOSR-JM) Volume 6, Issue 6 (May. - Jun. 2013), PP 14-2.
[8] Binod Chandra Tripathy and Diganta Jyoti Sarma, Generalized b-closed sets in ideal bitopological Spaces, Proyecciones Journal of Mathematics Vol. 33, No 3, pp. 315-324, September 2014.
[9] D.Andrijevic, on b-open sets,Mt.Vesnik 48(1996), no.1-2,59-64