Three Dimensional MHD Couette Flow in the Presence of Suction or Injection with Radiation Effect and Heat Source

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Abstract—This research is concerned with MHD three dimensional couette flow of a viscous incompressible electrically conducting fluid with radiation effect on temperature distribution in the presence of heat source. The lower stationary porous plate is subjected to a periodic injection velocity and the upper porous plate in uniform motion to a constant suction velocity. A magnetic field of uniform strength applied normal to the plane of the plates. Neglecting the induced magnetic field, an approximate solution for the flow field is obtained and discussed with the help of graphs.

Key words: MHD, Couette Flow, Radiation, Three Dimensional, Heat Transfer

I. INTRODUCTION

The flow between parallel plates is a classical problem that has important applications in magnetohydrodynamic power generators and pumps, accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil and fluid droplets and sprays. Hartmann and Lazarus [18], Jaffery [5], Chang and Lundgren [10], Cramer and Pai [22], Davidson [25] and Cowling [29] were studied the influence of a transverse uniform magnetic field on the flow of a conducting fluid between two infinite parallel, stationary and insulated plates. Then the problem was extended in numerous ways. Some exact and numerical solutions for the heat transfer problem are found by Nigam and Singh [27], Alpher [26] and Attia and Kotb [16]. Attia and Kotb [16] studied the steady MHD fully developed flow and heat transfer between two parallel plates with temperature dependent viscosity in the presence of uniform magnetic field. Latter Attia [17] extended the problem to the transient state. The phenomenon of MHD flow with heat transfer play an important role in various industrial applications. Some important applications are cooling of nuclear reactors, liquid metals fluid, power generation system and aerodynamics. An extensive contribution on heat transfer flow has been made by Gebhart [7] to highlight the insight on the phenomena. The viscous fluid flow and heat transfer in porous medium is a topic of research studied by researchers, e.g. Al-hadhrami et al. [2] and Kim et al. [28]. Gersten and Gross [21] studied the effect of slightly sinusoidal transverse suction velocity distribution on the flow and heat transfer over a plane wall. This suction velocity distribution leads to a cross flow and hence to a three dimensional flow over the surface. Effect of such a suction velocity on various flow and heat transfer problems along flat and vertical porous plates have also been studied extensively by Singh and Sharma [19], Singh [20], Jain and Gupta [24] and Sharma et al. [8]. Eckert and Drake[15] described the reduction of heat transfer in couette flow. Raptis [1] studied Radiation and free convection flow through a porous medium. The study of heat transfer in presence of radiation has acquired newer dimensions. Raptis and Perdikis [4] presented Viscoelastic flow by the presence of radiation. Sanyal and Adhikari [14] have investigated Effects of radiation on MHD vertical channel flow. Kar et al. [23] discussed Three –dimensional free convection MHD flow in a vertical channel through a porous medium with heat source and chemical reaction. Singh [9] analyzed MHD steady flow of liquid between two parallel plates. Chauhan and Shekhawat [11] investigated Heat transfer in couette flow of a compressible Newtonian fluid in the presence of a naturally permeable boundary. Chauhan and Vyas [12] analyzed Heat transfer in hydromagnetic couette flow of compressible Newtonian fluid. Chauhan and Kumar [13] examined Heat transfer effects in a coquette flow through a composite channel partly filled by a porous medium with a transverse sinusoidal injection velocity and heat source.

In view of the above-presented discussion, the objective of the present study is to investigate the effect of magnetic field on three dimensional couette flow of a viscous incompressible electrically conducting fluid between two infinite horizontal parallel flat plates with suction or injection in the presence of radiation and heat source. The governing equations of the flow field are solved by using series expansion method and the expressions for the velocity field and the temperature field are obtained. The effect of the flow parameters on the velocity field and temperature field have been studied and analyzed with the help of graphs.

II. FORMULATION OF THE PROBLEM

The flow under investigation has been modeled as the three dimensional couette flow of a viscous incompressible electrically conducting fluid bounded between two infinite horizontal parallel porous plates in presence of a uniform transverse magnetic field $B_0$. A coordinate system is chosen with its origin at the lower stationary plate lying horizontally on $x^* - z^*$ plane and the upper plate at a distance $d$ from it is subjected to a uniform velocity $U$. The $y^*$-axis is taken normal to the plane of the plates. The lower and the upper plates are assumed to be at constant temperatures $T^*_0$ and $T^*_1$ respectively with $T^*_1 > T^*_0$. The upper plate is subjected to a constant suction velocity $V_s$ whereas the lower plate to a transverse sinusoidal injection velocity of the form:

$$v^*(x^*) = V_s \left(1 + \epsilon \cos \frac{\pi x^*}{d}\right), \quad (2.1)$$

Where $\epsilon$ is a positive constant quantity and $d$ is the distance between the plates equal to the wavelength of the injection velocity. Due to this kind of injection velocity the flow remains three dimensional. All the physical quantities involved are independent of $x^*$ for this fully
developed laminar flow. Denoting the velocity components \(u, v, w\) in \(x, y, z\) directions, respectively and the temperature by \(T^*\), the problem is governed by the following equations:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]

(2.2)

\[
\rho \left( v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \frac{\partial T}{\partial y} + \mu \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) - \sigma B_0^2 \frac{\partial u}{\partial x}
\]

(2.3)

\[
\rho \left( v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)
\]

(2.4)

\[
\rho \left( v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - \frac{\partial q}{\partial z}
\]

(2.5)

\[
\rho C_p \left( v \left( \frac{\partial T}{\partial x} \right)_y + w \left( \frac{\partial T}{\partial y} \right)_z \right) = k \left( \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) - 4I(T^* - T_o)
\]

(2.6)

where

\[I = \int_0^\infty K \left( \frac{\partial e}{\partial T} \right)_0 dR.\]

Denoting velocity components \(u, v, w\) in the directions \(x, y, z\), respectively and temperature by \(\theta\), the flow is governed by the following dimensionless equations:

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0
\]

(2.9)

\[
\frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \frac{1}{R} \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) - \frac{M^2}{R} u
\]

(2.10)

\[
\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \frac{1}{R \mu} \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \frac{M^2}{R} v
\]

(2.11)

\[
\frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = -\frac{1}{R} \left( \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - \frac{\partial q}{\partial z}
\]

(2.12)

\[
\frac{\partial \theta}{\partial y} + \frac{\partial \theta}{\partial z} = \frac{1}{R \Pr} \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} - F\theta + \frac{Q}{\Pr} \theta.
\]

(2.13)

The boundary conditions to the problem in the dimensionless form are

\[
y = 0: u = 0, v = 1 + \varepsilon \cos \pi z, w = 0, \theta = 0, \quad y = 1: u = 1, v = 0, w = 0, \quad \theta = 1.
\]

(2.14)

### III. Solution of the Problem

Since the amplitude of the injection velocity \(\varepsilon(\parallel 1)\) is very small, we now assume the solution of the following form:

\[
f(y, z) = f_1(y) + \varepsilon f_2(y, z) + \varepsilon^2 f_3(y, z) + \ldots
\]

(3.1)

where \(f\) stands for any of \(u, v, w, p, \theta\). When \(\varepsilon = 0\), the problem is reduced to the two dimensional flow with constant injection and suction at both plates and are governed by the following equations:

\[
\frac{\partial u}{\partial y} = 0
\]

(3.2)

\[
\frac{\partial u_0}{\partial y} = \frac{1}{R} \frac{\partial^2 u_0}{\partial y^2} - \frac{M^2}{R} u_0
\]

(3.3)

\[
\frac{\partial \theta_0}{\partial y} = \frac{1}{R \Pr} \frac{\partial^2 \theta_0}{\partial y^2} - F\theta_0 + \frac{Q}{\Pr} \theta_0.
\]

(3.4)

with boundary conditions:

\[
y_0: u_0 = 0, v_0 = 1, w_0 = 0, \theta_0 = 0,
\]

\[
y_0: u_0 = 1, v_0 = 0, w_0 = 0, \theta_0 = 1.
\]

The solution of this two dimensional problem is

\[
u_0(y) = e^{-\gamma y} - e^{\gamma y}, \quad v_0 = 1, w_0 = 0, \quad p_0 = \text{constant},
\]

(3.5)

\[
\theta_0(y) = e^{\gamma y} - e^{-\gamma y}.
\]

(3.6)

When \(\varepsilon \neq 0\), substituting (3.1) to (2.9)-(2.13) and comparing the coefficients of identical powers of \(\varepsilon\),
neglecting those of $\varepsilon^2, \varepsilon^3$ etc., the following first order equations are obtained with the help of (3.6)

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0,$$  (3.7)

$$\frac{\partial u_1}{\partial y} + \frac{\partial v_1}{\partial z} = \frac{1}{R} \left( \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} \right) - \frac{M^2}{R} u_1,$$  (3.8)

$$\frac{\partial v_1}{\partial y} - \frac{\partial p_1}{\partial z} + \frac{1}{R} \left( \frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2} \right) = \frac{M^2}{R} w_1,$$  (3.9)

$$\frac{\partial w_1}{\partial y} - \frac{\partial p_1}{\partial z} + \frac{1}{R} \left( \frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2} \right) = \frac{M^2}{R} v_1,$$  (3.10)

$$\frac{\partial \theta_1}{\partial y} + \frac{\partial \theta}{\partial z} = \frac{1}{Pr} \left( \frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2} \right) - F \theta_1 + \frac{Q}{Pr} \theta_1.$$  (3.11)

The corresponding boundary conditions reduce to

- When $y = 0$: $u_1 = 0, v_1 = \cos \pi z, w_1 = 0, \theta_1 = 0$.
- When $y = 1$: $u_1 = 0, v_1 = 0, w_1 = 0, \theta_1 = 0$.

These are the linear partial differential equations describing the three dimensional couette flow.

To solve these equations, we shall first consider (3.7), (3.9) and (3.10), independent of the main flow component $u_1$ and the temperature field $\theta_1$. We assume

$$v_1(y,z) = v_1(y) \cos \pi z,$$  (3.13)

$$w_1(y,z) = -\frac{1}{\pi} v_1'(y) \sin \pi z,$$  (3.14)

$$p_1(y,z) = p_1(y) \cos \pi z,$$  (3.15)

where the primes denote differentiation with respect to $y$. Equations (3.13) and (3.14) have been chosen so that the continuity equation (3.7) is satisfied. Substituting these equations into (3.9) and (3.10) and applying the corresponding transformed boundary conditions, we get

$$v_1(y,z) = \left( B_1 e^{\pi y} + B_2 e^{\pi y} + B_3 e^{\pi y} \right) \cos \pi z,$$  (3.16)

$$w_1(y,z) = -\frac{1}{\pi} \left( B_1 e^{\pi y} + B_2 e^{\pi y} + B_3 e^{\pi y} \right) \sin \pi z,$$  (3.17)

$$p_1(y,z) = \frac{1}{R \pi^2} \left( A_2 B_1 e^{\pi y} + A_3 B_2 e^{\pi y} \right) + A_1 B_3 e^{\pi y} + A_2 B_4 e^{\pi y})$$  (3.18)

Now assume $u_1$ and $\theta_1$ as

$$u_1(y,z) = u_1(y) \cos \pi z,$$  (3.19)

$$\theta_1(y,z) = \theta_1(y) \cos \pi z.$$  (3.20)

Substituting these equations into (3.8) and (3.11), we obtain

$$u_1' - R u_1'' + (\pi^2 + M^2) u_1 = R v_1 u_1'.$$  (3.21)

$$\theta_1' - 2 R \theta' + (\pi^2 + R \pi F - R Q) \theta_1 = R Pr v_1 \theta_1'.$$  (3.22)

With corresponding boundary conditions

- $y = 0$: $u_1 = 0, \theta_1' = 0$.
- $y = 1$: $u_1 = 0, \theta_1 = 0$.

where the primes denote differentiation with respect to $y$.

Solving (3.21) and (3.22) under the boundary conditions (3.23), we get

$$u_1(y,z) = \left[ L_1 e^{(\pi + M) y} + L_2 e^{(\pi - M) y} + L_3 e^{(\pi + M) y} + L_4 e^{(\pi - M) y} \right] \cos \pi z,$$  (3.24)

$$\theta_1(y,z) = \left[ L_5 e^{(\pi + M) y} + L_6 e^{(\pi - M) y} + L_7 e^{(\pi + M) y} + L_8 e^{(\pi - M) y} \right] \cos \pi z.$$  (3.25)

IV. RESULTS AND DISCUSSION

The problem of MHD three dimensional couette flow with radiation effect and heat source in the presence of suction or injection is analyzed. Solutions for the velocity and temperature fields are obtained analytically and then evaluated for different values of parameters appeared in the equation. The variation of the physical quantities with flow parameters are shown graphically. Figure 1 shows the profile of the main flow velocity component $u$. It is observed that the velocity component $u$ decreases with increasing the Hartmann number $M$ since it increases the damping force on $u$. Also $u$ decreases with increasing injection parameter $R$ due to the convection of the fluid from lower region to the upper region.

Fig. 1: Main Flow Velocity Profile Against $y$ For Different Values Of $M$ And $R$ With $\varepsilon = 0.2$ And $z = 0.5$.

Figure 2 presents the temperature profile for different values of radiation parameter $F$ and heat source parameter $Q$. It is clear from figure that temperature increases with increasing $Q$ and decreases with increasing $F$ since the effect of radiation decrease the rate of energy transport to the fluid, thereby decreasing the fluid temperature.

Fig. 2: Temperature profile against $y$ for different values of $F$ and $Q$ with $Pr = 0.71$, $R = 0.5$, $\varepsilon = 0.2$ and $z = 0.5$. 
Figure 3 shows the profile of temperature for the effect of Prandtl number \( \text{Pr} \) and injection parameter \( R \). It is depicted that an increase in \( \text{Pr} \) temperature decreases since \( \text{Pr} \) increases, the thermal diffusivity decreases and this phenomenon lead to the decreasing manner of the energy transfer ability that reduces the thermal boundary layer. As the value of \( R \) increases, more amount of fluid is pushed into the channel through the lower plate, causing a decrease in temperature at all points in the channel.

Figure 4 shows the variation of the cross flow velocity component \( w \) for the injection parameter \( R \). We see that the injection parameter \( R \) enhances the cross flow velocity. This is due to the fact that there is injection at the stationary plate and suction at the plate in uniform motion, which are two exactly opposite processes.

**References**


