A Study of Waiting time of Multiple Server Queuing Model in a Children Hospital

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Abstract— Waiting lines of queue are experienced in our daily activities. Waiting in lines or queues causes inconvenience to customers (patients) and economically loss to firms and organization (hospitals). Customers (patients) wait for minutes, hours to receive medical services- waiting before or during being served. Queuing theory is a mathematical approach to the study of waiting in lines/queues. This paper presents the result of a study that evaluates how to decrease a waiting period before served from the point of view of patients in children hospital Rajkot. How many servers (doctors) added by the hospital management so patients have not to wait in queue and get service immediately. We also suggest that Numbers of servers (doctors) added into the hospital are also cost effective from the hospital management side. We use WINQSB software to measure the performance of queue system analyst for multiple server models. 

Key words: Servers, Utilization Factor, Queue System

I. INTRODUCTION

A common situation that occurs in everyday life is that of queuing or waiting in line. Queues (waiting lines) are usually seen at bus stops, hospital, bank counters and so on. Customer Satisfaction is very important to this type of service premise. If customer has to spent long time for getting service then he would go other competitive service premises. So waiting in line or in a system is to lose our customer. Hence Waiting time is very important in health care sector for getting services like consulting with doctor, medical reports, and admission in emergency department. Also from the side of hospital management how to reduce customer waiting time to get above type of services and also manage the staff (nurses, receptionist), doctor’s availability and report rooms available for patients as cost point of view. If there is no waiting period then the staff and doctors are in idle situation then it is not good for hospital management. So how to balance both the situation that hospital management can arrange sufficient staff for patient so patient has spent less waiting time in queue for getting service.

Operation Research existed as a scientific discipline since1930’s. It is a study of applying appropriate analytical methods of decision making. OR has been studied in health care sector since 1952. One of the major uses of Operation Research in health care is in the form of Queuing theory. Queues or Queuing theory was first analyzed by A.K. Erlang in 1913 in the context of telephone facilities. It is extensively practiced or utilized in industrial setting or retail sector operations management and falls under the purview of decision Sciences. There are many problems in health care system which can be solved by using queuing theory in Operation Research. Few of them are long waiting time at outpatient clinic, ticket reservation counter, fuel station, bank cash counter etc. Queuing theory is the mathematical study of waiting lines or queues. In queuing theory a model is constructed so that queue lengths and waiting times can be predicted. Queuing theory is generally considered a branch of operation Research because the results are often used when making business decision about the resources needed to provide service.

II. RESEARCH METHODS

Data were collected from children hospital Rajkot. The data were gathered for 30 days in two shifts morning and afternoon shifts. Data were collected by direct observation, interview with administrator staff and asking questions by researcher on a daily basis. The following assumptions were made for queuing system at the hospital which is in accordance with the queue theory. They are:

1) Arrival follows a Poisson Probability Distribution at an average rate of \( \lambda \) customer (patients) per unit of time

2) The queue discipline is First-Come First-Served (FCFS) basis by any of the servers. There is no priority classification for any arrival

3) Service times are distributed exponentially with an average of \( \mu \) customer (patients) per unit of time

4) There is no limit to the number of the queue i.e. infinite queue

5) The service providers are working at their full capacity

6) Servers here represent only doctors but not other medical personals

7) Service rate is independent of line length, service providers do not go faster because the line is longer.

III. THE M/M/S MODEL

The model adopted in this work is the M/M/S: (FCFS) Multi server Queuing Model. For this queuing system, it is assumed that the arrival follows a Poisson probability distribution at an average of \( \lambda \) customers (patients) per unit of time. It is also assumed that they are served on a first come, first served basis by any of the servers (doctors). The service times are distributed exponentially with an average of \( \mu \) customers (patients) per unit of time and number of servers S. If there are \( n \) customers in the queuing system at any point in time, then the following two cases arise:

1) If \( n \leq S \) (number of customers in the system is less than the number of servers), then there will be no queue. However, \( (S-n) \) number of servers will not be busy. The combined service rate will then be \( \mu n = n\mu ; n < S \)

2) If \( n > S \), (number of customers in the system is more than or equal to the number of servers then all servers will be busy and the maximum number of
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From the model the probability of having n customers in the system is given by

\[ P_n = \left\{ \begin{array}{ll}
\frac{\lambda^n}{n!} \mu^n P_0 & \text{for } n \leq s \\
\frac{\lambda^n}{n!} \mu^n P_0 & \text{for } n > s, \quad \rho = \frac{s \lambda}{s \mu}
\end{array} \right. \]

We now proceed to compute the performance of the queuing system.

The expected number of the customer (patients) waiting on the queue (length of line) is given as:

\[ L_q = \left[ \frac{1}{(s-1)} \left( \frac{\lambda}{\mu} \right)^s \frac{\mu^s}{(s \mu - \lambda)} \right] P_0 \]

Expected number of customer (patients) in the system is

\[ L_s = L_q + \frac{\lambda}{\mu} \]

Expected waiting time of customer (patients) in the queue

\[ W_q = \frac{L_q}{\lambda} \]

Average time a customer (patient) spends in the system:

\[ W_s = \frac{L_s}{\lambda} \]

Utilization factor i.e. the fraction of time servers (doctors) are busy

\[ \rho = \frac{s \lambda}{s \mu} \]

Where \( \lambda \) = the arrival rate of patients per unit time
\( \mu \) = the service rate per unit time
\( s \) = the number of servers

\( P_0 \) = the probability that there are no customers (patients) in the system

\( L_q \) = Expected number of customers in the queue

\( L_s \) = Expected number of customers in the system

\( W_q \) = Expected time a customer (patient) spends in the queue

\( W_s \) = Expected time a customer (patient) spend in the system.

IV. ANALYSIS OF THE DATA

There are 1735 customers arrive in 30 days. So average arrival per day is 57 customer per day and average arrival rate per hour is \( \lambda = 5.78 \) patient per hour and average service rate per hour is \( \mu = 7.22 \) patient per hour. Using above equation we calculate the performance measures of multi server Queuing system at children hospital Rajkot with the help of WINQSB software.

![Fig. 1: Using WINQSB software we get the performance measure for different server given in TABLE 1](image)

<table>
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<th>NO</th>
<th>SERVER</th>
<th>( \lambda )</th>
<th>( \mu )</th>
<th>( \lambda ) eff</th>
<th>( P_0 )</th>
<th>( L_s )</th>
<th>( L_q )</th>
<th>( W_q )</th>
<th>( W_s )</th>
<th>( \rho )</th>
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<tr>
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<td>0.0000</td>
<td>0.1385</td>
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</tbody>
</table>

Table 1: Performance Measure of Multi Server Queuing Model at Children Hospital
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Fig. 2: Utilization Factor (ρ) against Average waiting time of Patients in the System (Ws)

Fig. 3: Utilization Factor (ρ) against Average waiting time of patients in the queue (Wq)

V. DISCUSSION OF RESULTS

From Table 1, it was observed that the number of server is necessary to serve the patient is 4 in the case study for zero waiting period in queue. It was proved from customer point of view in Table 1 that 4 servers are appropriate to serve the patient without waiting in queue and get service from the doctors. We also suggest to hospital management that the number of servers 2 is appropriate to reduce the long waiting time 0.69 hour to 0.1679 hour.

REFERENCES