A Literature Survey on Multi Depot Vehicle Routing Problem
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Abstract— Due to the development of communication and information technology, and the increase pressure of transportation cost, many enterprises select the joint distribution of multiple depots instead of traditional fixed zone service of single depot. This paper presents a survey on multi depot vehicle routing problem (MDVRP). This article providing an up-to-date overview of the research papers on several variants of the MDVRP: - capacitated MDVRP, heterogeneous fleet MDVRP, time window MDVRP, delivery and pickup MDVRP.

Key words: Vehicle Routing Problem, Multi depot, heuristics and meta-heuristics

I. INTRODUCTION

The Vehicle Routing Problem (VRP) is a classical Combinatorial Optimization (CO) and Integer Programming (IP) problem that was proposed in the late 1950’s and it is still one of the complex and most studied in the field of Operations Research (OR), Transportation, Distribution and Logistic.

In VRP, where vehicle start their routes from the depots (distribution centres or manufacturing plant) through transportation network to the customer who have placed orders for such goods and return back to the depots after completion of routes. The Objective of VRP is to minimizing the total cost, here cost is represented in the form of travel time and travel distance of a vehicle for a route.

This problem was firstly described in 1959 [1] and it derives from the traveling salesman problem or arc routing problem. After then, considerable number of variants has been Considered: - capacitated, time windows, maximum route length, pickup and delivery, backhauls, multi-depot, multiple trip, periodic etc.

In capacitated vehicle routing problem (CVRP) maximum capacity of vehicle on a route is given. In CVRP no any customer has greater demand or greater pickup (in case of delivery and pickup) than any vehicle capacity.

In vehicle routing problem with time window (VRPTW) each customer has consist a time window in which delivery of product to the customer must be done.

In vehicle routing problem with simultaneously delivery and pickup (VRPSDP) where supplier may both send and receive goods simultaneously. In case of VRPMDP, where supplier may either deliver a product or receive a product but not both simultaneously.

MDVRP is one of the most widely used variations of VRP problem. Multi depot VRP (MDVRP) extends the basic problem in such a way, that there are multiple depots (manufacturing plant or distribution center) from where vehicles are starting a route (Fig. 2). Depending on the situation, vehicles are either obligated to return to the starting base (fixed) or they have no such limitation (non-fixed).

A. MDVRP formulation:

Mathematical model of MDVRP based on Renaud, Laporte, and Boctor (1996) [3]. Let G = (V,E) be a directed graph where V is a set of vertices and E is the set of edges and arcs connecting vertices of the graph. V is further partitioned into two sets V=\{Vc,Vd\}, here, Vc is a set of customers and Vd is a set of depots.

\[ Vc=\{c_1,c_2, \ldots,c_p\}, \text{ where } p \text{ is the number of the customers, and } Vd=\{d_1,d_2, \ldots,d_q\}, \text{ where } q \text{ is the number of the depots.} \]

The R set is a set of all of the routes between the depots and customers. Each route has its cost and it is also assumed that the cost of each route is known. As is standard with this problem type, the cost is represented in the length of the route. Ri is a route for one vehicle: R = \{(ri, rj) : ri, rj\in Vc U Vd, i /= j\}, where lij is the distance of the route (ri, rj).
The classic problem definition is that $l_{ij}$ is symmetric and therefore satisfies the triangle inequality: $l_{ij} = l_{ji}$ for each $i,j$. $R_i$ is the route of one vehicle, where it starts from a depot by vehicle, visits customers for satisfying their demands and return back to the starting depot: $R_i = \{d, c_1, \ldots, c_l, d\}$, where $d \in V_D$, $D$ is the demand of the $c_i$ customer, and $C_k$ is the capacity of the vehicle $k$. The sum of the vehicles for all the depots is $q$. In MDVRP, the following properties must be met:

- Sum of customer demands on one route must not exceed the capacity of vehicle servicing that route.
- Each customer is visited or serviced by a vehicle only once.
- Starting and ending of a route by a vehicle at a same point (depot).
- Demand or pickup of a customer does not exceeded to the vehicle capacity.

Goal of the MDVRP problem is to minimize the cost of all routes:

$$\text{min} \left( \sum_{i \in VCU VD} \sum_{j \in VCU VD})l_{ij} \right)$$
and to minimize the number of vehicles needed [4].

B. Paper Organization:

The rest of this paper is organized as follows. Section 2 present the review of papers solved by heuristics and meta-heuristics and important facts are highlighted. Section 3 present the analysis work on literature. Section 4 consist some concluded remark and also consist future scope.

II. LITERATURE REVIEW

Exact algorithms, heuristics and meta-heuristics have been developed to solve the VRP and its numerous extensions. The exact algorithms are time consuming and hardly ever applicable to problems concerned with more than 50 customers. In case of realistic situations heuristics are usually more applicable.

This section represents a literature survey of research papers which contained the solution methods of MDVRP based on heuristics and meta-heuristics.

A. Heuristics Approach:

The first heuristic approach to solve the multi depot vehicle routing problem (MDVRP) were developed by Tillman and Cain (1972) and Wren and Holliday (1972).

Tillman and Cain (1972)[5] introduced a heuristic approach based on the savings algorithm proposed by Clarke and Wright (1964). Single customers tours are constructed initially with each tour bound to its the closest depot. The algorithm then merges tours if there is a subsequent decrease (a “savings”) in route length. The assignment of customers to depot can change in the route merging procedure. This heuristic, as many other heuristics does not include an improvement stage on the initial solution to obtain a better solution.

The heuristic introduced by Wren and Holliday (1972)[6] does include an improvement stage; in the first stage an initial feasible solution is obtained and in the second stage several refinement heuristics are applied to this solution. This procedure of obtaining an initial solution first and then improving it, is one of the two common structures of the heuristics developed to solve the MDVRP.

The second well known structure involves decomposing the MDVRP into sub-problems first and then solving these sub-problems separately before connecting them iteratively. Gillett and Johnson (1976)[7] introduced an algorithm based on this structure, also known as the cluster first route second principle. It applies the two-stage solution technique by assigning the customers to the depots in the first stage and determining the routes in the second stage. Hence, in the first stage MDVRP is divided into single-depot problems and in the second stage the optimal routes corresponding to these sub-problems are obtained.

Golden et al. (1977)[8] also developed an algorithm using the cluster first - route second principle. This algorithm make a cluster of customers in the first stage according to the distances to the depots closest and second closest to the customers.

Raft (1982)[9] developed an algorithm for which the problem is decomposed in five, instead of two, stages. The five stages are solved separately and then connected iteratively.

Min, Current, and Schilling (1992)[10] studied the version of the MDVRP with backhauling and proposed a heuristic procedure based on problem decomposition.

et al. (1993)[11] applied the cluster first - route second principle presented by Golden et al.(1977) to obtain an initial solution and improve this solution by changing the depot assignments of the customers.

Salhi and sari(1997)[12] proposed a heuristic method approach for MDVRP with three levels. The first level was the construction of an initial feasible solution. The second and the third levels were to improve the routes in each depot, that is, intra-depot and the routes in all depots, that is, inter-depot respectively.

Hadjiconstantinou and Baldacci (1998)[13] formulated the problem of providing maintenance services to a set of customers as the multi-depot PVRP (MDPVRP). The authors decomposed the MDPVRP into four levels, and then used a heuristic method to solve the problem. The first level was to assign which depots serve which customers. The second level was to solve a PVRP for each depot. At the third level, a classical VRP for each depot for each day of the given period was solved. At the last level, a classical TSP for each route was tackled.

Su (1999)[14] proposed a dynamic vehicle control and scheduling system to solve the MDVRP. All the control decisions were made according to the real time status of the system, such as the location, quantity, and due date of the demand.

During the 21st century research has primarily been dedicated to extensions of the MDVRP but the classical MDVRP has been investigated as well.

Giosa at el.(2002)[15] investigated the multi-depot VRPTW (MDVRPTW), which is also an extension of the MDVRP. The authors designed and compared six heuristics for assigning the customers to depots while using the same VRP heuristic for each depot.

Salhi and nagy (2005)[16] presented a number of heuristic methods to solve the single-depot VRPPD. The methods can be modified to tackle the multi-depot VRPPD (MDVRPPD).

Pisinger and Ropke (2007)[17]developed a unified heuristic to solve five extensions of the classical VRP
including the MDVRP. This heuristic uses the adaptive large neighborhood search framework designed by (Ropke and Pisinger, 2006). This framework expands and contracts the search for a better solution by choosing adaptively among a number of heuristics that are able to insert and remove customers in a route.

The work of Gulczynski, Golden, and Wasil (2011)[18] developed an integer programming-based heuristic. The objective of this study was to determine the reduction in travelled distance that can be achieved by allowing split deliveries among vehicles based at the same depot and vehicles based at different depots. The multi-depot capacitated vehicle routing problem with split delivery (MDCVRPSD) is studied by Liu, Jiang, Fung, Chen, and Liu (2010)[19].

More recent works on the application of dedicated heuristics include the work of Vahed, Crainic, Gendreau, and Rei (in press)[20] for the case of a MDVRP with the objective of determining the optimal vehicle fleet size, and the work of Afshar-Nadjafi and Afshar-Nadjafi (in press)[21] for the study of the time-dependent MDVRP with heterogeneous vehicles and time windows.

B. Meta-heuristics Approach:

There are several meta-heuristic approaches to solving VRP and its variant transportation problems. State of the art solutions include: particle swarm optimization approach, ant colony optimization, genetic approach, simulated annealing and tabu search.

Here classification of different variants of MDVRP is based on constraints.

The most studied variant of the problem has been the capacitated MDVRP. Different meta-heuristics proposed in literature, we can highlight the Simulated Annealing algorithms of Wu,Low, and Bai (2002)[22] and Lim and Zhu (2006)[23], the Variable Neighborhood Search procedure proposed by Polacek, Hartl, Doerner, and Reimann (2005)[24], Polacek, Benkner, Doerner, and Hartl (2008)[25].

Tabu Search algorithms from Lim and Wang (2005)[26] and Maischberger and Cordeau (2011)[27] . Genetic Algorithms has been proposed as well for this problem variant, as illustrated in the works of Ba, Hwang, Cho, and Goan (2007)[28], Vidal, Crainic, Gendreau, Lahrichi, and Rei (2010)[29].

All of these works seek for the minimization of total route distance or cost. Rahimi-Vahed, Crainic, Gendreau, and Rei (2013)[30] employed path re-linking for the case of capacitated MDVRP with route duration constraint.


This paragraph deals with the meta-heuristics algorithms proposed for the heterogeneous fleet (HFMDVRP). We can highlight the works of Jeon, Leep, and Shim (2007)[32], who proposed a hybrid genetic algorithm that minimizes the total distance travelled, and that of Flisberg, Lidén, and Rönqvist (2009)[33] who considered a Tabu Search procedure. Simulated Annealing (SA) has been employed as well. Wu et al. (2002)[34] coupled SA with TS to solve the heterogeneous fleet case of the integrated location routing problem. In their problem, location of depots, routes of vehicles and client assignment problems are solved simultaneously.

The multi-depot heterogeneous vehicle routing problem with time windows (MDHVRPTW) was studied by Dondo and Cerdá (2009)[35], who proposed a MILP and a Local Search Improvement Algorithm that explores the neighborhood in order to find the lowest cost feasible solution.

Vidal, Crainic, Gendreau, and Prins (2014)[36] proposed a hybrid genetic algorithm with iterated local search and dynamic programming was presented for the classical MDVRP with unconstrained vehicle fleet.

In this paragraph literature deals with time windows (MDVRPTW). The first meta-heuristics reported in literature was the TS procedure of Cordeau et al. (2001)[37] in which the objective function is the minimization of the number of vehicles. Polacek, Hartl, Doerner, and Reimann (2005)[38] proposed a Variable Neighborhood Search (VNS) algorithm for the MDVRP with time windows and with fixed distribution of vehicles. This problem was also studied by Lim and Wang (2005)[39] with the characteristic of having exactly one vehicle at each depot.

Ting and Chen (2009)[40] presented a hybrid algorithm that combines multiple ant colony systems (ACS) and Simulated Annealing (SA). Zarandi, Hemmati, and Davari (2011)[41] presented a SA procedure to minimize routing cost, while Wang, Zhang, and Wang (2011)[42] coupled SA with a modified Variable Neighborhood Search algorithm, and a clustering algorithm is used to allocate customers in the initial solution construction phase.

The variants with split delivery (MDVRPSD) or with pickup & delivery (MDVRPPD) have been considered by very few authors in the scientific literature. The work of Wasner and Zapfel (2004)[43] presents an application to postal, parcel and piece goods service provider in Austria. The model employed is the MDVRPPD (MDVRP with pickup and deliveries) with the objective of determining the number and location of depots. Also, the client assignment problem is addressed. Schmid, Doerner, Hartl, and Salazar-González (2010)[44] studied a realistic case inspired from companies in the concrete industry, and presented a mixed integer linear program (MILP) and a Variable Neighborhood Search (VNS) procedure to minimize routing cost for the variant with split deliveries.

Mirabi, Fatemi Ghomi, and Jolai (2010)[45] addressed the problem of minimizing the delivery time of vehicles. They compared three hybrid heuristics, each one combining elements from both constructive heuristic search and improvement techniques. The improvement techniques are deterministic, stochastic and simulated annealing (SA) methods.

Subramanian, Uchoa, and Ochi (2013)[46] proposed a meta-heuristic procedure for the cases of the MDVRP and MDRVP with mixed pick up and deliveries. Their algorithm is based on iterated local search and exploits set partitioning models at certain stages of the procedure to obtain competitive solutions.
III. ANALYSIS WORK

The first publication on multi-depot vehicle routing problem appeared in 1984. Since there, more than 145 papers have been published up to 2014 in the research for the problem of MDVRP and its variants.

![Fig. 3 Distribution of paper published on MDVRP.](image)

Fig. 3 Represent the interest of researchers for this problem and its variants. With the slow start interest of researchers increases year by year and more rapid growth shows between 2006 and 2010 and between 2011 and 2014 with a total 52 and 51 publication. Giving an average of 10.4 papers per year in the period 2006–2010 and 12.75 paper per year in the period 2011–2014 inclusive.

![Fig. 4 Distribution of objective function](image)

Fig. 4 shows the distribution of paper based on the objective function. Commonly MDVRP both single objective and multiple objective. As seen in the previous review, majority of work has to be done on single objective function, 88% and only 12% work have to be done on multiple objective function.

![Fig. 5 distribution based on solution technique](image)

Fig. 5 shows the distribution based on solution technique for the MDVRP and its variants. MDVRP is, as is VRP, a NP hard combinatorial optimization problem and therefore it is difficult to find its optimal solution. While exact methods solve small problems quite efficiently, issues still exist for the larger problems or the special types of the VRP variants. On the other hand, meta-heuristic methods find solutions in less time.

Exact method includes branch and bound, mathematical programming are employed in 25% of reviewed papers.

33% of reviewed papers deals with the heuristics. Heuristic algorithm consist many different algorithm and ad-hoc methods. For the other 42% a meta-heuristic algorithm is proposed. Meta-heuristics consist different technique like Tabu Search (TS), genetic algorithms (GA), simulated annealing (SA), ant colony optimization and variable neighborhood search[49].

IV. CONCLUSION AND FUTURE WORK

In this review work, the MDVRP was studied because the number of depots is not limited to one in many real-world situations. In this paper, we deals with the MDVRP and also its variants.

Early works on MDVRP mainly proposed exact algorithms, which is quite efficiently. With the development of computers and the increasing popularity of meta-heuristics, researches then focused on proposing genetic algorithms, tabu search procedures, simulated annealing, ant colony optimization algorithms or even variable neighbourhood procedures to efficiently solve the problem.

In future we are dealing with the one of the variant of MDVRP, MDVRPSDP is a very important problem in practical applications. MDVRPSDP is an extension of VRPSDP, and it is more complicated than the VRPSDP considering that it needs to tackle customer assignment and the VRPSDP problem simultaneously. To the best of our knowledge, genetic algorithm (meta-heuristic technique) are not applied on MDVRPSDP. Our next paper we are dealing with MDVRPSDP solving by Genetic Algorithm.

REFERENCES


