Seismic Analysis of Transformer using Different Mode Combination Methods

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Abstract—A transformer is a major equipment of electrical power system. Its proper functioning is essential for uninterrupted power supply. Certain past observations have revealed that transformers are more vulnerable to earthquakes as compared to other power equipment. An in depth study of the behavior of transformer, its stressing, and deformation in critical parts is extremely essential. A 33kVA transformer was analyzed for seismic effects using the Response Spectrum Method and the design response acceleration curve. The finite element based software ANSYS was used to analyze the structure; using three different mode combination methods, i.e. SRSS, CQC and ROSENBLUETH. The main purpose of the study, being to compare the relative predictions of the three mode combination methods for the state of stress and component deformation in the transformer.

Key words: Transformer, Response Spectrum, ANSYS, SRSS, CQC, ROSENBLUETH

I. INTRODUCTION

A Transformer is a static device that transfers electrical energy from one circuit to another by electromagnetic induction without change in frequency [1]. Transformers play the lead role in transmission and distribution of electrical energy. They can be regarded as the prime components of any utility of industry.

A transformer design should inevitably consider the effect of earthquakes. Performance of transformers during past earthquakes has not been satisfactory and significant damage has been observed. As compared to other power equipment or machinery, transformers have proved to be more vulnerable to earthquakes because shaking of internal components of a power transformer has adverse effect on its long term performance as there are large gaps between the core-coil assembly and the tank and this increases the possibility of dislocation, deformation and rocking of the assembly during the event of an earthquake [2].

As an example, in the incident of San Fernando earthquake of 1971 the LADWP Slymer Plant affected in the earthquake, not just suffered failure of few transformers during the occurrence, but also lost many transformers in subsequent years.

Later in 1999 the ‘JIJI earthquake’ that hit central Taiwan, resulted in shutdown of electric power supply for about a week owing to the subsidence of transformer and electrical transmission towers [3].

II. THEORY

There are various mode combination methods used widely. The authors have adopted three methods for analysis which are briefed below:

A. Square Root Sum of Squares Method (SRSS):

In the square root sum of squares, the peak responses are evaluated using the following expression [4]:

\[ r_{\text{max}} = \sqrt{\sum_{i=1}^{n} r_i^2} \quad \text{... ... ... ...} \quad (1) \]

where:

\[ r_i = \text{response of } i^{\text{th}} \text{ mode} \]

\[ r_{\text{max}} = \text{maximum response} \]

It is known that this method yields poor results when the frequencies are closely distributed. Therefore SRSS method is used only when the frequencies are well separated.

B. Rosenblueth (ROSE):

Based on the application of random vibration theory, this method of mode combination uses a more practical approach. A correlation factor is used which takes into account the mode interactions. Hence the modal responses with presence of close spaced natural frequencies can be evaluated accurately. The relevant equation based on US Nuclear Regulatory Commission (NRC) Guide 1.92 Rev 2, July 2006, can be written as below [5]:

\[ r_{\text{max}} = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} r_{ij} \rho_{ij} \rho_{ij}} \quad \text{... ... ... ...} \quad (2) \]

where:

\[ r_i \& r_j = \text{peak responses for the } i^{\text{th}} \text{ and } j^{\text{th}} \text{ modes, respectively} \]

\[ \rho_{ij} = \text{correlation coefficient between the } i^{\text{th}} \text{ and } j^{\text{th}} \text{ modes} \]

\[ \rho_{ij} = \frac{1}{1 + \left( \frac{\omega_i - \omega_j}{\zeta_i \omega_i + \zeta_j \omega_j} \right)^2} \quad \text{... ... ... ...} \quad (3) \]

where:

\[ \omega_i \text{ and } \omega_j = \text{undamped natural frequencies for the } i^{\text{th}} \text{ and } j^{\text{th}} \text{ modes respectively} \]

\[ \omega_i' \text{ and } \omega_j' = \text{damped natural frequencies for the } i^{\text{th}} \text{ and } j^{\text{th}} \text{ modes, respectively} \]

\[ \zeta_i = \text{damping ratio for the } i^{\text{th}} \text{ mode} \]

\[ \zeta_i' = \text{modified damping ratio for the } i^{\text{th}} \text{ mode} \]

The relationships between modified and raw natural frequencies as well as modified and raw damping ratios are given as:

\[ \omega_i' = \omega_i \left( 1 - \zeta_i^2 \right)^{\frac{1}{2}} \quad \text{... ... ... ...} \quad (4) \]
\[ \zeta_i' = \zeta_i + \left(\frac{2}{t_0 \omega_i}\right) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots (5) \]

where:
\[ t_0 = \text{earthquake duration} \]

**C. Complete Quadratic Combination (CQC):**

This method which is similar to the ROSE method is again based on the application of random vibration theory. The basic CQC expression can be written as [4]:

\[ r_{\text{max}} = \sum_{i=1}^{n} \sum_{j=1}^{n} r_i \rho_{ij} r_j \quad \ldots \ldots \ldots (6) \]

where:
\[ r_i \text{ and } r_j = \text{peak responses for the } i^{\text{th}} \text{ and } j^{\text{th}} \text{ modes, respectively} \]
\[ \rho_{ij} = \text{correlation coefficient between the } i^{\text{th}} \text{ and } j^{\text{th}} \text{ modes} \]

When the damping ratios are different it can be shown that the correlation coefficient, \( \rho_{ij} \) is given as [6]:

\[ \rho_{ij} = \frac{8(\zeta_i \zeta_j)^{1/2}(\zeta_i + \beta \zeta_j)^{3/2}}{(1 - \beta^2)^2 + 4(\zeta_i \zeta_j^2 + \beta^2) + 4(\zeta_i^2 + \zeta_j^2)\beta^2} \quad \ldots \ldots \ldots (7) \]

where:
\[ \zeta_i \text{ and } \zeta_j = \text{damping ratios of the } i^{\text{th}} \text{ and } j^{\text{th}} \text{ modes, respectively} \]
\[ \beta = \frac{\omega_i}{\omega_j} \text{ with } \omega_1 \text{ and } \omega_i \text{ being the } j^{\text{th}} \text{ and } i^{\text{th}} \text{ eigen values, respectively} \]

When damping ratios for both the modes are same, the correlation coefficient can be shown to be [6]:

\[ \rho_{ij} = \frac{8\zeta^2(1 + \beta)\beta^{3/2}}{(1 - \beta^2)^2 + 4\zeta^2\beta(1 + \beta)^2} \quad \ldots \ldots \ldots (8) \]

where:
\[ \zeta = \text{modal damping ratio} \]

It is to be noted that while the CQC method assumes an infinite duration of white noise, the ROSE method assumes finite duration of white noise [7, 8].

**III. SEISMIC ANALYSIS OF TRANSFORMER**

**A. Solid Model:**

A 33kVA distribution transformer is analyzed by the response spectrum analysis method using ANSYS. ANSYS offers a comprehensive software suite that spans the entire range of physics, providing access to virtually any field of engineering simulation that a design process requires [9]. It is a Finite Element based software.

The entire seismic analysis simulation procedure can be broadly divided into four parts:

1) **Modeling**
2) **Meshing**
3) **Modal Analysis**
4) **Response Spectrum Analysis**

The 33kVA transformer was modeled in SOLIDWORKS as thirty eight separate components. The integrated solid model of the transformer is shown in Fig 1.

![Fig. 1: Solid Model of Transformer](image)

For conducting the analysis all the connections were assigned to behave as frictional joints with asymmetric nature. Further the components were meshed using tetrahedral elements.

ANSYS has two methods for generating Tetrahedron elements, i.e. “Patch conforming” and “Patch Independent” Methods. In this work the Patch Conforming Method was adopted. Patch conforming method essentially uses bottoms – up approach, where the surface meshing is done first followed by volume meshing [10].

The transformer was divided into 1,94,584 elements with a total of 2,88,276 nodes.

![Fig. 2: Meshed Solid Model of Transformer](image)

Before conducting the modal analysis the optimal pre – tension was assigned to all the bolts in the equipment. The pre – tension can be applied either by using the adjustment option and assigning the axial displacement or by assigning the compressive axial load to the cylindrical face of the bolt.

**B. Modal Analysis:**

The modal frequencies were computed for the first 50 modes. It was seen that the required mass participation ratio of more than 90% was achieved using these 50 modes, making it sufficiently accurate for the analysis.

Graphical plot of the eigenvalues as a function of mode number are presented in Fig 3.
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Fig. 3: Frequency versus Mode Number for the First Fifty Modes

The natural frequencies values for the lowest ten modes of the transformer are summarized in Table - I.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>45.761</td>
</tr>
<tr>
<td>2</td>
<td>78.917</td>
</tr>
<tr>
<td>3</td>
<td>103.52</td>
</tr>
<tr>
<td>4</td>
<td>133.19</td>
</tr>
<tr>
<td>5</td>
<td>154.05</td>
</tr>
<tr>
<td>6</td>
<td>169.31</td>
</tr>
<tr>
<td>7</td>
<td>194.33</td>
</tr>
<tr>
<td>8</td>
<td>327.59</td>
</tr>
<tr>
<td>9</td>
<td>420.55</td>
</tr>
<tr>
<td>10</td>
<td>496.6</td>
</tr>
</tbody>
</table>

Table 1: First ten Natural Frequency Values versus Mode Number

C. Response Spectrum Analysis:
The response spectrum analysis was done using all the three methods of mode combination using ANSYS. The Design Response Acceleration Curve for soft soil as given in IS 1893(Part4):2005 was used as input for the analysis.

Fig. 4: Design Response Acceleration Curve for Soil Type – II (IS 1893-2005)

IV. RESULTS AND DISCUSSIONS

The results of the response spectrum analysis using the three mode combination methods are presented in Table - II. The results depict the responses for the complete transformer assembly. It presents values of various stresses and the deformations at equivalent locations obtained using the three modal summation methods. The results predicted by the SRSS method for the equivalent stress, normal stress and shear stress in the complete transformer are presented graphically in Fig. 5. The X and Y direction deformations predicted by the SRSS method are shown in Fig.6. Equivalent stress maps are plotted in the transformer components such as bottom clamp, vertical plate and core assembly using the SRSS technique and are shown in Fig.7.

<table>
<thead>
<tr>
<th>Mode Combination</th>
<th>Maximum Stress ( MPa )</th>
<th>Location of Maximum Stress</th>
<th>Location of Minimum Stress</th>
<th>Maximum Deformation (mm)</th>
<th>Location of Maximum Deformation</th>
<th>Location of Minimum Deformation</th>
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<td></td>
<td></td>
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</tr>
<tr>
<td>SRSS</td>
<td>10.8</td>
<td>6.1</td>
<td>5.2</td>
<td>2.8</td>
<td>VPlate-6</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.23</td>
<td>Terminal-PVC</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Bottom Clamp-MS</td>
</tr>
<tr>
<td>CQC</td>
<td>11.5</td>
<td>6.05</td>
<td>4.8</td>
<td>3.0</td>
<td>VPlate-6</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.23</td>
<td>Terminal-PVC</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Bottom Clamp-MS</td>
</tr>
<tr>
<td>ROSE</td>
<td>11.5</td>
<td>6.05</td>
<td>4.8</td>
<td>3.0</td>
<td>Vplate-6</td>
<td></td>
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Table 2: Stress State and Deformation State Predictions
In Fig 8, bar graphs are presented for equivalent stress, normal stress (X), normal stress (Y), shear stress, deformation (X) and deformation (Y). It is seen that all the three mode summation techniques predict nearly identical values for various stress and deformation parameters computed. The maximum variance among the three methods for the stress parameters is computed to be 6.67% while that for the deformation parameters is computed as 0.6%. Further the predictions for the locations of the peak and minimum stresses and deformations are identical for all the three methods used.

Clearly the fact that the eigen modes for the transformer are uniformly distributed and well separated has played a significant role in ensuring that the predictions of the SRSS method are close to the predictions of the more rigorous CQC and ROSENBLUETH methods.

V. CONCLUSIONS

Based on the results obtained, the following conclusions are derived:

1) The eigenvalues of the transformer are found to be uniformly distributed in the frequency space and do not show clustering, enabling the use of SRSS method of mode summation without loss of accuracy.

2) Response spectrum analysis was conducted using the design response acceleration curve recommended in IS 1893(Part 4):2005 using the SRSS, CQC and ROSENBLUETH methods of mode summation.

3) All the three mode summation methods predict near identical responses in terms of various stress tensor elements, stress invariants, deformation vector elements, total deformation etc. The maximum variation between the predicted stress state was...
computed as 6.67% while that for the deformation state was computed as 0.6%. It may be noted however the SRSS predicts slightly lower response for equivalent stress in comparison to predictions of the CQC and ROSE methods. In contrast for the normal stress (X) the SRSS prediction is higher.

4) Further all three mode summation methods predicted identical locations for the peak and minimum values of the stresses. Maximum stresses are seen to occur in the internal components of the transformer such as vertical plates and the core assembly. It is pertinent also to point out that stresses in the windings are comparatively lower.

VI. ACKNOWLEDGMENT

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