

# In-Network Stabilizations of Dynamical Systems

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**Abstract**— Maintenance of wireless network system is the major problem now a days. computational methods in wireless networks degrade the performance and stability of the network. The problem of stabilizing a linear system over a wireless network using simple in-network computation method. Wireless Control Network is a simple network of nodes where each node maintains its position details and gets frequent updates from other nodes in linear combination of neighbor plant outputs. In WCN structural system have less computation overhead and beneficial scheduling with compositionality properties. This system proposes basic characteristics of topological conditions which stabilize the network. The coupling between plant's dynamics and nodes dynamic nature in network structure is analyzed by this system. To achieve this the system stabilizes the control inputs even if vertex connectivity of the network is very larger than the geometric multiplicity of unstable eigenvalues of the plant, this is analogous to typical min-cut condition that is required in traditional information distribution problem. Proposed system overcome this problem by topological conditions which imposes communication through plant's sensors and decentralized controllers at actuators in the network employed with traditional code on it.

**Key words:** Networked control systems, decentralized control, wireless sensor networks, structured systems, in-network control, network coding, cooperative control

## I. INTRODUCTION

Today's technologies in wireless communication, the sensors and actuators are available with updated features. Embedded computing along with multi-hop wireless networks standards with low cost is becoming a disruptive technology today. Wireless multi-hop mesh network replaces the earlier wired interconnection of plant sensors, controllers and actuators. This saves cost and space required to perform necessary computation on plants and provide more efficient and robust communication. Introducing feedback loops in wireless communication provides accurate and reliable routing but it gives several challenges to feedback control. Previously, information transmission in multi-hop network and routing should be schedule carefully which relays on the feedback decisions to reduce packets dropouts due to collisions between neighbor nodes. Stability of interconnected systems in multi-hop network is degraded due to collision and packet dropouts and the stability of interconnected system is not identified. Proposed system forecasts these problems in beginning stages and avoid it by including some dedicated controllers or state estimators in specified locations and observe closed loop interconnection of sensor-estimator and/or controller-actuator communication channels are unreliable.

The standard Architecture uses dedicated controllers for routing information along with one or more fixed paths to overcome end to end delay within the network. These delays are optimized by adding control loops to controllers. Flexible transmission is achieved by

scheduling computation in controller which regulates the communication at run time without affecting the actual system. Congestion complexity needs to be avoided by alternate routing approaches to carry information over the network. For this an effective routing approach defined with all possible routes within the wireless network. Wide use of wireless technology for communication and feedback process introduces delays because of multi-hop network and packet dropouts due to collision between neighbor nodes while transmitting information in the network. The goal of this system is to detect collisions and maintain stability of the system with necessary distributed algorithm for resource constrains node and perform collective control laws in the network.

## II. WIRELESS CONTROL NETWORK

WCN Architecture includes sensor, controller and actuator are interconnected through channels and control laws are defined to control the network. There is a problem with this system, how come the network itself acts as controller as well as stabilizer to control the plant communication by using distributed algorithm. All these things introduces a new setup of wireless nodes are deployed in proxy of plant in which few nodes directly access sensor outputs and some other few nodes are placed in listening range of plants actuators.

Proposed system develops a feasible approach to receive coefficients of each node connected in linear manner to stabilize the network by applying actuator. this scheme has low packet dropouts rate. This approach also benefits easy scheduling of route to carry the information within composable path and ability to handle geographically separated sensors and actuators of the network.

The decentralized fixed modes of a linear dynamical system are Eigen values of the plant that cannot be moved by static output feedback. WCN handles arbitrary feedback patterns and to enable a graph-theoretic analysis problem. WCN acts as a structured controller. There is no stabilizing configuration for the WCN when each node maintains a scalar state.

The set is connected to the network considers only the interconnections between the plant state variables, but does not consider the numerical values of those interconnections. This allows to characterize WCN properties that would guarantee stabilization of almost any plant having a certain structure. WCN stabilize both structured and numerically specified plants.

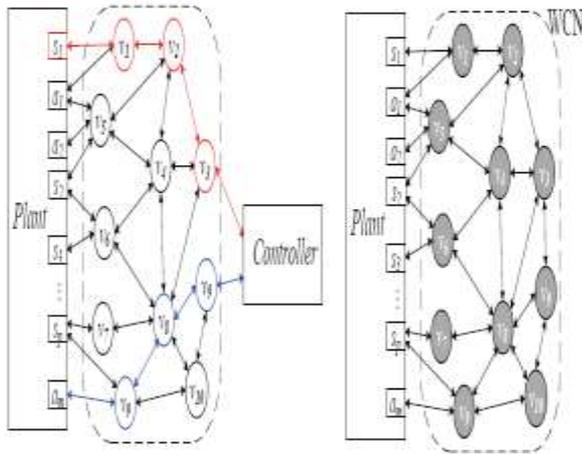


Fig. 1: Standrad architectures used for control over wireless network; A multi-hop wireless control, network acts as a distributed controller

### III. MULTIHOP WCN

Analysis initially disregard the effects of the actuators on the plant; i.e., by assumption that at each time-step the plant actuators do not use transmissions from the nodes in the set  $V_A$  to actuate the plant[5]. This allows to consider the plant  $\Sigma = (A,B,C)$  and the WCN together as a linear system  $\hat{\Sigma}$ , where the outputs of the plant are injected into the WCN. If the transmissions of the nodes in  $V_A$  as the output of the  $\hat{\Sigma}$  system can be specified as

$$\hat{x}[k+1] = \begin{bmatrix} x[k+1] \\ z[k+1] \end{bmatrix} = \underbrace{\begin{bmatrix} A & 0 \\ HC & W \end{bmatrix}}_{\hat{A}} \begin{bmatrix} x[k] \\ z[k] \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{\hat{B}} u[k].$$

$$\hat{y}[k] = \underbrace{\begin{bmatrix} 0 & E_{V_A} \end{bmatrix}}_{\hat{C}} \begin{bmatrix} x[k] \\ z[k] \end{bmatrix}.$$

Here,  $E_{V_A} = [e_{i1} \ e_{i2} \ \dots \ e_{it}]'$  selects the state values from the set  $V_A = \{v_{i1}, v_{i2}, \dots, v_{it}\}$ , where  $t = |V_A|$ . the vector  $\hat{Y}[k]$  contains the state transmitted by the wireless nodes closet to the actuators at time- step  $k$ .  $X$  is the set of state vertices corresponding to the states of the plant,  $U$  is the set of  $P$  input vertices corresponding to the actuators, and  $V$  is the set of vertices corresponding to the network nodes. The set  $E_A$  represents the edges between state vertices given by the matrix  $A$ , and  $E_B$  represents edges from the plant inputs to the states given by the matrix  $B$ . The set  $E$  represents the topology of the network, and the set  $E_O$  captures how the state vertices influence the vertices in the wireless network. Specifically, the states of the plant affect the outputs of the plant (via the edge set  $E_C$ ), and each plant output connects to one or more nodes (via the edge set  $E_{out}$  defined in [2]). The output vertices simply pass the information about the state vertices through to the wireless network, this can remove the output vertices from the representation and introduce

connections directly from the state vertices to the wireless vertices as follows:

$$E_o = \{ (x_i, v_j) \in X \times V_s \mid \exists v_k \in Y, (x_i, y_i) \in E_c, (y_k, v_j) \in E_{out} \}$$

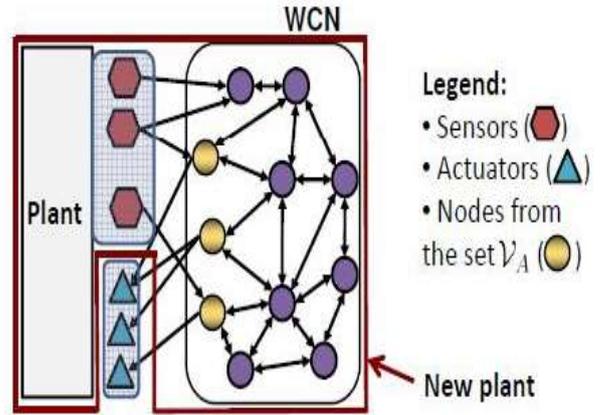


Fig 2: System Architecture

To model resource constrained nodes, assumed that each node is capable of maintaining only a limited internal state. Then distributed algorithm presented in the form of a linear iterative strategy for each node to follow, each node periodically updates its state to be a linear combination of the states of the nodes in its immediate neighborhoods. The actuators of the plant also apply linear combinations of the states of the nodes in their neighborhoods. Given a linear plant model and the network's topology. Devise a design-time procedure to derive the coefficients of the linear combinations for each node and actuator to apply in order to stabilize the plant. The method could also handle a sufficiently low rate of packet dropouts in the network to maintain stability. The computation of the control law is done in-network as in Wireless Control Network (WCN). The scheme has several benefits, including easy scheduling of wireless transmissions, compositional design, and the ability to handle geographically separated sensors and actuators. The use of the WCN in industrial process control applications is illustrated.

Advantage of the system architecture for the wireless network system model.

- (1) WCN in industrial process control applications
- (2) Less time should be taken and cost will be low saving place
- (3) Easy scheduling of wireless transmissions

A fixed mode will be introduced with Each WCN state vertex  $Z_i$  that does not belong to a strong component in

the graph  $G_{\hat{\Sigma}_{in}} = (V_{\hat{\Sigma}}, E_{\hat{\Sigma}} \cup E_{in})$  with an edge from  $E_{in}$  this might happen if the network is disconnected. By setting zero to all the weights associated with the links outgoing from  $Z_i$ , WCN state vertex is effectively removed from the network. In this system, due to the state vertex  $Z_i$  the system has a structured fixed mode in the origin. In both cases the closed-loop system does not have structured fixed-modes outside of zero, that almost every system with this structure will be stabilizable using the WCN.

Delays may be introduced if a multi-hop wireless network is used to route information. Transmissions in the network must be scheduled carefully information between the plant sensors, actuators and controllers. To avoid packet dropouts due to collisions between neighboring nodes. This can be detrimental to the goal of maintaining stability of the

closed loop system These works typically adopt the convention of having one or more dedicated controllers or state estimators located in the system and the stability of the closed loop system achieved by assuming that the sensor estimator and/or controller-actuator communication channels are unreliable (dropping packets with a certain probability) This standard architecture use the dedicated controllers imposes a routing requirement along one or more fixed paths through the network along with strict end-to-end delay constraints to ensure stability.

#### IV. EXTENSION OF PEER TO PEERS NETWORK

Wireless Control Network can be extended to control over networks with wired communication links. The problem of network synthesis for peer to peer system where network coding over point-to-point communication links is use. The goal is to provide topological conditions that guarantee that there exist linear dynamical controllers at the actuators that can stabilize the plant. In this case when the network delay over each link in the network is equal to the sampling period of the plant,when an idealized, delay-free network is used. It is worth noting that this scenario can be used to model closed-loop systems where the speed of the network is much higher than the sampling period of the plant.

$G_c = (V_c, E_c \cup Y_{1,p} \cup U_{1,m})$  is a network with point-to-point links, where  $Y_{1,p} = \bigcup_{i=1}^p Y_i$  represents the links coming into the network from the plant's sensors, and  $U_{1,m} = \bigcup_{j=1}^m U_j$  represents the set of links coming out of the network into the plant's actuators. As in standard linear network coding, the information sent on each outgoing edge from a given network node is a linear combination of information carried on the edges entering that node. That in the wired communication model, the linear combinations on each outgoing edge are allowed to be different. As in paper [12] the graph  $G_c$  can obtain the unique directed labeled line graph  $B = (V_B, E_B)$ , where  $V_B = E_c \cup Y_{1,p} \cup U_{1,m}$ , and for all  $e_i, e_j \in V_B$ ,  $(e_i, e_j) \in E_B$  if and only if there exist  $v_1, v_2, v_3 \in V_c$  such that  $e_i = (v_1, v_2)$  and  $e_j = (v_2, v_3)$  (i.e.,  $\text{head}(e_i) = \text{tail}(e_j)$ ). Each link  $(e_i, e_j) \in E_B$  is labeled With the coefficient (i.e., weight) assigned to the information received over edge  $e_i$  in the linear combination that is used to produce information over  $e_j$ . An illustration of this procedure where the labeled line graph is given for the network from that each link in the initial graph corresponds to a unique vertex in the labeled line graph.

If each link in the initial network introduces a fixed communication delay as in Time-Triggered networks, the labeled line graph directly corresponds to the WCN model. The matrices contain the gains between network links, between inputs and network links, and network links and the outputs. Which system is able to derive a stabilizing configuration for the corresponding WCN, the same configuration (i.e., the network coding parameters and parameters of the controllers) would guarantee stability when network coding is used in the initial point-to-point network  $G_c$ . Specify sufficient conditions for the WCN topology to ensure that such a configuration exists. These conditions require a sufficient vertex cut (i.e., linking) for the WCN topology. Since each vertex in the WCN corresponds to a specific edge in the initial network (and vice versa) that directly obtain sufficient topological

conditions for a network that uses network coding over point-to-point links.

Similar results can be obtained with delay-free communication networks, where the information injected in the network by the plant's sensors is expected to be instantaneously available at the actuators. In this case, as in [12], for the can define  $W$ —the adjacency matrix of the labeled graph  $w_{ij}$  is the weight assigned to the edge  $e_j$  in the linear combination used to derive  $e_j$  (if  $\text{head}(e_i) \neq \text{tail}(e_j)$  then  $w_{ij} = 0$ )<sup>9</sup>. Using the matrix  $W$ , as in [12] it can be shown that for any set  $I \subseteq M$ ,  $E_F (W-I)^{-1}H$  is the transfer matrix of the network, from the input edges from sensor to the output edges. This is equal to the WCN transfer function, evaluated at  $\lambda = 1$ , even for delay-free networks that use network coding over point-to-point links, specifies sufficient conditions for the existence of network coding parameters for which the plant can be stabilized via controllers at the actuators.

The conditions for a given system to not have structural fixed modes when controlled using a WCN, where each node in the network maintains only a scalar state, and the actuator nodes maintain vector states. Start analysis by initially disregarding the effects of the actuators on the plant; i.e., assume that at each time-step the plant actuators do not use transmissions from the nodes in the set  $V_A$  to actuate the plant. In decentralized control systems, a set of non-interacting local controllers is used to control a dynamical system (plant); each of the controllers generates the appropriate plant inputs by observing only a subset of the plant's outputs. Due to these limitations imposed on each of the local controllers, it is possible that even a controllable and observable system cannot be stabilized with the aforementioned setup. As shown in the problem of decentralized control can be formulated as a static output feedback control problem, where the feedback matrix potentially has some sparsity constraints. Furthermore, introduced the notion of fixed modes to derive conditions for the existence of a stabilizing set of decentralized controllers. The concept of fixed modes was generalized in to handle arbitrary feedback patterns, and to enable a graph-theoretic analysis of the problem.

#### V. CONCLUSION

Stabilizing a network is a critical problem with devices mobility and dynamic changes in infrastructure. The conventional approaches completely concentrate on routing mechanism to regulate transmission through controllers with some dynamic strategies to get accurate inputs to actuators. The proposed scheme regulates the communication in plants with end to end connection requests the neighbor nodes to transmit the information to nearest node in desired timestamp. Distributive algorithm maintains this by network dynamics. Along with this, topological condition stabilizes the network by auto correcting the nodes and actuators are arranged in linear passion with iterative strategy chosen appropriate weights to stabilize the plant network in dynamic manner rather than no of source nodes.

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