

# Artificial Bee Colony –Fuzzy Approach for Controlling Nonlinear System

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**Abstract**— The paper deals with the optimization of fuzzy membership function for nonlinear controlled system. The designed fuzzy controller was Takagi-Sugeno type with triangular membership function. The successful application of fuzzy control largely depends on some subjectively decided parameters, such as fuzzy membership functions. In this paper, we proposed to design and optimize the fuzzy membership functions by Artificial Bee Colony algorithm.. Here in this paper the CSTR model is considered to design a fuzzy controller.

**Key words:** Artificial Bee Colony; Fuzzy Logic Control; Membership Function; Takagi-Sugeno Fuzzy Approach

## I. INTRODUCTION

In Industrial applications most of the systems are nonlinear. A nonlinear system is one which contrasts to a linear system or which does not satisfy the superposition principle Nonlinear problems are very common in many applications so engineers and physicists are very much interested in them. So it is important to study behaviour of nonlinear system and control of nonlinear system. The most important consideration in nonlinear system is time delay. Time delay is the property of an actual system by which the response to an applied action is delayed in its effect. Time delay is very common and occurs. Time-delayed characteristics are frequently encountered in a variety of dynamic systems, such as chemical processes, various industrial processes, transmission lines, rolling mill systems and so on and can be cause of instability, the stability of control problem related with the time-delay systems have been studied in the past decades(1-3), There is less progress in nonlinear time-delay systems as compared with the linear time-delay systems. Fuzzy logic control is used to obtain nonlinear control systems, when there is incomplete knowledge of the plant. This paper deals with study of T-S Fuzzy controller (FLC) approach for nonlinear time delayed systems(4,5). T-S Fuzzy models with time delay are presented for continuous time systems with time delay. The design procedure aims for getting the stable fuzzy controllers for the nonlinear system which has time delay in it here fuzzy rules are optimized with the help of Artificial Bee Colony (ABC) approach. Usually, the main drawback of nonlinearity is that they have undesirable effects, and control systems are to be designed such that it will compensate for them.

Fuzzy Logic Control (FLC) is a powerful controller tool, even if the system is a non-linear or accurate mathematical model is unavailable. But, FLC have drawback of tuning its parameters (number of membership functions MFs type, formulating rules, etc.). The tuning of scaling factors for this parameter can be done either interactively method (trial and error) or human expert. Therefore, the tunings of the FLC parameters are necessitated to an effective method for tuning. Nowadays, several new intelligent optimization techniques have been emerge.

ABC optimization technique has lightened considerable attention among various modern optimization techniques (11). In this paper, optimized fuzzy controller using ABC is intended.

## II. NON-LINEAR TIME-DELAY SYSTEM

Ordinary differential equations can be written in the form of,

$$\dot{x} = f(t, x(t))$$

It has been a prevalent model description for dynamical systems. In this description, the variables  $x(t) \in R^n$  are known as the state variables, and the differential equations characterize the evolution of the state variables with respect to time. In other words, the value of the state variables  $x(t)$ ,  $t_0 < t < \infty$ , for any  $t_0$ , can be found once the initial condition  $x(t_0) = x_0$  is known (8). In practice, many systems cannot be satisfactorily modelled by an ordinary differential equation. For a particular class of many systems, the future evolution of the state variables  $x(t)$  not only depends on their current value  $x(t_0)$ , but also on their past values, say  $x(\xi)$ ,  $t_0 - r < \xi < t_0$ . Such a system is said to be a time-delay system (5,6).

## III. SYSTEM MODEL FOR FUZZY CONTROLLER

### A. Continuously Stirred Tank Reactor (CSTR):

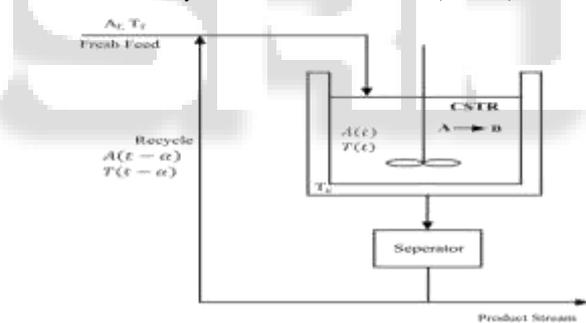


Fig. 1: Continuously Stirred Tank Reactor Model

The process, shown in Fig.1, consists of a reactor and a separator. The reaction is of the form  $A \rightarrow B$ , where A is considered as reactant and B is the product. This reaction is exothermic. The process has first ordered reaction rate given by

$$r = K_0 \exp \left[ \frac{-E}{RT(t)} \right] A(t) \quad (1)$$

Where,  $K_0$  pre-exponential constant and E is activation energy of the reaction. T Is temperature and  $A(t)$  is concentration of species A in the reactor.

A process can be represented from mass and energy balances equations. The two nonlinear differential difference equations are given below

$$V \frac{dA}{dt} = \lambda q A_0 + q(1 - \lambda) A(t - \alpha) - q A(t) - V K_0 \exp \left[ \frac{-E}{RT(t)} \right] A(t) \quad (2)$$

$$VC\rho \frac{dA}{dt} = qC\rho[\lambda T_0 + (1 - \lambda)T(t - \alpha) - T(t)] + V(-\Delta H)K_0 \exp\left[\frac{-E}{RT(t)}\right]A(t) - U(T(t) - T_w) \quad (3)$$

Where  $A(t) = \varphi_1(t)$  and  $T(t) = \varphi_2(t)$  for  $t \in [-\alpha, 0]$ ,  $A(t)$  is concentration of chemical A.  $T(t)$  is temperature and the remaining constants  $\alpha, \lambda, q, A_0, V, K_0, -E/R, C, \rho, (-\Delta H), U$ , and  $T_w$  are all positive and defined in Table 1. The constant  $\lambda$  is the recycle coefficient, which satisfies the conditions  $\lambda \in [0,1]$ . The objective is to control the concentration of A,  $A(t)$ . They can be reduced to dimensionless form using the notation

$$x_1(t) = \frac{A_0 - A(t)}{A_0}, \quad x_2(t) = \frac{T(t) - T_0}{T_0} \left( \frac{-E}{RT(t)} \right) \quad (4)$$

$$\theta_1(t) = \frac{A_0 - \varphi_1(t)}{A_0}, \quad \theta_2(t) = \frac{\varphi_2(t) - T_0}{T_0} \left( \frac{-E}{RT(t)} \right),$$

$$t_{\text{new}} = \frac{t}{v} = \frac{V}{q\lambda} \tau = \frac{\alpha}{v} \quad (5)$$

$$\gamma_0 = \frac{E}{RT(t)} \beta = \frac{Uv}{VC\rho} D_a = K_0 v \exp(-\gamma_0) \quad (6)$$

$$H = \frac{(-\Delta H)A_0 E}{C\rho T_0^2 R} u(t) = \frac{T_0 - T_w}{T_0} \left( \frac{-E}{RT(t)} \right) \quad (7)$$

$A(t)$	Chemical concentration of chemical specie A
$T(t)$	Reactor temperature
$\alpha$	Recycle delay time
$V$	Reactor volume
$\lambda$	Coefficient of recirculation
$q$	Feed flow rate
$A_0$	Feed concentration
$K_0$	Reaction velocity constant
$E/R$	Ratio of Arrhenius activation energy to the gas constant
$\rho$	Density
$C$	Specific heat
$-\Delta H$	Heat of reaction (positive)
$U$	Heat transfer coefficient times the surface area of reactor
$T_0$	Feed temperature
$T_w$	Average coolant temperature in reactor cooling coil

Table 1: CSTR Parameter

Then (5) and (6) in dimensionless variables become

$$\dot{x}_1(t) = f_1(x) + \left( \frac{1}{\lambda} - 1 \right) x_1(t - \tau) \quad (8)$$

$$\dot{x}_2(t) = f_2(x) + \left( \frac{1}{\lambda} - 1 \right) x_2(t - \tau) + \beta u \quad (9)$$

Where,

$$x(t) = [x_1(t) \ x_2(t)]^T \quad (10)$$

$$f_1(x) = \frac{-1}{\lambda} x_1(t) + D_a (1 - x_1(t)) \exp\left( \frac{x_2(t)}{1 + x_2(t)/\gamma_0} \right) \quad (11)$$

$$f_2(x) = -\left( \frac{1}{\lambda} + \beta \right) x_2(t) + H D_a (1 - x_1(t)) \exp\left( \frac{x_2(t)}{1 + x_2(t)/\gamma_0} \right) \quad (12)$$

$$x_i(t) = \theta_i(t) \text{ for } i = 1, 2$$

$$t \in [-\tau, 0].$$

The state  $x_1(t)$  is the conversion rate of the reaction  $0 \leq x_1(t) \leq 1$ ;  $x_2(t)$  is the dimensionless temperature. Clearly, we must restrict  $\lambda \neq 0$ . Constants  $H, \beta, D_a, \gamma_0$  and  $\tau$  are all positive. We assume that only the temperature can be measured on line, i.e.

$$y(t) = [0 \ 1]x(t) \quad (13)$$

the parameters are given as

$$\gamma_0 = 20; H = 8; D_a = 0.072; \lambda = 0.8; \tau = 2$$

### B. Phase Plane Analysis of CSTR Model:

Some practical control systems can indeed be adequately approximated as second-order systems, and the phase plane method can be used easily for their analysis. It is easy to find the three steady-states when  $u = 0$  are;

$$x^1 = (0.1440 \ 0.8862),$$

$$x^2 = (0.4472 \ 2.7520),$$

$$x^3 = (0.7646 \ 4.7052).$$

### C. Linearizing the System around Operating Point:

In this part of the paper, we will consider simple fuzzy control law designing for the system. For this we use the analytical approach. Consider the nonlinear system with time delay described by (8) and (9), which can be linearized around the stationary point given as,

$$\dot{x}(t) = A_1(x^s, u^s)(x(t) - x^s) + A_2(x(t - \tau) - x^s) + B(u - u^s) \quad (14)$$

Where,  $x_s = [x_1^s \ x_2^s]^T$

$$A_1(x^s, u^s) = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_2}{\partial x} \end{bmatrix}^T (x^s) \quad (15)$$

$$A_2(x^s, u^s) = \begin{bmatrix} \frac{1}{\lambda} - 1 & 0 \\ 0 & \frac{1}{\lambda} - 1 \end{bmatrix} \quad (16)$$

$$B(x^s, u^s) = \begin{bmatrix} 0 \\ \beta \end{bmatrix} \quad (17)$$

### D. Fuzzy State Feedback Controller Design

We consider the following fuzzy control law for the fuzzy model

IF  $\theta_1(t)$  is  $\mu_{i1}$  and ... and  $\theta_p(t)$  is  $\mu_{ip}$  THEN

$$u(t) = -F_i x(t), \quad i = 1, 2, \dots, r.$$

The overall state feedback fuzzy control law is represented by

$$u(t) = -\frac{\sum_{i=0}^r w_i(\theta(t)) F_i x(t)}{\sum_{i=1}^r w_i(\theta(t))}$$

### E. Fuzzy Rules

Rule 1: If  $x_2(t)$  is low (i.e.,  $x_2(t)$  is about 0.8862) THEN

$$\delta \dot{x}(t) = A_1^1 \delta x(t) + A_2^1 \delta x(t - \tau) + B^1 \delta u(t)$$

$$\delta u(t) = -F_1 \delta x(t)$$

Rule 2: If  $x_2(t)$  is Middle (i.e.,  $x_2(t)$  is about 2.7520) THEN

$$\delta \dot{x}(t) = A_1^2 \delta x(t) + A_2^2 \delta x(t - \tau) + B^2 \delta u(t)$$

$$\delta u(t) = -F_2 \delta x(t)$$

Rule 3: If  $x_2(t)$  is High (i.e.,  $x_2(t)$  is about 4.7052) THEN

$$\delta \dot{x}(t) = A_1^3 \delta x(t) + A_2^3 \delta x(t - \tau) + B^3 \delta u(t)$$

$$\delta u(t) = -F_3 \delta x(t)$$

Where,

$$\delta x(t) = x(t) - x_d, \delta x(t - \tau) = x(t - \tau) - x_d,$$

$$\delta u(t) = u(t) - u_d$$

And  $F_1, F_2, F_3$  are to be designed.

Where,

$$A_1^1 = \begin{bmatrix} -1.4274 & 0.0757 \\ -1.4189 & -0.9442 \end{bmatrix}$$

$$A_1^2 = \begin{bmatrix} -2.0508 & 0.3958 \\ -6.4066 & 1.6168 \end{bmatrix}$$

$$A_1^3 = \begin{bmatrix} -4.5279 & 0.3167 \\ -26.2228 & 0.9837 \end{bmatrix}$$

$$A_2^1 = A_2^2 = A_2^3 = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}$$

$$B^1 = B^2 = B^3 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix} \quad F_1 = [-8.4129 \quad 2.3021]$$

$$F_2 = [-3.5455 \quad 2.9766] \quad F_3 = [-18.2132 \quad 3.6272]$$

#### IV. ARTIFICIAL BEE COLONY OPTIMIZATION

The ABC algorithm is a swarm based, meta-heuristic algorithm based on the foraging behavior of honey bee colonies. The artificial bee colony contains three groups: scouts, onlooker bees and employed bees. The bee carrying out random search is known as scout. The bee which is going to the food source which is visited by it previously is employed bee. The bee waiting on the dance area is an onlooker bee. The onlooker bee with scout also called unemployed bee. The employed and unemployed bees search for the rich food sources around the hive. The employed bees store the food source information and share the information with onlooker bees. The number of food sources is equal to the number of employed bees and also equal to the number of onlooker bees. Employed bees whose solutions cannot be improved through a predetermined number of trials (that is “limit”) become scouts and their solutions are abandoned. (11)The number of food sources in ABC algorithm represents the number of solutions in the population. The position of a good food source indicates the position of a promising solution to the optimization problem and the quality of nectar of a food source represents the fitness cost of the associated solution.

##### A. Initialization phase

In the first step, the food sources, whose population size is SN, are randomly generated by scout bees. Each solution  $X_i$  is a Dimensional vector scouted by bees. Here, D is the number of optimization parameters. The following definition might be used for initialization purposes

$$X_i^j = X_{min}^j + rand[0,1] * (X_{max}^j - X_{min}^j) \quad (18)$$

Where  $X_{min}^j$  and  $X_{max}^j$  are the lower and upper bounds of parameter  $X_i$ , respectively. And  $i = 1, 2, \dots, SN$  and  $j = 1, 2, \dots, D$ .

##### B. Employed bee phase

After initialization, employed bee flies to a food source and finds a new food source within the neighbourhood of the food source. The higher quantity food source is memorized by the employed bees. The food source information stored by employed bee will be shared with onlooker bees. A neighbour food source  $V_{ij}$  is determined and calculated by the following equation

$$V_{ij} = X_{ij} + \varphi_{ij}(X_{ij} - X_{kj}) \quad (19)$$

Where  $k = \{1, 2, \dots, SN\}$  and  $j = \{1, 2, \dots, D\}$  are randomly chosen indexes. Although  $k$  is determined randomly, it has

to be different from  $i$ .  $\varphi_{ij}$  is a random number between  $-1$  and  $1$ . If the generated parameter  $V_{ij}$  value is out of the boundaries, it is shifted into the boundaries. After production of the new food source  $V_{ij}$ , its fitness is calculated and a greedy selection is applied between  $V_{ij}$  and  $X_{ij}$ . The fitness value of the solution  $fit_i(X_{ij})$  may be calculated using the following equation:

$$fit_i(X_{ij}) = \frac{1}{1+fit_i(X_{ij})} \quad \text{if } fit_i(X_{ij}) \geq 0$$

$$= 1 + \text{abs}(fit_i(X_{ij})) \quad \text{if } fit_i(X_{ij}) < 0 \quad (20)$$

Where,  $fit_i(X_{ij})$  is the objective function value of solution  $X_{ij}$ .

##### C. Onlooker bee phase

Onlooker bees calculates the profitability of food sources by observing the waggle dance in the dance area and then select a higher food source randomly. After that onlooker bees carry out randomly search in the neighbourhood of food source. The quantity of a food source is evaluated by its profitability and the profitability of all food sources.  $P_i$  is determined by the formula

$$P_i = \frac{fit_i}{\sum_{i=1}^2 fit_i} \quad (21)$$

Where  $fit_i$  is the fitness value of the solution, which is proportional to the nectar amount of the food source in position  $i$ , and SN is the number of food sources, which is equal to the number of employed bees.

##### D. Scout bee phase

The unemployed bees that choose their food sources randomly are called scouts. The abandoned food source is replaced with a new food source by the scouts. In the ABC algorithm, this is simulated by producing a position randomly and using it to replace the abandoned one. Providing that a position cannot be improved further through a predetermined number of cycles, then that food source is assumed to be abandoned (12, 13).

The main steps of the algorithm are as follows.

*Initialization phase:* Send the scouts to the initial foodsources

REPEAT

*Employed bee phase:* Send the employed bees to the food sources and determine their nectar amounts

*Onlooker bee phase:* Calculate the probability value of the sources that are preferred by the onlooker bees

*Scout bee phase:* Stop the exploitation process of the sources abandoned by the bees and send the scouts into the search area to discover new food sources randomly  
Memorize the best food source found so far  
UNTIL (Cycle = Maximum Cycle Number)

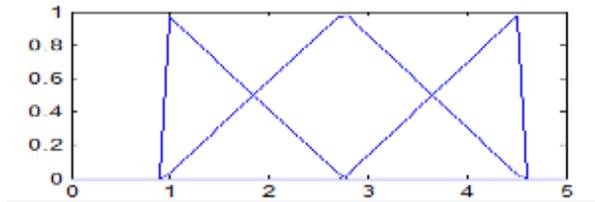
##### E. Optimization for triangular membership functions

The objective function of first triangular MF is  $Y = (2.7520 - X1) / (2.7520 - X2)$ . For this objective function we get  $X1=2.752$  and  $X2=0.893$  base values by ABC algorithm. The objective function of second triangular MF is  $Y = (X1 - 2.7520) / (X2 - 2.7520)$ . For this objective function we get  $X1=4.643$  and  $X2=2.752$  base values by ABC algorithm. Similarly we calculate the base values of third triangular MF [9]. We choose the control parameters of ABC are taken as per table 2.

Parameters	ABC
Colony Size	50
Maximum Cycle	500
Limit	20
Dimension	2

Table 2: Control Parameter of ABC

We can get the base values of first second and third membership function by ABC and we get overall membership function as follow in figure

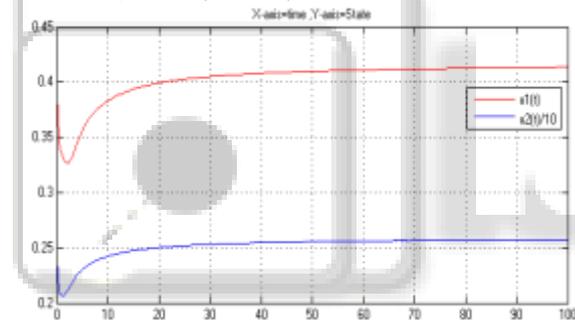


### V. SIMULATION AND RESULT

The SIMULINK model of the Continuously Stirred Tank Reactor (CSTR) is simulated in MATLAB /SIMULINK environment. The simulation results with constant time delay (5 second) using SIMULINK block diagram are given below.

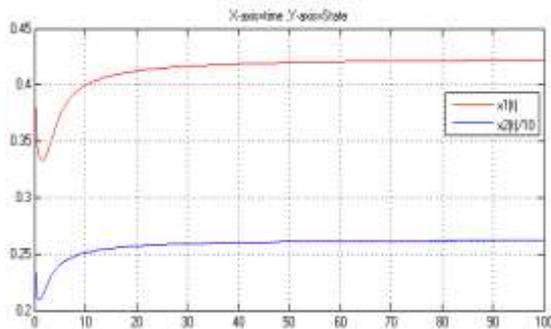
#### (1) Simple Fuzzy Controller

Desired operating Point  $x_d(t) = (0.4472, 2.7520)$ , Initial Conditions  $x_0(t) = (0.4, 2.5)$ .



#### (2) Fuzzy-ABC Controller

Desired operating Point  $x_d(t) = (0.4472, 2.7520)$ , Initial Conditions  $x_0(t) = (0.4, 2.5)$ .



Controller	Fuzzy	Fuzzy-ABC
X2(Temp.)	2.575	2.598

Table 3: Comparison of Results

### VI. CONCLUSION

In this paper, T-S fuzzy and Fuzzy-ABC approach is used to control nonlinear system. The continuous stirred tank system is developed with energy and mass balance

equations. The control performance is analyzed by using simulation of various controllers. At the end simulation fuzzy controller is compare with the Fuzzy-ABC controller. The simulation results confirmed Fuzzy-ABC has better result than traditional Fuzzy controller

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