A Study on Additive White Gaussian Noise Level Estimation in SVD Domain for Images

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Abstract— The precise estimation of Gaussian noise level in vision and image processing applications like Radar, medicine imaging, Biometrics etc are critical to the processing techniques that we follow in conventional algorithms. On the basis of study of singular values of noise corrupted images a new novel algorithm is proposed. The novelty of proposed algorithm is that its addresses major problem in noise estimation. a) The use of the tail of the of singular values for noise for noise estimation to alleviate the influence of the signal on the data basis for the noise estimation process. b) The addition of known noise to estimate the content-dependent parameter, so that the proposed scheme is adaptive to visual signals, thereby enabling a wider application scope of the proposed scheme.

Key words: Additive White Gaussian Noise, Noise Estimation, Singular Value Decomposition (SVD)

I. Introduction

Noise causes the random changes or variations in the brightness or color information in images and it is unavoidable during visual data acquisition (i.e., capturing of the real world images using digital camera), processing and transmission. Random noise sources mainly include various sensors and circuits of digital equipment (e.g., a scanner, digital camera or photon detector), signal quantization and communication channels. Denoising is one of the very important step to improve the accuracy or performance of many image processing techniques, such as image segmentation [12], [13] and recognition [14], [15].

Many denoising techniques are proposed in literature such as wavelets, bilateral filtering etc and results are promising in theoretical way but noise level estimation in practical way quite different and difficult to analysis. The noise level estimation is very useful in many computer vision and other image processing algorithms. The algorithms that use noise level estimation are denoising, super-resolution, shape-from-shading, and feature extraction. The Estimation of the Gaussian noise level present in a single image is a difficult task, because we need to decide whether local image variations are due to texture, color and llightlighting variations of the image itself, or due to the noise. In the image denoising literature, noise is often assumed to be zero-mean additive white Gaussian noise (AWGN). An observed noisy image $\mathrm{A}(\mathrm{i},\mathrm{j})$

Variations in lighting a noisy image $\mathrm{A}(\mathrm{i},\mathrm{j})$ is expressed as

$$\mathrm{A}\big((\mathrm{i},\mathrm{j}) = \mathrm{A}_0(\mathrm{i},\mathrm{j})\big) + \mathrm{N}(\mathrm{i},\mathrm{j}) \qquad (1)$$

Where $\mathrm{A}_0(\mathrm{i},\mathrm{j})$ represents the original image and $\mathrm{N}(\mathrm{i},\mathrm{j})$ represents the signal-independent noise. The amplitude of noise is of Gaussian distribution is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad (2)$$

Where μ represents the mean value and σ represents the noise standard deviation. For zero-mean AWGN (i.e., μ = 0), the key parameter to be determined is only σ.

There are two major challenges in estimating noise from a single image:

1) preparing a data basis required for estimating the noise level present in the image with minimum influence of the image signal itself (otherwise, we need to estimate noise level based upon signal data) and
2) Making the algorithm adaptive to visual content so that it is suitable for different images.

Very important property of the SVD is its statistical representations of image details in subspaces of decreasing importance, while the influence of noise remains same in all subspaces. Therefore the later SVD subspaces(i.e., tail of the singular values) can be used as the proper data basis for noise level estimation and the influence of image details do not have significant influence in that part of subspaces. Making use of the SVD technique we can separate the image details and noise in a single image and such separation is difficult otherwise.

In noise estimation using SVD technique we first divide the singular values S of a noisy image into two parts in SVD. One part is contributed by image structure(S_s) and other part contributed by noise (S_n). The contribution of image details is very little to the later part of the singular values, as S_s and on the other hand noise dominates the later part of the singular values, as S_n. When noise level *(σ)* become lower, the contribution of noise will be smaller but the tail of the singular values is still dominated by noise. The later 80% of singular values is the best range for estimating the noise, because this represents the data to which noise is the dominant factor.

II. SVD for Images and the Influence of AWGN

A. *Singular Values and Noise Levels*

The SVD is based on the theory in linear algebra with which a rectangular matrix A can be decomposed into the product of three matrices - an orthogonal matrix U, a diagonal matrix *S*, And the transpose of another orthogonal matrix V. The SVD of an m×n image A (assume r is the rank of A*)* can be written as:

$$A = U \times S \times \mathrm{V}^{\mathrm{T}} \qquad (3)$$

Where $U^TU = I_{mm}; V^TV = I_{nn};$ (I_{mm} and I_{nn} denote the m-square and n-square identity matrices); m and n represent the dimensions of *A*. The columns of U are orthonormal eigenvectors of AA^T, the columns of V are orthonormal eigenvectors of A^TA, and *S* is a diagonal matrix containing the square roots of eigenvalues of $\mathrm{AA^T}$ or $\mathrm{A^TA}$ arranged in the descending order. Let the singular values be denoted by$s(i)(i = 0,1, \dots\dots, r)$, and then $s(1 > s(2) > \cdots > \cdots > s(r)$.

To separate the contribution of image from that of noise, following the notations in the last section (Equation (1)), S_s and S_n are defined as the singular values due to the original image and the additive noise decomposed by singular vectors U and V:

$$S_s = U^{-1} \times A_0 \times (V^T)^{-1} = U^T \times A_0 \times V \quad (4)$$

$$S_n = U^{-1} \times N \times (V^T)^{-1} = U^T \times N \times V \quad (5)$$

Obviously $S = S_s + S_n$ or $S(i) = S_s(i) + S_n(i)$.

B. Analysis of AWGN

Let N be a m×n image with zero mean and standard deviation σ, and its SVD can be expressed as:

$$N = U \times S_n \times V^T \quad (6)$$

and

$$\sigma^2 = \sum_{i=1}^{r} S_n^2(i)$$

The parameter M is used to represent the number of the last singular values or tail of the singular values under consideration. Obviously, the average of the last M singular values is a function of σ, and can calculated as

$$P_M(\sigma) = \frac{1}{M} \sum_{i=r-M+1}^{r} S_n(i) \quad (7)$$

Where $1 \le M \le r$. When $M = 1$, only the last singular value (i.e., $S_n(r)$) is considered and when $M = r$, all singular values (i.e., $S_n(1)$ to $S_n(r)$) are considered. Where P_M denotes the average of the last M singular values or average of tail of the singular values

If P_M is linearly dependent on σ, two sufficient and necessary conditions must be satisfied:

$$\begin{cases} P_M(k\sigma) = k \times P_M(\sigma) \\ P_M(\sigma + \sigma_1) = P_M(\sigma) + P_M(\sigma_1) \end{cases} \quad (8)$$

Where σ represents the standard deviation of an additional noise N_1.

The P_M is almost linearly dependent on the noise level σ even if the process of AWGN changes:

$$P_M(\sigma) = \alpha\sigma \quad \text{when } M \gg 1 \quad (9)$$

Where α denotes the slope of the linear function which can be affected by the choice of M. If $M < r/4$, the randomness of AWGN will cause the value of P_M to be vibrated. In the proposed method for the AWGN images of size 512×512, we calculate P_M as the average of the 128th ~512th singular values. With change of the process of AWGN, the value of α changes slightly.

With the images of cartoons and real-world scenes, the average of the last M singular values P_M is calculated, and the corresponding relationship between P_M and σ become:

$$P_M = \sum_{i=r-M+1}^{r} S_i = \alpha\sigma + \beta \quad (10)$$

By comparing equations (9) and (10), we come to know that β is image content related. We can also separate P_M into two parts i.e. P_{M_s} and P_{M_n} for signal and noise respectively, and they are given by

$$P_{M_s} = \sum_{i=r-M+1}^{r} S_s(i) \quad (11)$$

$$P_{M_n} = \sum_{i=r-M+1}^{r} S_n(i) \quad (12)$$

Where P_{M_s} denotes the contribution of image structures to P_M, and P_{M_n} denotes the contribution of noise to P_M.

When we choose the value of M, We should not use a too large M because it causes the inclusion of the early part of singular values into the calculation for P_M and we have known that the image content dominates the early part of singular values. In estimating the noise levels the selection of the M value should not be greater then $\frac{4r}{5}$. On the other hand M should not be too small either since this would cause the data size for noise estimation too small and therefore affecting the estimation accuracy and reliability. Therefore M should be larger than or equal to $\frac{r}{4}$. In general the range of M should be $M\epsilon[\frac{r}{4}, \frac{4r}{5}]$.

III. NOISE ESTIMATION ALGORITHM USING SVD

The M value should be selected in range of $M\epsilon[\frac{r}{4}, \frac{4r}{5}]$, the slope parameter α is almost the same for images of the same size. That is for images of the same size, M is the only factor affect the value of α. we choose $M = 3r/4$; as we mentioned earlier, any choice in the range of $(\frac{r}{4}, \frac{4r}{5})$ is reasonable. For images of other size, we can determine α as following steps.

1) Calculate PM of pure AWGN images of the same size at different noise levels, we get a set of data like a row.
2) Calculate α by the least square fitting according to data acquired in the first step.

Assume that the noise deviation is σ in the input noise corrupted image. If we add known AWGN of σ_1 to the noise corrupted image, then it is easy to illustrate that the total noise σ_2 will still be AWGN with standard deviation of $\sqrt{\sigma^2 + \sigma_1^2}$. Assume that N is the AWGN sequence with standard deviation σ, and N_1 is the AWGN sequence that we added to the noisy image. For zero mean AWGN, the mean values are zero, i.e., $E(N) = 0$ and $E(N_1) = 0$

$$\sigma^2 = E[N - E(N)]^2 = EN^2 \quad (13)$$

$$\sigma_1^2 = E[N_1 - E(N_1)]^2 = E(N_1^2) \quad (14)$$

The variance of total noise becomes:

$$\begin{aligned} \sigma_2^2 &= E[(N + N_1) - E(N + N_1)]^2 \\ &= E(N^2) + E(N_1^2) + 2E(NN_1) \\ &= E(N^2 + E(N_1^2) \\ &= \sigma^2 + \sigma_1^2 \end{aligned} \quad (15)$$

From the above equations we can see that N and N_1 are not related, and this means that $E(N)N_1 = E(N)E(N_1) = 0$. Thus we can get Equation (4.9), and we can calculate the standard deviation σ_2 as:

$$\sigma_2 = \sqrt{\sigma^2 + \sigma_1^2} \quad (16)$$

We have two equations with two variable, β and σ from (10):

$$P_M = \alpha\sigma + \beta \quad (17)$$

$$P_{1M} = \alpha\sqrt{\sigma^2 + \sigma_1^2} + \beta \quad (18)$$

Since β is a content related parameter, it is derived by adding noise to the test image each time when we estimate the AGWN level. It is difficult to derive β based upon simulation with a group of images (because we would end up with the need to estimate the image content in order to determine β). Instead, we add known noise to the image under test every time to obtain two noising image versions of same content to derive β. So the above equation set is applicable to all kinds of images.

Solving the equation (17) and (18) we can figure out the value of σ

$$\hat{\sigma} = \frac{\alpha\sigma_1^2}{2(P_{1M} - P_M)} - \frac{P_{1M} - P_M}{2\alpha} \qquad (19)$$

Theoretically, σ_1 of the added AWGN can be arbitrary. In practice, the choice of σ_1 affects the precision of estimation. If σ_1 is too small, according to (19), the total noise level σ_2 will be too close to σ.

The proposed noise level estimation procedure for image A is therefore composed of 7 stages as follows:

1) Choosing a proper M value (the suggested M value is $r \times 3/4$), and calculate corresponding α.
2) Apply singular value decomposition to the noisy image A.
3) Calculate the average of the last M singular values P_M.
4) Add AWGN of σ_1 = 50 to noised image A to yield a new image A_1.
5) Perform singular value decomposition to the acquired image $A1$.
6) Calculate the average of the last M singular values P_{1M}.
7) Obtaining the estimated noise level by Formula (19).

IV. EXPERIMENTAL RESULTS

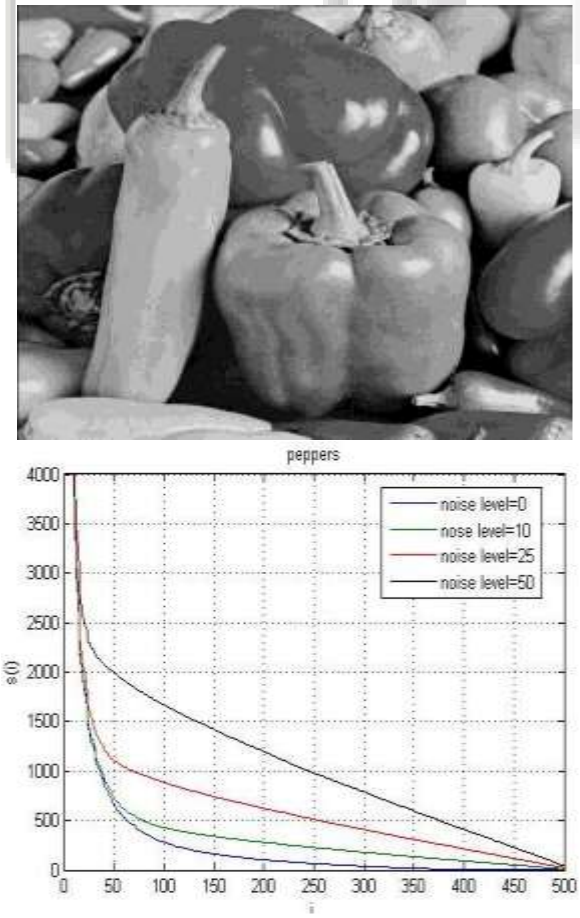


Fig. 1: Singular values of peppers (512×512)

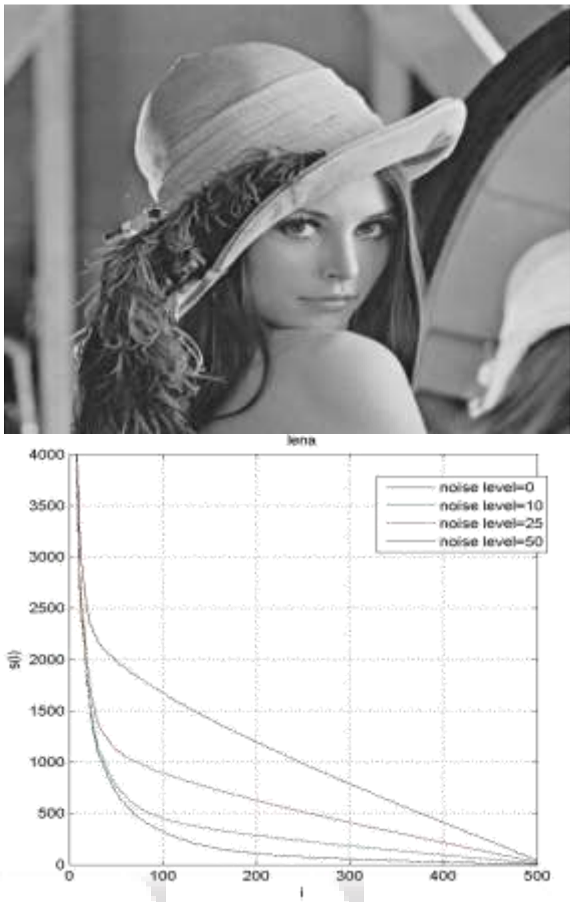


Fig. 2: Singular values of lean (512×512)

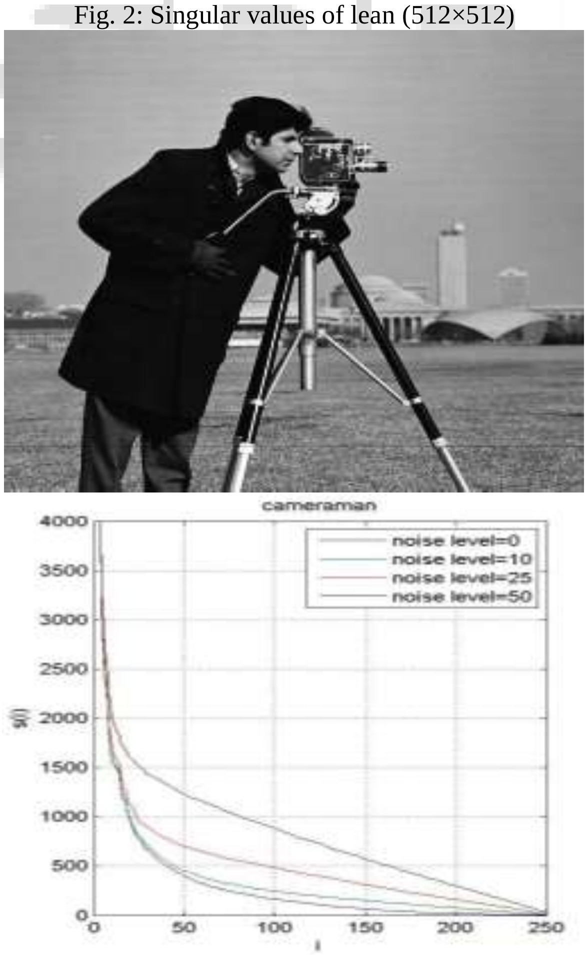
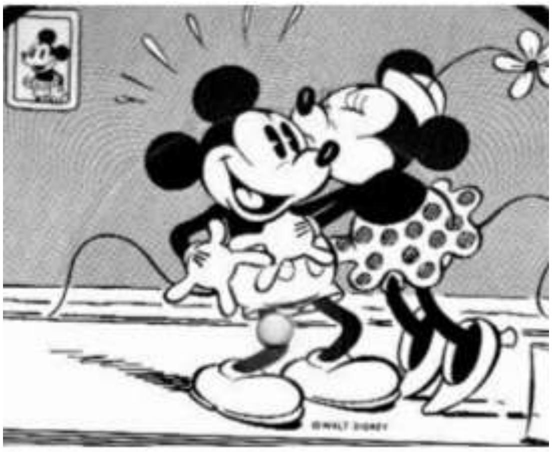


Fig. 3: Singular values of Cameraman (256×256)



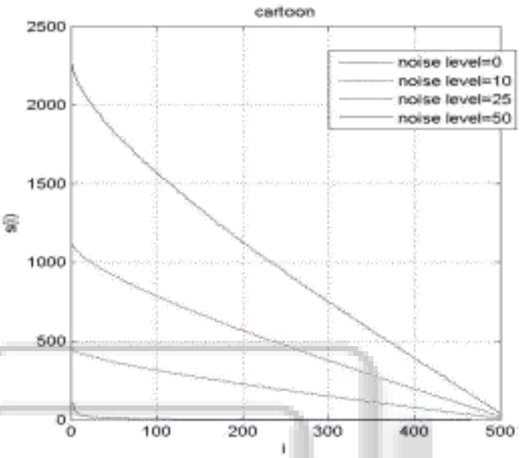


Fig. 4: Singular values of Cartoon (512×512)

The figure 1, 2, 3 and 4 shows Singular values of image peppers (512×512), Lena (512×512), cameraman (256×256) and cartoon (512×512) with different noise levels respectively. As we can see the addition of the noise to the images increases the image singular values. The X axis denotes the number of singular values and Y axis denotes the singular values. Higher the noise level is, the larger the singular values become.

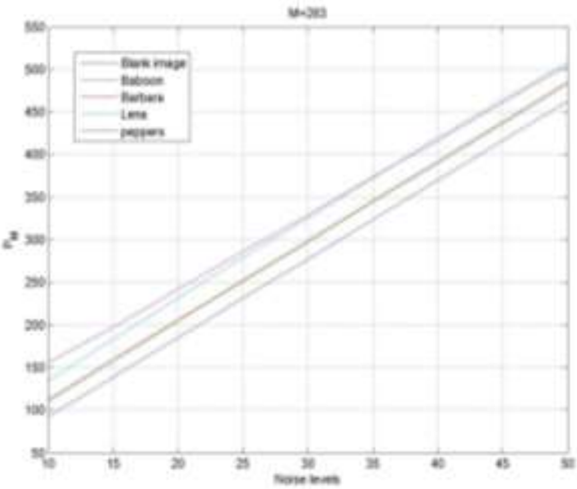


Fig. 5: Relationship between P_M and σ for 512×512 standard gray images

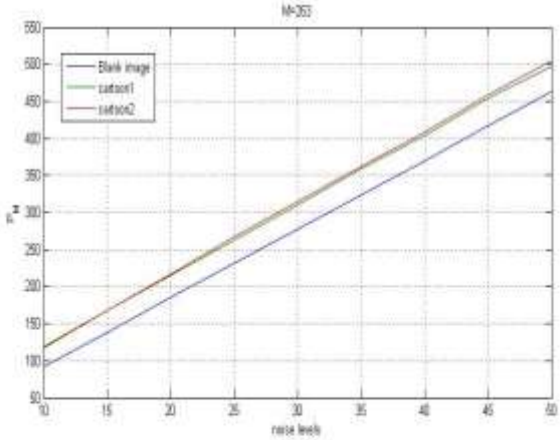


Fig. 6: Relationship between P_M and σ for 512×512 cartoon images

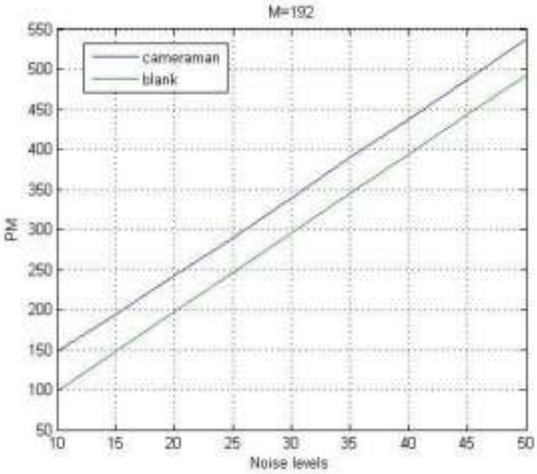


Fig. 7: Relationship between P_M and σ for 256×256 gray images

The figure 5 shows the experimental results from standard 512 × 512 gray scale test images. From the top to the bottom, the curves correspond to peppers, Baboon, Barbara, Lena and Blank image. The blank image is tested in order to find the relation of P_M and noise level without the influence of image content. The figure 6 shows the experimental results from 512×512 Cartoon2.We can see that the curve corresponding to Cartoon2 is also parallel to the curve of blank images of the same size. From the above figure we can see that Cartoon 1 and Cartoon 2 image are almost overlapping. The figure 7 illustrates the experimental results from 256×256 images. The bottom one is a 256× 256 blank image. We can see from Figure that the curves of the same size images are also linear and parallel.

From Fig. 5, 6 and 8 we can see that P_M is linearly dependent on the noise level under all these circumstances. Image details have little influence on slope α, but can increase β notably. In other words, image structures and details contribute to the value of β. Generally, for images with more details, the value of β is greater.

V. CONCLUSION

In the proposed method first we have shown how to infer the noise level according to the image singular values out of SVD, due to the fact that the image signal and noise can be separated well in SVD space. In addition, we have proposed to add new noise (and therefore, known noise) to images to be estimated, and analyze the change of singular values in order to determine the content related parameter in the model (so that the proposed method can be applied to any kind of images). Experiments results demonstrated that the proposed algorithm can determine noise levels better.

VI. FUTURE WORK

When noise level is low, the tail of singular values used for performing estimation may be dominated by both noise and image signal, especially for images of high activity. This is the reason for lower performance at low noise level .This will affect the estimation accuracy. One possible solution is to choose an adaptive length of the tail of singular values, so that all the chosen singular values are dominated by noise.

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