# Dimensionality Reduction of Images with Empirical Wavelet Transform and Householder Transform

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*Abstract*— Dimensionality reduction of an image is necessary for transmission and storage. Recently we are using Empirical Mode Decomposition and Empirical Wavelet Transform for the processing of non-stationary signal or image. Time – frequency transformation is necessary to analyze a nonstationary signal. This paper presents an application of the Empirical Wavelet Transform and Householder Transform method to reduce the dimensionality of an image for the reduction of cost and time while processing an image. Both the transform together will find out a minimum set of features that will give accurate classification result. The Empirical Wavelet Transform approach is used to decompose a signal into different harmonic modes (IMF) using appropriate wavelet filter banks accordingly the information presented in it. Spectral information can be taken by applying Empirical Wavelet Transform to each 2D image. Empirical Wavelet Transform along with Hilbert Transform gives good frequency decomposition. This will give better classification by providing better class seperability. Specifically, the EWT is applied to each band and the Householder transform is applied after applying EWT to all bands in order to get Orthogonal Features.

***Key words:*** Empirical Wavelets Transform, Hilbert Transform, Householder Transform

## I. INTRODUCTION

Image may be two dimensional or multi dimensional. They may be captured by optical devices such as cameras, telescope et.al. Sometimes we have to transfer these images from one point to another or we have to store the image. If the image size is less we can easily store and transfer the image. In this paper we are going to introduce a dimensionality reduction method. Dimensionality reduction methods require different signal analyzing methods. It is very important to analyze a signal to determine its characteristic such as frequency, phase et.al. Fourier Transform, Wavelet Transform et al, is used to analyze a signal. Usually Fourier transform is suitable for stationary signals because it cannot identify how the frequency is varying for a nonstationary signal. To overcome this short coming we adopted a new method that is called Short Time Fourier Transform (STFT). STFT uses a gliding window of certain length to determine the frequency variation with respect to time. But it has poorer frequency resolution. The Wavelet Transform (WT) provides the time frequency representation of the signal. But the disadvantages of this method are lack of directionality, oscillations, shift variance and aliasing.

The recent research on the adaptive technique developed new techniques called Empirical Mode Decomposition (EMD) and Empirical Wavelet Transform (EWT). The EMD proposed by Huang et al. [1]. EMD using two data in order to decompose the signal that are time and scale. It is highly efficient and suitable for both stationary and nonstationary signals but it does not have a theoretical base. EMD can detect small variation of frequency at any time that occurred in the signal.

EMD can act as an adaptive filter bank. From the Fourier point of view, a new method which is having theoretical base called Empirical Wavelet Transform (EWT). This method will extract sine or cosine components of the signal using filter banks. This method is proposed by Jerome Gilles [2].

The orthogonalisation of components in a vector will help to remove some unimportant data from the signal. The dimensionality reduction also aims to remove the unwanted information from an image. For that we are using an orthogonalisation method, Householder Transform.

In the paper below we present an improved method that can reduce the image dimensionality based on theoretical aspect. The purpose of this paper is therefore to contribute experimentally to a better understanding of the EWT method and Householder Transform for the dimensionality reduction of an image. EWT, using Hilbert Transform for the analytical signal. Hilbert Transform is proposed by the German Mathematician David Hilbert. EWT together with Hilbert Transform gives good frequency decomposition. This will reduce the difficulty to understand the signal [3]. Here the purpose is to apply Householder Transformation after applying the EWT to get more compressed result.

The sections of the paper are organized as follows. Section II has two distinct subsections, in section II-A, we recall some wavelet methods, in II-B, we recall the principle of the EMD algorithm; in III we discuss the EWT; In section IV, we are discussing the principles of Householder Transform. In section V the proposed empirical wavelets with Householder Transform is described. In section VI the experimental results are shown.

## II. EXISTING METHODS

### A. Wavelet Methods:

The Fourier Transform (FT) is one of the most commonly used signal processing tools in many applications. It transforms a signal in time domain to frequency domain. The FT cannot apply for signal, having frequency that varies with time. This limitation introduces STFT. The poorer frequency resolution of this method developed WT [8]. For analyzing a signal a set of wavelets are required. This is a more popular adaptive technique. Using WT we can analyze signal in terms of time and frequency. The WT is reversible process. So it is suitable in the field of image compression and decompression. Even though the wavelet packets are

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useful in many applications, they use a constant prescribed ratio in the subdivision scheme, which limits their adaptability [2].

*B. Empirical Mode Decomposition:*

This is a new method which overcomes the disadvantages of WT and it was proposed by Norden. E. Huang et al [1]. It is well work with both stationary and nonstationary signal. Decomposition of an image using EMD will produce Intrinsic Mode Function (IMF). An IMF is any harmonic function with the same number of extrema and zero crossings, whose envelopes are symmetric with respect to zero [6].

EMD uses an algorithm to extract the spatial information from an image called Sifting process. The process which is used to find out the IMF for ID signal is given below. Consider a signal x(t), first find out all maxima points form an upper envelop emax(t) using cubic spline line. Then find out all the minima's and form lower envelop emin(t). Then calculate the mean of the upper and lower envelop. After that, subtract the mean from the original signal. This is one iteration of the sifting process. Check whether the obtained result is an IMF and the stopping criterion for the checking process is for the envelope mean to be close to zero. If it is IMF then we have to find out the Residue. The EMD process is stopped when there is no extrema point in the residue. Reconstruction is possible by summing the IMFs. EMD is applicable for both 1D signal and 2D signal. The EMD algorithm is highly efficient and adaptable for all types of signals. However, it is very difficult to model mathematically.

## III. DIMENSIONALITY REDUCTION USING EMPIRICAL WAVELET TRANSFORM

For an on board preprocessing and hyper spectral imagery such as image interpretation, classification, and image transmission dimension reduction is essential. Two approaches feature selection and feature extractions are used to reduce the dimensionality of an image. If one method alone will result some compression. If we use two methods together it will result more compressed output. . Feature selection method finds a subset of the original variables as features. Feature extraction reduces an image dimensionality by transforming higher dimension to lower dimension. If we are using high dimensional data means that, it will result more cost like computational costs and equipment costs. Sometimes the multidimensional images contain only few features. So we are forced to do dimensionality reduction. Dimensionality reduction actually means a feature selection process in which we are selecting some orthogonal features for clustering process. Adaptive techniques like EMD and EWT gives the same result. But the short coming of EMD is that it does not have theoretical base. In order to overcome this we are using the method EWT. Through EWT we can process a signal according to some principle. This method will create a set of adaptive wavelets and the operation is equivalent to that of a set of band pass filters [2]. The EWT is defined in the same manner as that of wavelet transform. The main idea of this method is to extract the different wavelet tight frames or harmonic modes of a signal (IMF).This is shown in fig 1.

The property of an efficient compression algorithm is that reduced information rate for storage and transmission purposes. In this paper, EWT along with Householder Transform are used for compression. The segmentation of the Fourier spectrum is important because this will give the adaptability for the signal in our method. We propose to differentiate different parts of the Fourier spectrum using adaptive filter banks which gives harmonic modes with respect a frequency. Consider a normalized Fourier axis with a periodicity of 2πhere we considering only $\omega \in [0, \pi]$. In this project ,while applying EWT fix the number of segment as N, and we have to find out N +1 boundaries, 0 an π are already boundaries so we need to calculate N − 1 extra boundaries[2]. The procedure to find out the extra boundaries is explained [2].

1) Find out the local maxima in the Fourier spectrum and arrange them from lower order to higher order. Here no need to consider 0 and π. Suppose M is the no of maxima that present in the Fourier spectrum. Two cases can arise:
   – M ≥ N: the algorithm found enough maxima to define the needed number of segments, so we should keep only the first N −1 maxima,
   – M < N: the signal has less maxima than expected, then we keep all the detected maxima and reset N to the appropriate value.
1) Define the boundaries ωn of each segment as the center between two consecutive maxima.

The partitioning is shown in Fig.2.

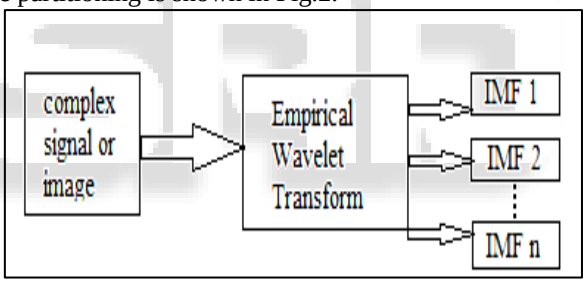


Fig. 1: Empirical Wavelet Transform

The Empirical Wavelet Transform is defined in the same way as for the classic wavelet transform. We can now define the Empirical Wavelet Transform (EWT), $Wf^0$ (n, t). The detail coefficients are given by the inner products between the signal f(t) and empirical wavelet $\psi_n$ (t-τ),

$$Wf^o(n,t) = \langle f, \psi_n \rangle = \int f(t)\, \overline{\psi_n (t-\tau)}\, dt \qquad (1)$$

And the approximation coefficients by the inner product with the scaling function:

$$Wf^e(0,t) = \langle f, \emptyset_1 \rangle = \overline{\int f(t)\, \emptyset_1 (t-\tau)}\, dt \qquad (2)$$

The reconstruction is obtained by

$$f(t) = Wf^e(0,t) \star \emptyset_1(t) + \sum Wf^o(n,t) = 1 \star \psi_n(t) \qquad (3)$$

After applying EWT we use Hilbert Transform inorder to get the strong analitical signal. The analytical form *u(t)* is represented as *X(t)* and is expressed as

$$X(t) = A(t)e^{j\omega(t)} \qquad (4)$$

A real signal *u(t)* and its Hilbert transform *Hu(t)* forms a strong analytic a signal that is given in "(4)".This will result both amplitude and frequency as a function of time. *A(t)* represents the varying amplitude and *ω(t)*

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represent the instantaneous phase. The Hilbert transform of a function or signal *u(t)* is defined using Cauchy principal value and is given by

$$H\,u(t) = p.v \int u(t)h(t-\tau)dt = p.v \int \frac{u(t)}{\tau - t}\,dt, \quad (5)$$

And we can use the *Hu(t)* to figure out the instantaneous frequency.

$$\text{Instantaneous Frequency} = (1/2\pi)\, d/dt\, \Theta, \quad (6)$$

$$\Theta = \tan^{-1}(X_H(t))/(u(t)) \quad (7)$$

$X_H(t)$ here represent to $H\,u(t)$ .

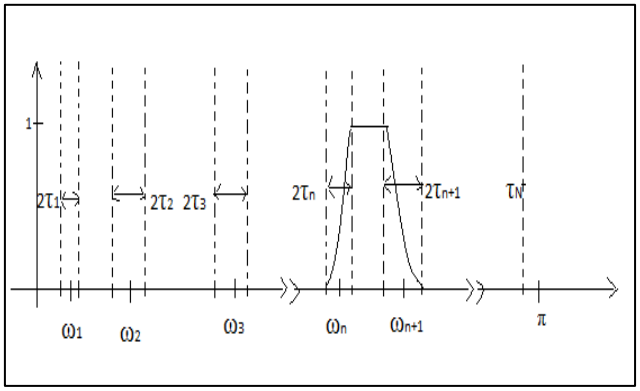


Fig. 2: Partioning of the Fourier Axis

## IV. HOUSEHOLDER TRANSFORM

The Gram-Schmidt orthogonalisation procedure is not generally recommended for numerical use. Gram-Schmidt orthogonalisation procedure sometimes results loss of orthogonality. This is not Good, while treating an image. We know the compression is more effective when the basis functions are orthogonal. After applying the EWT, the compression rate that may be less due to the non–orthogonality of the basis function. In order to increase the compression ratio or to reduce the dimensionality of an image here we are using a transformation called Householder Transformation. The Householder Transformation will transform the basis function into suitable orthogonal form. Thereby it will reduce the dimensionality of an image. Householder Transformations are orthogonal transformations that can be used to similar effect of the Gram-Schmidt orthogonalisation [10]. Householder transformation is nonlinear transformation. This is also known as Householder Reflection. It will find out the opposite component about a plain which contain the origin. Householder transformations are mostly used in linear algebra for the orthogonalisation of vectors and perform QR decompositions. The Householder transformation was introduced in 1958 by Alston Scott Householder [9]. The reflection hyper plane can be defined by a unit vector v (a vector with length 1) which is orthogonal to the hyper plane. The reflection of a point x about this hyperplane is:

$$x - 2\langle x,v\rangle v = x - 2v(v^H x) \quad (8)$$

Where v is given as a column unit vector with Hermitian transpose $v^H$. This is a linear transformation given by the Householder matrix:

$$P = I - 2\,v\,v^H \quad (9)$$

Where I is the identity matrix.

### A. *Algorithm:*

This method allows us to change a symmetric nxn matrix A = $(a_{ij})$ into a tridiagonal matrix. With the same set of Eigen values [9]. Let v be a column vector with $\|v\|_2 = 1$. The Householder transformation corresponding to the vector v is the orthogonal matrix

$$H = I_n - 2vv^t \quad (10)$$

1) Set k=1 and initialize B=A
2) Compute

$$s = \sqrt{\sum_{i=k+1}^{n} b_{ik}^2}. \quad (11)$$

If s=0, then set k=k+1 and recomputed s

3) Set:

$$SG = \begin{cases} -1 & \text{if } b_{k+1,k} < 0 \\ +1 & \text{if } b_{k+1,k} \geq 0 \end{cases} \quad (12)$$

4) Set:

$$z = \frac{1}{2}(1 + SG.b_{k+1,k}/s) \quad (13)$$

5) Set $v_i=0$ for i=1,2,.... k set $v_{k+1}=z^{1/2}$ and set

$$v_i = \frac{SG.b_{ki}}{2v_{k+1}s} \quad i = k+2, \cdots, n \quad (14)$$

6) Set $v=(v_1.v_2,....v_n)^t$ and let $H=I_n-2vv^t$.
7) Compute
A=HBH. (15)
8) If k=n-2, then output A and stop.

After getting the IMF from the given data using EWT, by applying Householder transform we can orthogonalise the components present in it.

## V. PROPOSED METHOD

Empirical Wavelet Transform is a method to decompose a signal into different modes. The modes are having slightly varying amplitude and phase. In this form we can't interpret the properties of signal. In order to interpret the properties of an analytical signal we need Hilbert Transform. EWT is designed for nonstationary signal, so it will produce a result which will suit for the analytical purpose. Consider a signal u(t) and it is splitted into different IMF through EWT method describe in the section III. From the obtained IMF we can find out the analytical signal which is in a complex form, using Hilbert Transform. This will result a simultaneous understanding of both the time and frequency. EWT has been used to obtain the IMFs of hyper spectral image bands. New features have been reconstructed by adding the lower order IMFs of all bands. In this paper, we propose a dimensionality reduction method which uses both EWT and Householder Transform. Wavelet based IMF features (WIMF) are introduced to get better classification accuracy. The method involves applying EWT to the signal, corresponding IMF's are obtained. Then we have to group the same scale of IMF's of all bands. The obtained IMF's stacks are subjected to apply Householder Transform. This

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will estimate the number of orthogonal components. The EWT is applied to the signal to obtain the WIMF. Householder Transformation is used to take the orthogonal features. Together the feature vector is determined. After that using SVM Classifier we are going to classify the tested data into two categories such as.

High frequency and lower frequency according to the data base already computed. Specifically, the proposed method is described as follows; here we are going to apply EWT on a 2D image. For that consider the matrix representation of the signal. The matrix consists of columns and rows which preserve the image aspect ratio.

*A. Algorithm:*

1) EWT is applied to each hyper spectral image band and M orders of IMFs are obtained,
   - Apply Fourier Transform to each of the row and column to get the Fourier Spectrum and smoothened.
   - Find out the peaks from the spectrum in row wise and column wise.
   - And compute the midpoint between two consecutive maxima. The mid points act as boundaries for the segments. This process is equivalent to Band Pass Filtering.
   - Then use a window say Tukey window to get segments in the form a rectangular window. (Above 4 steps together called EWT).
   - The Inverse Fourier Transform can be obtained by using the adjoint formulation (e.g applying the inverse 2D EWT with respect to the columns first and then with respect to the rows).
   - Apply Hilbert Transform, in "(5)", to calculate analytic signal.
2) Householder Transform for each IMF feature in all bands $IMF_l$,m(i,j),
3) Obtain M order of IMF features as a result of step 2, WIMFm, m = 1 : M,
4) Generate new wavelet based IMF features WIMF(1 : M) by summing up lower order WIMFm

$$WIMF1..M(i,j) = \sum_{m=1}^{M} WIMFm(i,j) \qquad (16)$$

5) Select R features from new wavelet based IMF features WIMF (1 : M).
6) Classify R features using an SVM classifier. The flow chart for the proposed method is shown in Fig. 3

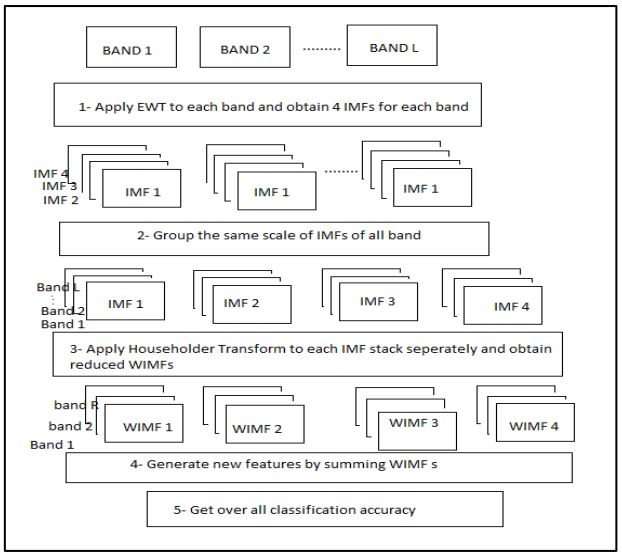



Fig. 3: Flow Chart of Proposed Method

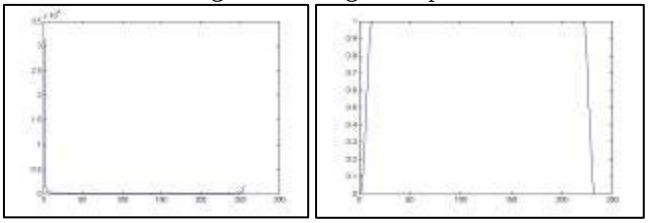
## VI. EXPERIMENTAL RESULT AND ANALYSIS

We proposed to test the EWT and Householder Transform on 2D image. In this paper, we tested different 2D images and their PSNR and CR are determined, the TABLE.I shows the details of different images. From the table it is clear that for every image the PSNR is lies between 40 to 50, and the CR is changing depending upon the image. We selected one image and the plots are shown below. It is experienced that the described method give better decomposition like EMD along with a perfect theoretical concept,and it give a analytical form a signal. Results are shown Fig.4-7.

| Test Images | PSNR(Peak Signal To Noise Ratio) | Compression Ratio |
|---|---|---|
| 1 | 42.7037 | 1.6766 |
| 2 | 43.0240 | 3.2026 |
| 3 | 43.0923 | 6.8163 |
| 4 | 42.4138 | 3.9761 |
| 5 | 42.5722 | 1.8957 |
| 6 | 40.8333 | 2.0063 |
| 7 | 42.1640 | 2.1934 |
| 8 | 41.2813 | 3.3749 |

Table 1: PSNR and CR of Test Images

Fig. 4: 2D Image as Input



(a) (b)

Fig. 5: (a) Fourier spectrum of the Input Image (b) Tukey window for the purpose of segmentation or filtering.

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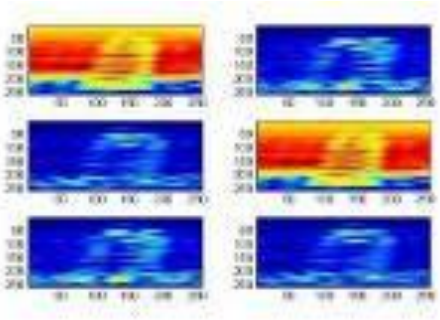
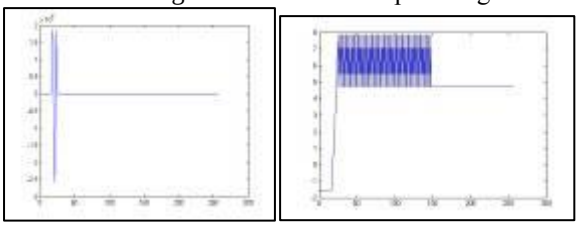


Fig. 6: IMF's Of the Input Image



(a) (b)

Fig. 7: Analytical Plot of the Image A) Magnitude B) Phase of the Real Part

| Test Image | Image Without Orthogonalisation | | Image with Householder Transformations | |
|---|---|---|---|---|
| 1 | 42.7037 | 1.6766 | 42.7037 | 2.2299 |
| 2 | 43.0240 | 3.2026 | 43.0240 | 4.2595 |
| 3 | 43.0923 | 6.8163 | 43.0923 | 9.0656 |
| 4 | 42.4138 | 3.9761 | 42.4138 | 5.2882 |
| 5 | 42.5722 | 1.8957 | 42.5722 | 2.5212 |
| 6 | 40.8333 | 2.0063 | 40.8333 | 2.6684 |
| 7 | 42.1640 | 2.1934 | 42.1640 | 2.9172 |
| 8 | 41.2813 | 3.3749 | 41.2813 | 4.4886 |

Table 2: PSNR and CR of Images With and Without Householder Transformation

After applying the EWT the CR and PSNR of the input image is given below

Compression Ratio= 1.6766

Peak Signal to Noise Ratio= 42.7037

Here the components in the images are not orthogonal. In order to orthogonalise each vector we are going to apply Householder Transformation.

*A. Experiment using Householder Transformation:*

After applying EWT we applied Householder Transform to the extracted IMF. The CR and PSNR for the same image shown in fig 4 are

Compression Ratio= 2.2299

Peak Signal to Noise Ratio= 42.7037

From TABLE II. It is very clear that after the orthogonalisation, the compression ratio will increase. So in order to achieve more compression orthogonalisation is required. The original image can be reconstructed from the orthogonalised vectors i.e. by adding all corresponding IMF's and the final residue and is given in Fig.8

Fig. 8: Reconstructed Output after Orthogonalisation

To classify the images two different categories are used. One is high frequency signal and another one is low frequency signal. Support machine vector (SVM) is very important tool in the case of classification, pattern reorganization and regression analysis. Support Vector Machines are based on the concept of decision planes that define decision boundaries[11]. A decision plane called hyperplane is one that separates between a set of objects having different class memberships. The classified result is shown in Fig.9.

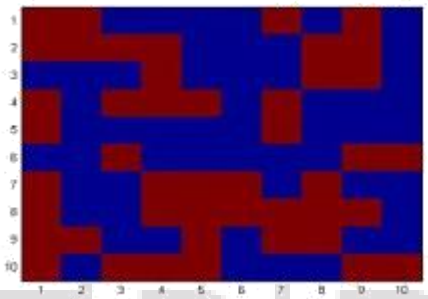


Fig. 9: Low Frequency and High Frequency Signal In An Image Low Frequency Signal High Frequency Signal

The proposed method combines the advantage of both EWT and Householder Transform in reducing the dimensionality of an image because it incorporates both spatial information and spectral information. The advantages of using IMF's feature stems from other ability to represent further detailed formation and more features of the image. It can be seen that for most of the images, the proposed method is better than using only EWT and Householder Transformation, and the original data.

## VII. CONCLUSION

This paper described an application of the 2D-EWT and Householder transform for the dimensionality reduction images in order to get enhanced classification accuracy. Our proposed method is based on theoretical aspects as compared to EMD. The main concept is to design a set of band pass filtering called wavelet filter bank; with the help of filter bank we extracted the information present in the spectrum. For more compressed result we applied Householder Transform on each IMF. This results more reduced dimensionality of an image. The compression ratio and peak signal to noise ratio also calculated for different type of images. Experiments are carried out on 2D images.

## VIII. FUTURE WORK

We proposed a method to compress 2D images using EWT and Householder Transform. Future work will consider the

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application of the proposed method in conjunction with multidimensional wavelet transform for the classification of multi-dimensional Image.

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