Review on Bending Solution of Thin Plates

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Abstract—Thin-Walled structures in the form of thin-plates and shells are important in many branches of technology like mechanical, aeronautical, civil, marine, chemical engineering and also a wide-spread use of thin plate arises due to its intrinsic characteristics. In this review paper various shapes and methods for bending solution of either numerical or analytical are discussed.

Key words: Bending, Plates, Thin, Analytical, Numerical, ANSYS

I. INTRODUCTION

Thin plates are initially flat structural members bounded by two parallel planes called faces and a cylindrical surface called an edge or boundary. The generators of the cylindrical surface are perpendicular to the plane faces. It will be assumed that the plate thickness is small compared with other characteristic dimensions of the faces (length, width, diameter, etc.). Geometrically, plates are bounded either by straight or curved boundaries. The static or dynamic loads carried by plates are predominantly perpendicular to the plate faces. Thin plate bending is widely utilized in the fields of aerospace, mechanics and civil engineering. These kinds of plates are formulated based on Kirchhoff's theory. In this theory, the shear deformation of the element is not considered [1].

A. Applications:

Because of the numerous advantages of thin plates. They are extensively used in all fields of engineering. Plates are used in architectural structures, bridges, hydraulic structures, pavements, containers, airplanes, missiles, ships, instruments, machine parts, etc (Fig1)[2].

B. Various Boundary Conditions:

Thin plates have free, simply supported and fixed boundary conditions, including elastic supports and elastic restraints or in some cases, even point supports (Fig.2). The loads carried by Thin-plates are perpendicular to the plate surface.

II. SHAPES OF PLATES

There are different shape of plates i.e. sector, elliptical, square, triangle, square with hole, circular with hole listed below are the plates and their approach to solve the bending problem.

A. Square Plate with Different Boundary Conditions:

Bending solutions of rectangular cantilever thin plates analytically. Using only the basic governing equations of the plates are used and there are no predetermined functions. By using the double integral transform of higher-order partial derivatives of W. he derived the double integral transform of the load function q(x, y). This mathematical approach employed does not require the pre-selection of a deformation function, which can scarcely be avoided in the traditional semi-inverse approaches [3]. Two higher order hybrid-Trefftz elements for thin plate bending problems and a triangular and a quadrilateral element are formulated based on the hybrid-Trefftz method. The triangular element is
named as THT -15, and it has 6 nodes and 18 degrees of freedom. The quadrilateral element is composed of 8 nodes and 24 degrees of freedom, and it is denoted as QHT-23. Two independent fields are introduced: one within the element and the other on the edges of the element. The internal field satisfies the governing equation of the thin plates. The boundary field is related to the nodal degrees of freedom by shape functions. For better capability, the shape functions of a 3 node Euler-Bernoulli beam are used for each edge of the element. The order of these functions for the deflection, rotation and torsion fields is equal to five, four and two, respectively. By depicting these fields in general coordinates, x−y, shape functions for the element’s edges are obtained. Several numerical tests are performed to assess the robustness of the suggested elements. The findings demonstrate the high accuracy of the proposed elements in analysing the thin bending plates and in the conclusion it should be added that the QHT-23 element is more accurate than the triangular one [4].

A wavelet-based stochastic finite element method is applied for bending analysis of thin plates. This is numerical method which gives approximate solution of the plate deflection. This wavelet theory was based on the notion that any signal function can be broken down a series of local basis functions called wavelet. Bending of square thin plates by using the developed spine wavelet thin plate element formulation and bending moments and central deflection are analyzed for simply supported and fixed supported. The method can achieve a high numerical accuracy and is very fast converging in solving the stochastic problem of thin plate bending [5]. A fast multi-pole boundary element method for solving the thin plate bending problem. The boundary element method (BEM) has been applied successfully to solve the thin plate bending problem since the late of 1970s and early 1980s. Many researchers derived the direct boundary integral equation (BIE) formulations for both linear and nonlinear responses. This method is also employ to the square plate with four holes [6].

The symplectic superposition method is developed to obtain the analytical bending solutions of clamped rectangular thin plates resting on elastic foundations. The solution approach reveals several advantages in handling the engineering problems such as the bending of rectangular plates on elastic foundations. Firstly, the symplectic superposition method provides a totally rational way to analytical solutions, which starts from the basic elasticity equations of the problem and proceeds without any preset solutions. The second advantage is that the method gives us a systematic solution procedure, which can be applied to other plate bending problems with classical edge support conditions [7]. Exact bending solutions of fully clamped orthotropic rectangular thin plates subjected to arbitrary loads are derived using the finite integral transform method. The integral transform has been used to obtain exact solutions of specific partial differential equations in the theory of elasticity, while it was employed in the analysis of certain engineering problems. In this mathematical method one does not need to predetermine the deformation function because only the basic governing equations of the classical plate theory for orthotropic plates are used in the procedure. Therefore, unlike conventional semi-inverse methods, it serves as a completely rational and accurate model in plate bending analysis. The applicability of the method is extensive, and it can handle plates with different loadings in a uniform procedure, which is simpler than previous methods. Which have been treated via semi-inverse or approximate methods [8].

A novel superposition method based on the symplectic geometry approach is presented for exact bending analysis of rectangular cantilever thin plates. The basic equations for rectangular thin plate are first transferred into Hamilton canonical equations. By the symplectic geometry method, the analytic solutions to some problems for plates with simply supported edges are derived. Then the exact bending solutions of rectangular cantilever thin plates are obtained using the method of superposition. The symplectic superposition method developed in this paper is completely rational compared with the conventional analytical ones because the predetermination of deflection functions, which is indispensable in existing methods, is dispelled. The method serves as a completely rational model in the analysis of plate problems, which have to be handled by the semi-inverse method or numerical methods [9]. The benchmark bending solutions of rectangular thin plates with a corner point-supported are obtained by an up-to-date symplectic superposition method within the framework of the Hamiltonian system. The developed method offers a rational way to obtain the solutions of corner point supported thin plates with sufficient accuracy. Appropriate extension of the method can also yield more benchmark solutions of the similar problems. In this paper bending solutions of a uniformly loaded square plate with a corner point-supported and both or one of its opposite edges clamped or simply supported are done using are obtained by the Hamiltonian system-based symplectic superposition method. The derivation is rigorous within the mathematical framework and no prior assumption of the deflection is employed, which demonstrates the advantages of the methods [10]. A theory is developed which includes transverse shear and direct stress effects, and solutions to this theories obtained using finite difference method and localized Ritz method and its application to sandwich plates is also done and results are obtained for case of practical shear stiffness to bending stiffness ratios [11].

A rectangular finite element based on the strain approach for plate bending .This new strain based rectangular plate element (SBRP) was then compared with the other plate elements such as DKT, DSTM, SBH8 and other type of elements for cantilever plate with edge moment and edge shear and found that SBRP convergence rate is very rapid, and free from shear locking and can be applied to thick and thin plates [12]. Analytical solution of a two variable refined plate theory for bending analysis of orthotropic Levy-type plates. The analytical solutions of refined plate theory for bending analysis of rectangular plates with various boundary conditions. The closed-form solutions of deflection and stress are obtained for rectangular plates with various boundary conditions by applying the state space concept to Levy-type approach. The obtained results are compared with those reported in the literature. The effects of thickness ratio, modulus ratio, and aspect ratio on deflection and stress of orthotropic plates are studied. Firstly by formulation the problem in terms of the
kinematics, Constitutive equations, and governing equations then applied Ney type solution procedure with clamped, simply supported and free edges [13, 14].

B. Circular Plate:
The theoretical and experimental study of a sandwich circular plate under pure bending equation of equilibrium was presented of the sandwich circular plate. Then deflection and stiffness of sandwich circular plate was derived by analytical solution and also include finite element method calculation then lastly experiment was performed on the sandwich circular plate. The mathematical model gives the possibility to determine the strains in the center of the upper face [15]. Nonlinear bending analysis of ring-stiffened circular and annular general angle-ply laminated plates with various boundary conditions. Elastic large deflection analysis of axisymmetric ring-stiffened circular and annular general angle-ply laminated plates subjected to transverse uniform load is studied. Based on first order shear deformation theory (FSKDT) and large deflection von-Karman relations, the governing equations are derived. In this work, the dynamic relaxation (DR) method has been applied. DR is an iterative method in which the basic idea is to convert a static problem into a dynamic one to obtain a steady state solution. To solve the plate equations using DR method they are transformed from a boundary value problem to an initial value format to facilitate the integration of the governing equations via a simple time-stepping iterative procedure [16].

The method for treating nonconforming thin plate bending problems. weak form for the thin plate bending problems and the boundary parameters included. The weak form is useful in dealing with and understanding about nonconforming problems in finite element methods and other numerical methods. It can be observed that finite element method (FEM) can also be formulated from the same weak form. In addition, a new discretization scheme is introduced, which is proven by numerical results to be highly accurate and has no nonconforming problems. The weak forms present a more general form for some discrete schemes, including FEM and the method given in this paper. Using internal moments as intermediate variables, boundary conditions can be introduced easily in weak forms. FEM can be pushed forward from the weak forms, and two reasons for nonconforming problems in FEM are presented, which may provide an alternative perspective for better understanding of the nonconforming problems in FEM. Based on the given weak forms, a new discrete scheme can be derived in the analysis of the discontinuity of the element interpolation functions. Furthermore, the boundary condition can be easily treated by the use of the equations. Comparing with FEM, the new discretization scheme has clear physical meaning and is not affected by nonconforming problems [17]. The regular hybrid boundary node method in the bending analysis of thin plate. Computational procedure based on the regular hybrid boundary node method (RHBNM) for solving the thin plate subjected to a concentrated load. The solution is decomposed into the particular solution arising from the concentrated load and the complementary solution for the homogeneous equation. The RHBNM is a promising boundary type meshless method based on a modified variational principle and the moving least squares (MLS) approximation, and exploits the mesh-less attributes of the MLS and the reduced dimensionality advantages of the boundary element method (BEM). In this paper, a modified variational functional of the thin plate is developed, in which the independent variables are the generalized displacements and generalized tractions on the boundary and the lateral deflection in the domain. The MLS method is employed to approximate the boundary variables whereas the domain variables are interpolated by a linear combination of fundamental solutions of both the bi-harmonic equation and Laplace’s equation. [18]

C. Sectorial Plate:
Bending analysis of thin annular sector plates using extended Kantorovich method. He presented approximate closed-form solution for bending of thin sector plates with clamped edges subjected to uniform and non-uniform loading using the extended Kantorovich method. The method employs the novel idea of Kantorovich to reduce the governing partial differential equation of a two-dimensional (2-D) elasticity problem to a double set of ordinary differential equations. According to the Kantorovich method, the deflection of the plate should be assumed as multi-term separated functions of both variables r and θ. However, due to iterative nature of the EKM technique, it was shown that even a single-term separated functions assumption for deflection can provide very accurate results after a few iterations. This method provides very fast convergence and highly accurate predictions for both annular and solid sectors by results. It is also show that the same formulation offers sufficiently accurate results for rectangular plates. Comparison of the deflection and moments at various points of various sector, annular and rectangular plates shows very good agreement with results of other analytical, numerical and finite element analysis [19]. Bending analysis functionally graded thick sectorial plate using Levinson plate theory and first-order shear plate theory. Levinson plate theory solution of functionally graded sectorial plates were expressed in in terms of the solution of the classical plate theory for homogeneous sectorial plates and then using direct method. Non homogeneous mechanical properties of plate, graded through the thickness, were described by a power function of the thickness coordinate [20].

D. Super Elliptical Plate:
Non-linear bending analysis for super elliptical thin plates with simply supported edge and clamped edge which based on classical plate theory. Approximate solutions of super elliptical thin plates are did by Ritz method. Here show that characteristics of non-linear bending are significantly influenced by different boundary condition and ratio of major to minor axis, as well as the power of super ellipse [21].

III. CONCLUSION
It was found that most of research papers are related bending of rectangular thin plate in context of various analytical (power series) and numerical (FDM, FEM, BEM, FMM, BEM, etc) methods. Bending of circular thin plate is also an important issue as rectangular thin plate. Circular
plates are common in many structures such as nozzle covers, end closures in pressure vessels, pump diaphragms, turbine disks, and bulkheads in submarines and airplanes. So it is very important to study their deformations and slopes under loads in order to understand their behaviour and possible conditions of failure, one of the important factors on which the bending depends on the load conditions and the support conditions.

Most validated methods are numerical methods, that give approximate solution but not exact bending solution also numerical methods can't satisfy the governing partial differential equation as well as the boundary conditions of the plate. Where analytical method gives exact result of bending of thin plates also it seems that current methods are basically semi-inverse or based on the semi-inverse methods.

REFERENCES


