Euclidean Plane Algorithm for Minimum Spanning Tree

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Abstract— The minimum spanning tree problem seeks for a minimum spanning tree interconnecting the n points so that there is only one path between any two points. One of the classic and frequently-used algorithms for minimum spanning tree problem is Prim's algorithm, but it consumes large time and space complexity for the plane minimum spanning tree problem is of O(n2) numbers of edges . Luckily, it was proved that the plane minimum spanning tree is a sub-graph of Delaunay triangulation for the given points in the plane, and the number of edges in the triangulation is O(n). This motivates us to efficiently compute the Delaunay triangulation of the given points and then find the minimum spanning tree in the triangulation. This paper presents an algorithm based on the divide and conquer for Delaunay triangulation together with the Prim's algorithm to produce an O(nlogn) algorithm for minimum spanning tree problem in the plane, implements the visual graphic interface with various selected algorithms for plane minimum spanning tree and compares their running time.

***Key words*:** Euclidean plane minimum spanning tree Algorithms

I. INTRODUCTION

Minimum spanning tree in plane space has been applied in many fields such as the typical problem of Steiner Tree [1]. Another typical example is that the most economic routing is designed for erection of pipes or networks among several places. Triangulation of points set is a vital technology of preprocessing. And Delaunay triangulation can complete many assignments with its unique optimization property. This paper will introduce how to complete the solution of EMST in the relatively small time complexity by using triangulation of Delaunay.

The definition for triangulation of points set :Suppose V is a Euclidean plane finite point set, while e is a focus point as the endpoint constitute the closed segment, E is set of e, then the set of points of a V triangulation (T = V, E) is planar graph G, the planar graph meet the conditions :(1) except the endpoint, edges in planar graph does not contain any point of points set; (2) without intersecting edges(3) all geometries are triangular in planar graph, and all of the triangular collection boundaries are convex hull of points set V[2]. In solving practical problems, Delaunay triangulation, a special triangulation with many excellent properties, is often used, so it is also called the optimal triangulation.

II. DELAUNAY TRIANGULATION ALGORITHM

A. Incremental Insertion for Delaunay Triangulation of Planar Point Set:

The basic idea of the algorithm is that: First create a large polygon surrounding all the points to seek a convex hull of points set, and constantly insert the remaining points. Each inserted point connected with other three vertices in triangle containing the point forms a new triangle. Then test the empty circum-circle one by one and see if it destroys the Delaunay triangulation characteristics. If it did so, correct the triangulation through the exchange of diagonal. The idea of the algorithm is simple and easy to program, but the time complexity is large and the actual performance is not very good.

B. Divide and Conquer Method for Delaunay Triangulation:

Divide and conquer method is an important algorithm which means to divide and rule a complex problem by dividing into two or more sub-problems until the final sub problem can be solved simply and directly. The solution of the original problem is the combined solution of sub-problems. For solving the problem the sequence of points in a plane has to be sorted according to the coordinates. According to horizontal coordinate or the longitudinal coordinate if horizontal coordinate is equal, Figure 1 shows a sample of points sorting. In this way the ordered sequence of planar point set is obtained. So the divide and conquer method for Delaunay Triangulation is a simple recursive procedure [9].

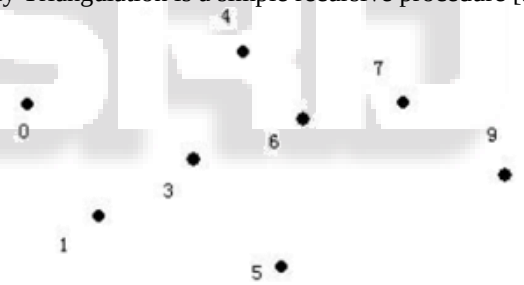

Fig. 1: Number the points from left to right and from bottom to top

III. PRIM ALGORITHM

A. Main Idea:

The main idea is that suppose N = (V, {E }) is China Unicom Network, TE is the set of edges in the minimum spanning tree of N. Algorithm begins from U = {U0} (U0V), TE = {}, the following operations is repeated: In all edges (u ,v) ϵ E for u ϵ U, v ϵ V-U a minimum cost edge (u0,v0) is merged into the TE set, at the same time V0 is merged into U until U = V; TE must have n-1 edges, then T = (V,{TE})is the minimum spanning tree for N [11].

B. Comparative Analysis:

Comparative analysis between the direct implementation of EMST and implementation of EMST based on Delaunay triangulation is conducted in this paper. In the situation of direct implementation Prim algorithm is called for the complete graph of planar point set. When adjacency list storage map and the small top storage candidate edge are

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adopted, time of O(Elogn)[12] (E is the number of sides, n is the number of fixed points)is consumed. It is obvious that the number of sides is n * (n-1) / 2 in the complete graph so that the solution of EMST consumes O (n2logn).But after Delaunay triangulation for planar point set the remaining numbers of sides will not exceed 3 * n so that the solution of MST only consumes O(nlogn).Through the analysis below divide and conquer method for Delaunay triangulation can be implemented in O (nlogn) and the solution of EMST can be implemented in O(nlogn).

IV. DESIGN AND ANALYSIS OF ALGORITHMS

A. Incremental Insertion Method:

1) The Algorithm Implementation of Incremental Insertion

- Identify 4 poles and respectively add them into 5 districts: LU Point, LD Point, RD Point, RU Point, Remain Point with (a1,a2) corresponding to left, (b1,b2) corresponding to up, (c1,c2) corresponding to right and (d1,d2) corresponding to down.
- Start from left to establish the convex hull in the upper left corner and preserve slope until the minimum or maximum slope is found.
- Points in Remain Point are inserted one by one. The first point and all the points of the convex hull construct N triangles (N for side number of convex hull). Respectively test the empty circum-circle of these triangles guaranteeing characteristics of Delaunay triangulation with Flip function.

Check the triangle while the remaining points are inserted one by one. The corresponding test for the empty circum-circle is conducted in the new triangles guaranteeing characteristics of Delaunay triangulation with Flip function.

2) Time of Incremental Insertion

In the process of finding convex hull, first of all the coordinates of point set have to be traversed from a to z consuming O(n) to get 4 extreme points. The set of points is divided into 5blocks namely LU Point, LD Point, RU Point, D Point, Remain Point. Except 4 poles the other points has to be calculated from one time to four times with consumption of O (n). Then four point sets namely LU Point, LD Point, RU Point, D Point, Remain Point are operated as followed: suppose the number of a point set is M, the first eligible point to be found needs about M slope operations with a slope determination, the second eligible point needs about m-1 slope operations, so the total is no more than m * (m-1) /2slope operations. So 4 point sets need about 2*m*(m-1)/2 slope operations consuming O(m2). The number of m and all point numbers of n are constructed at linear ratio, so the solution process of convex hull will consume O (N2).

B. Divide and Conquer Method for Delaunay Triangulation:

In order to find candidate points conveniently, two classifications of Angle and MyStack are introduced:
The classification of Angle has a serial number and a value, serial number SEQ is representative of the current point number(6,7,9 shown in Figure 1) ,Value represents the angle degrees between the point representative of the current serial number and the LR-edge. In the search for candidate point, first of all push all related Angles meeting the conditions of1 to MyStack, here a technique is used for push method, where each stack put the current Angle into the proper position ensuring that Angle in MyStack is ordered from big to small, so each stack can get the operating point, and a candidate point can be obtained when the top of the stack is taken again after the stack.

Time complexity analysis on divide and conquer algorithm for Delaunay : The first step of the combination is to calculate the support line of the convex hull, this can be implemented in O (n).The second step is to constantly identify the candidate point on the base of LR-edge ,judge and generate a new LRedge .In the triangle net, the number of the adjacent points for a point is about4-6(possibly other numbers), but the number of the adjacent points doesn't increase with the number of n. It is a constant. Find out the points and push the related information of Angle to MyStack, because the number of the adjacent points is a constant, if suppose this constant is num, that is to say, the left posterior candidate to be found out only needs 2* num operation. In a process of combination how many operations to find out the candidate point (it means how many times is needed to generate new LR-edge) and scale of points construct a linear relationship (It is obvious). Suppose the number is K * n (0 < K < n), that is to say, one combination needs about 2 * num * k * n operations. In the process of divide and conquer for Delaunay triangulation point set needs to be divided into two until the scale of the point set decreases to a certain extent (2 or 3), therefore, in the whole process the recursion relation of the operation times is:

C(n)=2*C(n/2)+M(n) M(n)=2*num*k*n n>3
C(n)=1 n=3
C(n)=1 n=2

For the convenience of calculation of operation times,
suppose n=2t,namely t=log2n,then

C(2t)=2*C(2t-1)+M(2t)
=2*(2*C(2t-2)+M(2t-1))+M(2t)
=…
=2t-1*C(2)+2t-2*M(22)+2t-3*M(23)+…+M(2t)
=2t-1+2t*2*num*k*(t-1)
Convert t into n,so
C(n)=n/2+n*2*num*k*(log2n-1)

In the above formula Num and K are both constants, time complexity of the entire process is O (nlogn).

C. Implementation of Prim Algorithm:

In order to easily find out the edge of minimum cost, MinInfo, Heap P Queue, edgeInfo, Heap are introduced.

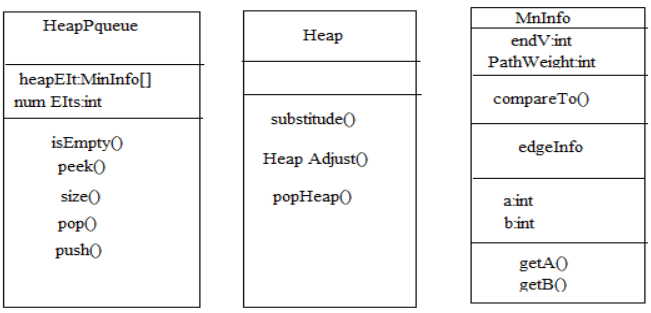

Fig. 2: classification graph of HeapPQueue Heap

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EndV of MinInfo is representative of the endpoint of this edge which exists in the points set of V, and attribute of path Weight is the weight of the edge. Then compare the weight between the current point and all adjacent points and the weights between the other points and the point to generate an object of MinInfo or a update object of MinInfo and put it into the HeapPQueue which is not the ordinary sense of the stack or queue but the smallest weight that the last element of pathWeight of private member heapElt in the HeapPQueue is made whenever a new object of MinInfo or an object of pop is pushed using the heap glide algorithm. In this way every time only invoking the method of pop can get the objects of MinInfo with the lowest pathWeight resulting in adding the point to U set and successfully completing Prim algorithm. Figure 4 shows the data structure of graph, vertex and edge.

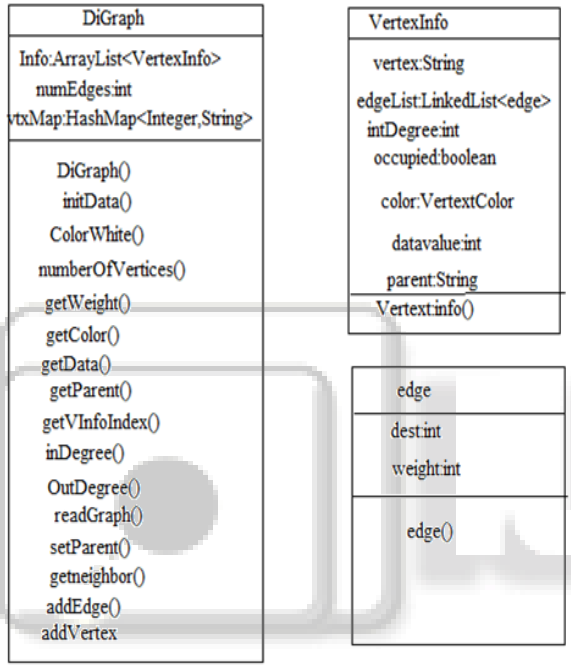

Fig. 3: the data structure of graph, vertex and edge.

The attribute of color in the VertexInfo class represents the state of the point. If the property of color is white, it indicates that the point doesn't join MST. If the property of color is black, it means the point has joined MST. The attribute of parent is the closest collected point .Data Value is a value, it indicates the distance between the point and parent.

D. Program Interface:

Program interface is shown in Figure 5. The upper left block :input the wanted point in the grey area by using the left button of the mouse and call Prim algorithm to generate a minimum spanning tree for the complete graph of all the points in the interface. The upper right block: click the button of Generate in left block to calculate Delaunay triangulation using divide and conquer method for point set and then mark the triangulation in the area with the green line. After Prim algorithm being invoked, EMST is calculated and marked with red line in the region. The lower right block : click the button of Generate in left block, then calculate Delaunay triangulation with Incremental Insertion for point set and mark the triangulation in the area with the blue line. After Prim algorithm being called, EMST is calculated and marked with red line in the region. The lower left block: Show the coordinates of all points and place the button. The button of Generate is used to generate the two graphics on the right, the button of Clear All is used to clear all points, the button of Back is used to remove the points recently inserted.

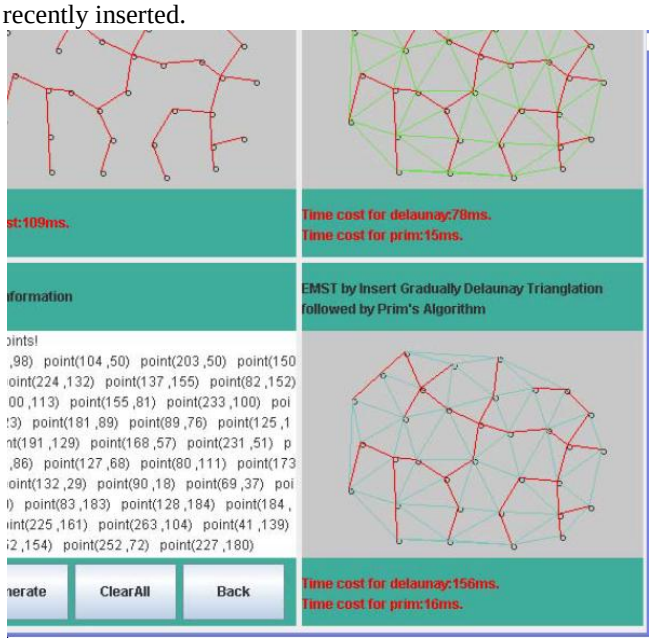

Fig. 4: The diagram of program interface

V. EXPERIMENTAL RESULTS AND ANALYSIS

From Figure 5 three different running times can be seen : directly run Prim algorithm: 109ms; run Prim algorithm after using divide and conquer method for triangulation:78ms (solving for triangulation net) + 15ms (PRIM) = 93ms; run Prim algorithm after using incremental insertion for triangulation:156ms(solving for triangulation net)+16ms (prim) =172ms.

The following table shows the operation under the situation of different numbers of points(unit: ms):

numbers of point	directly run Prim algorithm	divide and conquer method+ Prim algorithm	Incremental insertion + Prim algorithm
10	32	31+16	32+16
15	35	46+16	49+16
20	47	47+16	57+16
30	94	78+16	88+16
35	141	109+17	137+16
40	156	125+17	176+17
45	188	127+16	192+16
50	210	141+17	248+17

Because there is no special timing method, the results are not necessarily accurate, only the general trend of the time can be seen. According to the above table it indicates :when the point is very small (< 25), direct running prim will achieve better results, but when the point increases gradually, the advantage of divide and conquer will be revealed, while incremental insertion for Delaunay

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triangulation consumes too large time resulting in too large time for an entire solution process of EMST.

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