A Combine Nonlinear Companding Transform Technique and Clipping Technique for Efficiently Reducing PAPR of OFDM Signals

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Abstract— Orthogonal Frequency Division Multiplexing (OFDM) is one of most attractive technology in today's wireless communication systems. But its practical implementation is mainly limited by its high Peak-to-Average Power Ratio (PAPR). We are hereby proposing a combine nonlinear companding transform (NCT) technique and clipping technique for reduce PAPR. First, Nonlinear companding transform is performed, where statistics of OFDM signal is transformed into the desirable statistics by maintaining average power. After NCT, clipping is performed on an OFDM signal. At the receiver, original information is recovered by decompanding operation and demodulation. The proposed technique reduces PAPR compared to conventional techniques.

Key words: Orthogonal Frequency Division Multiplexing (OFDM), Peak-to-Average Power Ratio (PAPR), Nonlinear Companding Transform (NCT)

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) technique is one of the popular technologies in today's wireless communication systems because of its large number of advantages, such as high spectrum efficiency, robustness against the multipath fading effect and easy implementation using IFFT (Inverse Fast Fourier Transform). OFDM is used in a number of application such as Digital Audio Broadcasting (DAB), Terrestrial Digital Video Broadcasting (DVB-T), Digital Subscriber Line (DSL), Long Term Evolution (LTE) and Worldwide Interoperability for Microwave Access (Wi-max) [1].

However, OFDM has some drawbacks also. One of the major drawbacks is high Peak-to-Average Power Ratio (PAPR) of transmitted OFDM signals. When, OFDM signal pass through the High Power Amplifier (HPA) and Digital-to-Analog Converter (DAC), High peak is clipped. Which causes out-of-band radiation and in-band-distortion due to that performance of OFDM system is degraded. To avoid this type of degradation, large dynamic range of HPA and DAC required which is so expensive. Hence, PAPR reduction is necessary at the transmitter.

In order to reduce PAPR, Many PAPR reduction techniques are available in [2] such as, Clipping and Filtering, Selective Mapping (SLM), Partial Transmit Sequence (PTS), Block coding, Active Constellation Extension (ACE), Tone Reservation (TR) and Companding Transform [3]-[8].

Among all this technique, Companding transform technique is used due to its effectiveness and simplicity. Also, it can be used straight forwardly without any restriction on the system parameters, such as the size of subcarriers, frame format and constellation type [7].

First Nonlinear Companding Transform (NCT) technique was proposed by Wang which is known as µ-law companding. But, this technique increases the average power level of the signals after companding [3]. To overcome this problem Exponential Companding (EC) was proposed [4]. This scheme transforms the Rayleigh distributed OFDM signal into a uniformly distributed signal to achieve better performance compared to µ-law companding, while maintaining the output power level. However, the distribution of large signals is increased greatly by such transform, which makes its reduction in PAPR is very limited under certain Bit Error Rate (BER) performance constraint [7].

The Piecewise Companding (PC) scheme [5] offers better overall performance than EC, but it cannot perform flexibly in the companding form. Trapezoidal distributed Companding (TC) also developed to achieve a better PAPR reduction and BER performance [6]. For more efficient PAPR reduction Wang proposed two different Nonlinear Companding Transform schemes [7], [8].

In this paper, to reduce more PAPR clipping technique is used after the nonlinear companding transform. By using this technique after NCT we can further reduce PAPR as compared to NCT. The result of PAPR reduction is well verified by computer simulations.

The remainder of this paper is organized as follows. In section II, general OFDM system model and definition of PAPR is described. Section III explained nonlinear companding transform technique [8] with mathematical expression. Section IV introduces combine NCT and clipping technique. Simulation results are discussed in section V. last section contains conclusion.

II. OFDM SYSTEM MODEL AND PAPR DEFINITION

In OFDM system, input bit stream is modulated by PSK or QAM modulation. This complex modulated symbols X_k , $k \in \{0, 1, . . ., N - 1\}$ converted into OFDM symbols x_n after the Inverse Fast Fourier Transform (IFFT), where N is the number of subcarrier. The time-domain OFDM symbol can be represented by,

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k . exp\left(\frac{j2\pi nk}{N}\right) , \quad n = 0, 1 \ldots.., N-1. \tag{2.1}$$

Where n is time index. Since the Nyquist rate samples might not represent the peaks of the continuous time signal, hence true PAPR can be found from an oversampled signal. The oversampled time domain OFDM symbols x_n is obtained by performing a LN – point IFFT operation. This LN – point IFFT operations performed by padding $(L - 1)N$ zeros in middle of vector X. The oversampled time domain OFDM symbols x_n can be represented as,

$$x_n = \frac{1}{\sqrt{LN}} \sum_{k=0}^{LN-1} X_k . exp\left(\frac{j2\pi nk}{LN}\right), n = 0, 1 \ldots.., LN - 1. \tag{2.2}$$

The PAPR of continuous time signal oversampling factor preferable as $L \geq 4$. Based on the central limit theorem, when N is large ($N \geq 64$) the real and imaginary part of x_n becomes the Gaussian random variables with zero mean and $\sigma^2 = E\{|X_k|^2\}/2$ variance and amplitude of x_n follows Rayleigh distribution with probability distribution function (PDF),

$$f_{|x_n|}(x) = \frac{2x}{\sigma^2} exp\left(\frac{-x^2}{\sigma^2}\right) \quad , \quad x \geq 0. \tag{2.3}$$

The cumulative distribution function (CDF) of $|x_n|$ is as follows,

$$F_{|x_n|}(x) = \int_{-\infty}^{x} f_{|x_n|}(x)dx = \int_{0}^{x} \frac{2x}{\sigma^2} exp\left(\frac{-x^2}{\sigma^2}\right) dx$$
$$= 1 - exp\left(\frac{-x^2}{\sigma^2}\right) \quad , \quad x \geq 0 . \tag{2.4}$$

Generally PAPR of OFDM signal is define as follows,

$$PAPR = \frac{Peak\ Power}{Average\ Power}.$$

PAPR of OFDM signal x_n can be expressed as,

$$PAPR = 10log_{10}\left(\frac{max(|x_n|^2)}{E[|x_n|^2]}\right) \quad (dB). \tag{2.5}$$

where E [.] denotes the expectation operation.

The performance of PAPR reduction method is measured in reference with complementary cumulative distribution function (CCDF), which is defined as the probability that the PAPR exceeds an assigned level $\gamma_0 > 0$,

CCDF $x_n(\gamma_0)$ =Prob { PAPR $x_n > \gamma_0$} (2.6)

III. NONLINEAR COMPANDING TRANSFORM TECHNIQUE

A. Companding Transform:

In this section, NCT [8] is studied with some mathematical expression. Let C (.) is the companding transform and it is applied on the OFDM symbols after the IFFT. This NCT only changes the amplitude of OFDM symbols. Basic idea the NCT technique is to transform the amplitude statistics of input signal into a specific distribution form, whose PDF is defined by piecewise function given by,

$$f_{|yn|}(x) = \begin{cases} k_1 x & , \quad 0 \leq x \leq cA \\ k_2 x + (k_1 - k_2)cA & , \quad cA \leq x \leq A \end{cases} \tag{3.1}$$

Where two slopes $k_1 > 0$ and $k_2 < 0$ are variable parameters that determine the desired companding form as well as reduction in PAPR while controlling the output power level. cA ($0 < c < 1$) is an inflection point and A ($A > 0$) is a cut-off point and y_n is the signal after companding.

From this PDF, CDF of $|yn|$ is

$$F_{|yn|}(x) = \begin{cases} \frac{k_1}{2} x^2 & ,0 \leq x \leq cA \\ \frac{-(k_1 - k_2)}{2}(cA)^2 + (k_1 - k_2)cAx + \frac{k_2 x^2}{2} & ,cA \leq x \leq A \end{cases} \tag{3.2}$$

From the definition of CDF, we can obtain k_1,

$\therefore\ F_{|yn|}(A) = 1$

$$k_1 = \frac{2 - A^2 k_2 (c-1)^2}{A^2 c(2-c)} \tag{3.3}$$

The inverse of CDF is

$$F^{-1}{}_{|yn|}(x) = \begin{cases} \sqrt{\frac{2x}{k_1}} & , x \leq \frac{k_1}{2}(cA)^2 \\ \frac{1}{k_2}\left((k_2 - k_1)cA + \sqrt{(k_1 - k_2)k_1 c^2 A^2 + 2k_2 x} \right) & , x > \frac{k_1}{2}(cA)^2 \end{cases} \tag{3.4}$$

Given that $C\ (x)$ is a strictly monotonic increasing function and has the inverse function,

$$F_{|xn|}(x) = prob\{|xn| \leq x\}$$
$$= prob\ \{C(|xn|) \leq C(x\ )\}$$
$$= F_{|yn|}(C(x)) \tag{3.5}$$

By considering the phase of original signal, the companding transform $C\ (x)$,

$$C(x) = sgn(x). F^{-1}{}_{|yn|}\left(F_{|xn|}(x)\right) \tag{3.6}$$

Where sgn (x) is the sign function. By substituting (2.4) and (3.4) into (3.6), the companding function $C\ (x)$ is given in bottom of this page.

Where $\chi = \sigma\sqrt{-\ ln\left(1 - \frac{k_1}{2}(cA)^2\right)}$ is the range of companding transform This companding transform is applied at the transmitter side. At the receiver side, inverse companding transform is applied on the received signal. Inverse companding transform is also given in bottom.

In order to maintain average power level of input signal and companded signal by companding transform,

$$\therefore\ E|xn|^2 = E|yn|^2 = \sigma^2 \tag{3.9}$$

Average power of companded signal is obtain by,

$$E|yn|^2 = \int_{0}^{cA} k_1 x^3\ dx + \int_{cA}^{A} (k_2 x^3 + (k_1 - k_2)cAx^2)dx \tag{3.10}$$

From (3.9) and (3.10) value of A,

$$\therefore\ A = \left(\frac{1}{2\zeta_2}\left(\left({\zeta_1}^2 - 4\zeta_2\zeta_0\right)^{\frac{1}{2}} - \zeta_1\right)\right)^{\frac{1}{2}} \tag{3.11}$$

where

$\zeta_0 = 12\sigma^2(c-2)$
$\zeta_1 = -2(c^3 - 4)$
$\zeta_2 = k_2(c^3 - 3c + 2).$

B. Achievable PAPR:

The PAPR of this technique can be calculated from (2.5),

$$PAPR = 10log_{10}\left(\frac{max(|y_n|^2)}{E[|y_n|^2]}\right) \quad (dB)$$
$$PAPR = 10log_{10}\left(\frac{A^2}{\sigma^2}\right) \quad (dB)$$

From (3.11), put the value of A,

$$PAPR = 10log_{10}\left(\frac{\left({\zeta_1}^2 - 4\zeta_2\zeta_0\right)^{\frac{1}{2}} - \zeta_1}{2\zeta_2\sigma^2}\right) \quad (dB). \tag{3.12}$$

IV. PROPOSED SCHEME DESCRIPTION

In this paper, we proposed combine NCT-clipping technique for PAPR reduction. At the transmitter, clipping is performed after NCT technique. After that this companded-clipped signal is transmitted. By using this technique we can get more PAPR reduction compared to nonlinear companding technique.

$$C(x) = \begin{cases} sgn(x).\sqrt{\frac{2}{k_1}\left(1-exp\left(\frac{-|x|^2}{\sigma^2}\right)\right)} & ,x \le \chi \\ sgn(x).\frac{1}{k_2}\left((k_2-k_1)cA+\sqrt{(k_1-k_2)k_1c^2A^2+2k_2\left(1-exp\left(\frac{-|x|^2}{\sigma^2}\right)\right)}\right) & ,x > \chi \end{cases} \quad (3.7)$$

$$C^{-1}(x) = \begin{cases} sgn(x).\sigma\sqrt{-\ln\left(1-\frac{k_1|x|^2}{2}\right)} & ,x \le cA \\ sgn(x).\sigma\sqrt{-ln\left(-\frac{k_2|x|^2}{2}+(k_2-k_1)|x|cA+1-\frac{(k_2-k_1)c^2A^2}{2}\right)} & ,x > cA \end{cases} \quad (3.8)$$

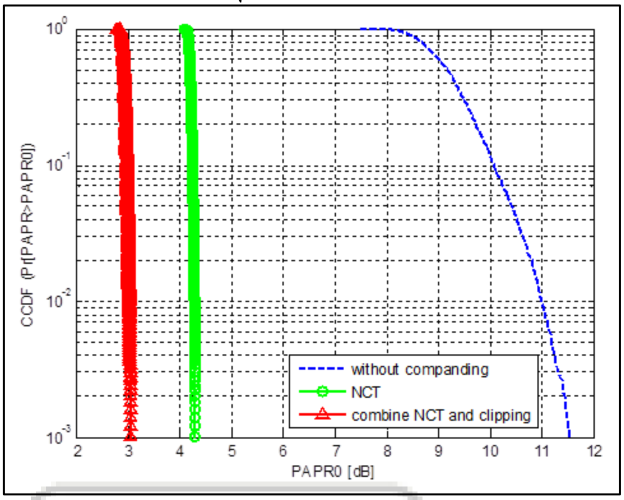

Fig. 1: PAPR reduction performance for case (1) c = 0.25, k2 = -0.001 with N = 1024, QPSK modulation and oversampling factor L = 4.

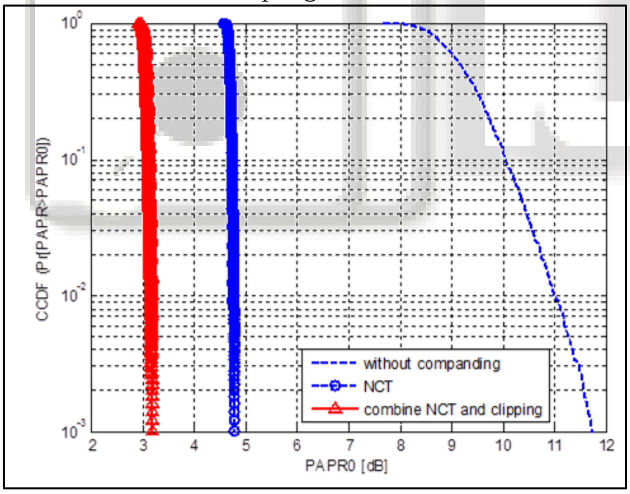

Fig. 2: PAPR reduction performance for case (2) c = 0.25, k2 = -0.45 with N = 1024, QPSK modulation and oversampling factor L = 4.

V. SIMULATION RESULTS

To evaluate result of nonlinear companding technique and proposed combine NCT-clipping technique number of subcarriers N = 1024 is considered. Randomly generated bit stream is modulated by QPSK modulation and this modulated complex data symbols are oversampled by oversampling factor L = 4. This complex symbols are converted into OFDM symbols using LN – point IFFT. Then NCT and clipping is applied. NCT is simulated for three cases (1). c = 0.25, k2 = -0.001 (2). c = 0.25, k2 = -0.45 and (3). c = 0.001, k2 = -0.20. After NCT, clipping is applied for the threshold value = 0.950. To compare PAPR reduction performances for NCT technique and combine NCT-clipping technique three different cases are simulated. Fig. 1 shows case (1) in which PAPR of original signal is in between 11-12 dB for CCDF = 10-3. Fig. 2 shows case (2) and Fig. 3 shows case (3). For these three cases PAPR in combining NCT and clipping technique is minimum compared to NCT technique.

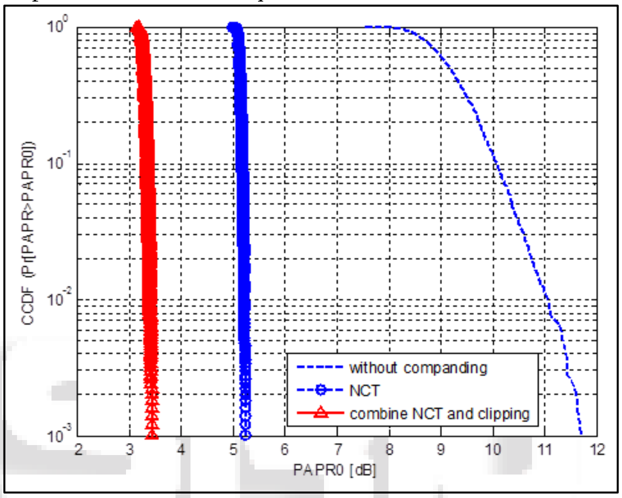

Fig. 3: PAPR reduction performance for case (3) c = 0.001, k2 = -0.20 with N = 1024, QPSK modulation and oversampling factor L = 4.

VI. CONCLUSION

In this paper, first we studied NCT technique for PAPR reduction. Then, we applied clipping technique for PAPR reduction which is performed after NCT technique. After that, both this techniques are simulated. Simulation results show the combine NCT-clipping technique reduce more PAPR compared to NCT technique.

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