Design and Analysis of Multilayer High Pressure Vessels and Piping
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Abstract—The cylindrical vessels are used for storing fluids at high pressure. In cylinder the stress distribution is not uniform. Distribution of stress in the juncture area and the rest will differ vessel cause a geometric discontinuity of the vessel wall. So a stress concentration is created around the opening. The junction may fail due to these high stresses. Hence a detailed analysis is required to determine the stress distribution along the junction. Using the ASME formulae the discontinuity forces and moments are determined from which displacements and stresses are found out for individual as well as whole juncture. The stresses developed in pressure vessel is analyzed by using ANSYS. As a thick cylinder storing fluid with large internal pressure has second order non-linear variation in the hoop stress across the wall. For more uniform hoop stress distribution, compound cylinders are formed by shrinkage process where outer cylinder is heated until it will slide freely over inner cylinder thus exerting the required shrinkage pressure on cooling. Using the calculated shrinkage pressures between two contacting cylinders, it is possible to reduce the hoop stress and make it more or less uniform over the thickness. The methodology for minimization of volume of shrink-fitted three layer compound cylinder and to get equal maximum hoop stresses in all the cylinders. The analytical results are validated in comparison with FEM in ANSYS Workbench.

Key words: Discontinuity, Multi-Layer Cylinders, Hoop Stress, Shrink Fit, Contact Pressure, Optimum Design, ANSYS

I. INTRODUCTION
Pressure vessels are equipment’s employed to store or transport the fluid under high pressures. They usually contain regions where abrupt changes in geometry, material, (Figure 1) or loading occur. These regions are known as discontinuity areas, and the stresses associated with them are called discontinuity stresses. The codes have outlined a general procedure for analyzing the discontinuity stresses. Numerical analysis is done for the calculation of magnitude of deflection and stress in the individual components as well as in the junction due to pressure loads and edge loads on the vessel. Secondly, the junction is analysed by means of same loads using FEM package ANSYS, and finally validating results with numerical results.

II. METHODOLOGY IN DESIGN OF PRESSURE VESSEL
The free body diagrams for each of the components that make up the pressure vessel. In this when the parts are separated each part must be in equilibrium under the action...
of the forces. The applied loading, in this case the internal pressure, is clearly an external force. The loading at the junctions are internal forces and moments that is, when the parts are re-assembled these forces cannot be drawn on a force diagram because they are internal.

Fig. 3: Free Body Diagram of Pressure Vessel

1) Input Data:
- Design pressure \( p \) = 11.72 N/mm²
- Design Code - ASME Sec VIII Division 1
- Length \( L \) = 393.7 mm
- Thickness \( t \) = 6.35 mm
- Poisson’s ratio \( \nu \) = 0.21
- Radius \( R \) = 101.6 mm
- Young’s modulus \( E \) = 9.9E4 N/mm²

A. Edge Bending Solution for Displacement and Rotation for Hemispherical End:

For the cylindrical end

Element A

\[
\delta = - \left( \frac{2Rm\lambda}{Et} \right) Q_o + \left( \frac{2\lambda^2}{Et} \right) M_o
\]

\[
\theta = - \left( \frac{2\lambda}{Et} \right) Q_o + \left( \frac{4\lambda^2}{RmEt} \right) M_o
\]

For hemispherical end

Element C

\[
\delta = - \left( \frac{2Rm\lambda}{Et} \right) Q_l + \left( \frac{2\lambda^2}{Et} \right) M_l
\]

\[
\theta = \left( \frac{4\lambda^3}{RmEt} \right) M_l
\]

There is no rotation of the sphere or the cylinder at the junction due to pressure so we may write the rotation equal to zero. Axisymmetric edge moment \( M \) and shear \( Q \) acting on the edge of shell of radius \( a \), thickness \( t \), produce the following deflections and slopes.

Fig. 4: Edge Loaded Shell

Equations obtained are

\[
f_1(\beta x) = e^{-\beta x} \cos \beta x
\]

\[
f_2(\beta x) = e^{-\beta x}(\cos \beta x - \sin \beta x)
\]

\[
f_3(\beta x) = e^{-\beta x}(\cos \beta x + \sin \beta x)
\]

Where \( \beta, D \) are

\[
\beta = \left( \frac{3(1 - \nu^2)}{(R + 0.5t)^2} \right)^{0.25},
\]

\[
D = \left( \frac{Et^3}{12(1 - \nu^2)} \right)
\]

B. Edge Deformation Due To Internal Pressure For Hemispherical:

\[
\delta_p sp = \frac{pR^2}{2Et} (1 - \nu)
\]

For Cylindrical

\[
\delta_p cy = \frac{pR^2}{2Et} (1 - \nu)
\]

Compatibility must be maintained at the junction of the hemisphere and cylinder so that the total deformation of each part is equal.

\[
\delta sp(Qo, Mo) + \delta sp = \delta cy(Qo, Mo, QL, ML) + \delta cy
\]

\[
\theta sp(Qo, Mo) = \theta cy(Qo, Mo, QL, ML)
\]

As from the above equations, equating the appropriate expressions, we get four equations as

\[
\left( - \frac{2Rm\lambda}{Et} - \frac{1}{2\beta^3 D} \right) Q_o + \left( \frac{2\lambda^2}{Et} - \frac{1}{2\beta^2 D} \right) M_o - \left( \frac{f_1(\beta x)}{2\beta^2 D} \right) Q_l - \left( \frac{f_2(\beta x)}{2\beta^2 D} \right) M_l = \left( \frac{pR^2}{2Et} \right) ML
\]

\[
\left( - 2\lambda \right) Q_o + \left( \frac{4\lambda^2}{RmEt} \right) M_o + \left( \frac{f_3(\beta x)}{2\beta^2 D} \right) Q_l + \left( \frac{f_2(\beta x)}{2\beta^2 D} \right) M_l = \left( \frac{pR^2}{2Et} \right) \beta ML
\]

\[
\left( - \frac{2\lambda}{Et} \right) Q_o + \left( \frac{4\lambda^2}{RmEt} \right) M_o + \left( \frac{f_3(\beta x)}{2\beta^2 D} \right) Q_l + \left( \frac{f_1(\beta x)}{2\beta^2 D} \right) M_l = \left( \frac{pR^2}{2Et} \right) \beta Q_o + \left( \frac{f_2(\beta x)}{2\beta^2 D} \right) \beta Q_l + \left( \frac{f_1(\beta x)}{2\beta^2 D} \right) \beta M_l = 0
\]

These four algebraic equations are obtained.

Four algebraic equations are thus assembled to solve for four unknowns, namely \( Q_o, M_o, QL, \) and \( ML \).

These equations can be written in the matrix form.

\[
\begin{bmatrix}
\left( \frac{2Rm\lambda}{Et} \right) & \left( \frac{2\lambda^2}{Et} \right) & \left( \frac{4\lambda^3}{RmEt} \right) & \left( \frac{f_3(\beta x)}{2\beta^2 D} \right) & \left( \frac{f_2(\beta x)}{2\beta^2 D} \right) & \left( \frac{f_2(\beta x)}{2\beta^2 D} \right) & \left( \frac{f_1(\beta x)}{2\beta^2 D} \right) & \left( \frac{f_1(\beta x)}{2\beta^2 D} \right) & \left( \frac{pR^2}{2Et} \right) ML
\end{bmatrix}
\]

Solving the above four equations as represented in matrix form. We get unknown values as:

\( Q_o = -27.4765 \) N mm

\( M_o = 5.50002 \) E-7 N mm

\( QL = -27.4765 \) N mm

\( ML = 1.649E-6 \) N mm
C. Calculation of Discontinuity Stresses Due To Edge Loads:
The principal stresses developed at the surfaces of a cylindrical shell at any location due to uniformly distributed edge loads are given
\[ \sigma_l = \pm \frac{6M(x)}{t^2} \]
\[ \sigma_t = \frac{E\delta(x)}{(R + \frac{t}{2})^2} \pm \frac{6vM(x)}{t^2} \]
\[ \sigma_r = 0 \]
In these formulas, where terms are preceded by a double sign ±, the upper sign refers to the inside surface of the cylinder and the lower sign refers to the outside surface.

D. Validation of Total Stress:
The stress is computed at any juncture by combining the stresses due to redundant shear forces and moments and with stresses due to internal pressure. The stresses in the cylindrical shell due to internal pressure may be computed from the expressions:
\[ \sigma_l = \frac{pR}{2t} \]
\[ \sigma_t = \frac{pR}{t} + \frac{Ew(x)}{(R + \frac{t}{2})^2} \pm \frac{6vM(x)}{t^2} \]
\[ \sigma_r = 0 \]
The stresses due to the redundant shear forces and moments were computed at both junctions O, L

III. STRESS ANALYSIS OF THREE LAYER COMPOUND CYLINDER
Consider three cylinders have the same material. The method of solution for compound cylinders constructed from similar materials is to break the problem down into four separate effects. Thus for each condition the hoop and radial stresses at any radius can be evaluated.

A. Radial and Hoop Stress in Cylinder 1:
If \( P_i \) is no internal pressure, radial stress in cylinder 1 is given by using Lame’s equation,
\[ \sigma_r = -P_{s12} \frac{r_2^2}{r_2^2 - r_1^2} \left( 1 - \frac{r_2^2}{r_1^2} \right) \]
(2.1)
or is maximum at outer radius \( r_2 \) of cylinder 1, using equation (1)
\[ \sigma_r \text{ max (at } r_2) = -P_{s12} \]
(2.2)
Hoop stress in cylinder 1 is given by using Lame’s equation
\[ \sigma_\theta = -P_{s12} \frac{r_2^2}{r_2^2 - r_1^2} \left( 1 + \frac{r_2^2}{r_1^2} \right) \]
(2.3)
Hoop stress at outer radius \( r_2 \) is
\[ \sigma_\theta (\text{at } r_2) = -P_{s12} \frac{r_2^2}{r_2^2 - r_1^2} \]
(2.4)
While hoop stress at inner radius \( r_1 \) is
\[ \sigma_\theta \text{ max (at } r_1) = -P_{s12} \frac{r_2^2}{r_2^2 - r_1^2} \]
(2.5)
In the shrink-fitting problems, considering long hollow cylinders, the plane strain hypothesis (in general, \( E \) ≠ 0) can be regarded as more natural. Hence as per the relation.
\[ \sigma_\theta = \theta (\sigma_r + \sigma_\theta) \]
The expression for the hoop strain is given by:
\[ \varepsilon_\theta = \frac{1}{E} \left[ \varepsilon_r - \nu \varepsilon_\theta \right] - \frac{1}{E} \left[ \varepsilon_r - \nu \varepsilon_\theta \right] \left( \sigma_r + \sigma_\theta \right) \]
Using equations (2.2) and (2.4), assuming plane strain condition the hoop strain at the outer wall \( r_2 \) of cylinder 1 is
\[ \varepsilon_\theta = \frac{1}{E} \left[ \frac{1}{r_2^2} - \frac{1}{r_1^2} \right] \left( -P_{s12} \frac{r_2^2}{r_2^2 - r_1^2} \right) \]
(2.6)
Radial displacement \( U_{r10} \),
\[ U_{r10} = -\frac{P_{s12} r_1 (1 + \nu)}{E} \left( -P_{s12} \left[ r_2^2 + r_1^2 \right] \right) \]
(2.7)
B. Radial and Hoop Stress in Cylinder 2:
Contact pressure \( P_{s12} \) is acting as internal pressure and contact pressure \( P_{s23} \) is acting as external pressure on cylinder 2.
Using Lame’s equation, radial stress in the cylinder 2 at inner radius \( r_2 \) is given by
\[ \sigma_r (\text{at } r_2) = -P_{s12} \]
(2.8)
While radial stress in the cylinder 2 at outer radius is given by
\[ \sigma_r (\text{at } r_3) = -P_{s23} \]
(2.9)
Hoop stress in the cylinder 2 at inner radius \( r_2 \) is given by
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While hoop stress in the cylinder 2 at outer radius \( r_2 \) is given
\[
\sigma_{\theta_{\text{max}}(at \ r_2)} = \frac{P_{s2}(r_2^2 + r_2 r_3)}{r_2^2 - r_2 r_3} - \frac{2P_{s3} r_2^2}{r_2^2 - r_2 r_3} \]  \( \text{(2.10)} \)

Using equations (2.8) and (2.9), assuming plane strain condition the hoop strain at the inner wall \( r_1 \) of cylinder 2 is
\[
\varepsilon_{\theta_1} = \frac{1}{E} \left[ (1 - \nu) \sigma_{\theta_1} - \nu \sigma_{r_1} \right] \frac{1}{r_1} \left( \frac{r_1^2 + r_1 r_2}{r_2 - r_1} \right) - \nu \left( \frac{P_{s2}}{r_1^2 - r_1 r_2} \right) \]  \( \text{(2.11)} \)

Radial displacement \( U_{r_{21}} \)
\[
U_{r_{21}} = \frac{r_1}{E} \left( \frac{P_{s2}^2}{r_1^2 - r_1 r_2} \right) \left( 1 - \nu \right) \frac{r_1^2 + r_1 r_2}{r_2 - r_1} - \nu \frac{P_{s2}}{r_1^2 - r_1 r_2} \]  \( \text{(2.12)} \)

Referring figure 3 and using equations (2.7) and (2.13), total interference \( \delta_{12} \) at the contact between cylinder 1 and 2 is:
\[
\delta_{12} = U_{r_{21}} - U_{r_{10}} \]  \( \text{(2.13)} \)

Radial stress in the cylinder 3 at inner radius \( r_3 \) is given by
\[
\sigma_{r_3} = \frac{P_{s3}^2}{r_3^2 - r_3 r_4} \]  \( \text{(2.17)} \)

Hence radial displacement \( U_{r_{32}} \)
\[
U_{r_{32}} = \frac{r_3}{E} \left( \frac{P_{s3}^2}{r_3^2 - r_3 r_4} \right) \left( 1 + \nu \right) \frac{r_3^2 + r_3 r_4}{r_4 - r_3} - P_{s3} \left( 1 + \nu \right) \frac{r_3^2 + r_3 r_4}{r_4 - r_3} \]  \( \text{(2.18)} \)

Using equations (2.9) and (2.11), hoop stress in the outer wall \( r_4 \) of cylinder 2 is given by
\[
\sigma_{\theta_4} = \frac{1}{E} \left[ (1 - \nu) \sigma_{\theta_4} - \nu \sigma_{r_4} \right] \frac{1}{r_4} \left( \frac{r_4^2 + r_4 r_3}{r_3 - r_4} \right) - \nu \left( \frac{P_{s3}}{r_4^2 - r_4 r_3} \right) \]  \( \text{(2.19)} \)

Using equations (2.8) and (2.9), hoop strain at inner wall \( r_3 \) of cylinder 3 is given by
\[
\varepsilon_{\theta_3} = \frac{1}{E} \left[ (1 - \nu) \sigma_{\theta_3} - \nu \sigma_{r_3} \right] \frac{1}{r_3} \left( \frac{r_3^2 + r_3 r_4}{r_4 - r_3} \right) - \nu \left( \frac{P_{s3}}{r_3^2 - r_3 r_4} \right) \]  \( \text{(2.20)} \)

C. Radial and Hoop Stress in Cylinder 3:

Hoop stress at any radius \( r \) in compound cylinder due to internal pressure only is given by
\[
\sigma_{\theta} = - \frac{P_{s3}^2}{r_3^2 - r_3 r_4} \left( r_3^2 + r_3 r_4 \right) + \frac{1}{r_3^2 - r_3 r_4} \]  \( \text{(2.23)} \)

IV. PRINCIPLE OF SUPERPOSITION

After finding hoop stresses at all the radii, the principle of superposition is applied, the various stresses are then combined algebraically to produce the resultant hoop stresses in the compound cylinder subjected to both shrinkage pressures and internal pressure \( P_i \).

A. Resultant Hoop Stress in Cylinder 1:

Fig. 7: Superposition of Hoop Stress Due To \( P_i \) & Residual Stress Due To \( P_{s_{12}} \) in Cylinder 1

Using equations (2.23) and (2.5), maximum hoop stress at the inner surfaces of cylinder 1 at \( r_1 \)
\[
\sigma_{\theta_{1\text{max}}} = \frac{P_{s3}^2}{r_3^2 - r_3 r_4} \left( r_3^2 + r_3 r_4 \right) - 2P_{s_{12}} \left( r_3^2 - r_3 r_4 \right) \]  \( \text{(3.1)} \)
B. Resultant Hoop Stress in Cylinder 2:

Using equations (2.23) and (2.10), maximum hoop stress at the inner surfaces of cylinder 2 at \( r_2 \)

\[
\sigma_{\theta 2} = \frac{P_{s12} r_2^2 + r_3^2}{r_2^2 - r_3^2} + P_{s23} \left[ \frac{r_4^2 + r_2^2}{r_4^2 - r_2^2} \right]
\]

(3.2)

C. Resultant Hoop Stress in Cylinder 3:

Using equations (2.18) and (2.23), maximum hoop stress at the inner surfaces of cylinder 3 at \( r_3 \)

\[
\sigma_{\theta 3} = \frac{P_{s12} r_2^2 + r_3^2}{r_2^2 - r_3^2} + P_{s23} \left[ \frac{r_4^2 + r_3^2}{r_4^2 - r_3^2} \right]
\]

(3.3)

V. OPTIMUM DESIGN METHODOLOGY FOR THREE LAYER COMPOUND CYLINDER

To obtain optimum values of the contact pressures \( P_{s12} \) and \( P_{s23} \) which will produce equal hoop stresses in all the three cylinders, maximum hoop stresses given by the equations (3.1), (3.2) and (3.3) must be equated.

Equating equations (3.1), (3.2) and (3.3) and rearranging,

\[
P_{s12} \left[ \frac{2r_2^2}{r_2^2 - r_3^2} + \frac{r_3^2 + r_2^2}{r_2^2 - r_2^2} \right] = P_{s12} \left[ \frac{r_2^2 + r_3^2}{r_2^2 - r_3^2} \right] + P_{s23} \left[ \frac{2r_2^2}{r_3^2 - r_2^2} \right]
\]

(4.1)

Let the following relations

\[
t_1 = \frac{r_2}{r_1}, t_2 = \frac{r_3}{r_2}, t_3 = \frac{r_4}{r_3}, \frac{r_3}{r_2}
\]

(4.2)

Where \( d_1, d_2, d_3, d_4 \) are diameters corresponding to radii \( r_1, r_2, r_3, r_4 \).

Hence

\[
t_1 t_2 = r_2 \frac{r_3}{r_1}, t_2 t_3 = r_3 \frac{r_4}{r_2}, t_1 t_2 t_3 = r_2 \frac{r_3}{r_1} \frac{r_4}{r_3}
\]

(4.3)

\[
k_1 = \frac{2t_1 t_1^2}{t_2 t_2^2 - 1} + \frac{t_2 t_2^2}{t_1 t_1^2 - 1} + \frac{1}{t_3 t_3^2}
\]

(4.4)

\[
k_2 = \frac{t_2 t_2^2 + t_1 t_1^2}{t_3 t_3^2 - t_1 t_1^2 - t_2 t_2^2 - 1} + \frac{1}{t_3 t_3^2}
\]

(4.5)

Hence equation (27) becomes

\[
P_{s12} = P_i \left[ \frac{k_1}{k_2} + P_{s23} \frac{k_2}{k_1} \right]
\]

(4.6)

Equating equations (3.2) and (3.3) \( \sigma_{\theta 2} = \sigma_{\theta 3} \) and rearranging

\[
P_{s12} \left[ \frac{r_2^2 + r_3^2}{r_2^2 - r_3^2} \right] + P_{s23} \left[ \frac{2r_2^2}{r_3^2 - r_2^2} \right]
\]

(4.7)

Let

\[
k_4 = \frac{t_2 t_2^2 + t_1 t_1^2}{t_3 t_3^2 - t_1 t_1^2 - t_2 t_2^2 - 1} + \frac{1}{t_3 t_3^2}
\]

(4.8)

\[
k_5 = \frac{t_2 t_2^2}{t_3 t_3^2 - 1} + \frac{t_1 t_1^2}{t_3 t_3^2 - 1} + \frac{1}{t_3 t_3^2}
\]

(4.9)

\[
k_6 = \frac{t_1 t_1^2}{t_2 t_2^2 - 1} + \frac{t_2 t_2^2}{t_2 t_2^2 - 1} + \frac{1}{t_2 t_2^2}
\]

(4.10)

Hence equation (33) becomes

\[
P_{s12} = P_i \left[ \frac{k_5}{k_6} \frac{k_2}{k_1} \frac{k_4}{k_3} \frac{k_3}{k_2} \frac{k_1}{k_4} \right]
\]

(4.11)

Equations (4.6) and (4.10) have been solved to get \( P_{s12} \) and \( P_{s23} \) in terms of \( P_i \) as

\[
P_{s12} = P_i \left[ \frac{k_5}{k_6} \frac{k_2}{k_1} \frac{k_4}{k_3} \frac{k_3}{k_2} \frac{k_1}{k_4} \right]
\]

(4.12)

\[
P_{s23} = P_i \left[ \frac{k_5}{k_6} \frac{k_2}{k_1} \frac{k_4}{k_3} \frac{k_3}{k_2} \frac{k_1}{k_4} \right]
\]

(4.13)

Putting the values of \( t_1, t_2, t_3 \) the equations (2.14) and (2.22) can be written

\[
\delta_{12} = \frac{t_1 (1 - \theta^2)}{E} \left[ P_{s12} \left( \frac{t_2^2 + 1}{t_2^2 - 1} + \frac{t_2^2 + 1}{t_2^2 - 1} \right) \right] - 2P_{s23} \left( \frac{t_2^2}{t_2^2 - 1} \right)
\]

(4.12)

\[
\delta_{23} = \frac{t_2 (1 - \theta^2)}{E} \left[ P_{s23} \left( \frac{t_2^2 + 1}{t_2^2 - 1} + \frac{t_2^2 + 1}{t_2^2 - 1} \right) - 2P_{s12} \right] - \frac{t_2^2}{t_2^2 - 1}
\]

(4.13)

VI. ANALYTICAL METHOD

A. Methodology:

Material for all the three cylinders is assumed to be the same i.e. steel. For the given volume of fluid to be stored, the internal diameter of cylinder 1 (d1) is known. Here it is assumed as 100 mm. Yield strength of the steel material is \( \sigma_y = 250 \text{ MPa} \). In case of pressure vessels it is observed that failure occurs across the thickness of cylinder where the hoop stress is acting. Although the von Mises stress is more than the hoop stress, it is acting on larger area than hoop stress. So chances of failure due to hoop stress is more in pressure vessel. Hence maximum hoop stress criteria is used.
for material suffering. Maximum hoop stresses in all the cylinders should not exceed the yield stress of the material to avoid the failure of the compound cylinder. Optimum material volume can be calculated using the following steps.

1) Steps:
   1) Assume internal diameter of cylinder 1 (d₁) say 100 mm.
   2) Select the ratios
      \[ t_1 = \frac{d_2}{d_3}, t_2 = \frac{d_3}{d_4}, t_3 = \frac{d_4}{d_5} \]
   3) For the given internal pressure Pᵢ, one can find contact (shrinkage) pressures Pₛ₂ and Pₛ₃ in terms of ratios t₁, t₂, t₃.
   4) Find the volume of the compound cylinder using
      \[ V = \pi \left( d_4^2 - d_1^2 \right) / 4 \]
   5) Minimize the volume subjected to the constraints, 
      i) \( \delta_{θ₁} = σ_y \) ii) \( σ_{θ₂} ≤ σ_y \) iii) \( σ_{θ₃} ≤ σ_y \) iv) \( δ_{θ₂} > 0 \) v) \( δ_{θ₃} > 0 \)
   6) Optimized parameters t₁, t₂, t₃ and \( δ_{θ₂}, δ_{θ₃} \) are used for the design.

   The values of t₁, t₂, t₃ are selected from 1.1 to 2.4 with the increment of 0.10, 0.05, and 0.002. Thus with lot of combinations of t₁, t₂, t₃ material volume is found. A number of combinations of t₁, t₂, t₃ satisfy the condition of equal maximum hoop stresses in all three cylinders which is less than yield stress of the material. Out of these combinations some important combinations are presented in this paper for comparison. However there is one unique combination where volume is minimum. In programming, equations (4.12) and (4.13) are used to find contact (shrinkage) pressures Pₛ₂ and Pₛ₃ for given internal pressure Pᵢ resp. Also equations (4.12) and (4.13) are used to find interferences \( δ_{θ₁}, δ_{θ₂} \) resp. These interferences are later used in section 4 for modeling in Finite Element Method.

### Table 1: Analytical Results of Numerical Method Using Computer Program (MS Excel) (D₁ = 100 Mm and = 250 M, Pa)

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<th>Parameters</th>
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<th>Set 3</th>
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<td>Pₛ₁₂</td>
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<td>25.04</td>
<td>29.51</td>
</tr>
<tr>
<td>Pₛ₂₃</td>
<td>20.64</td>
<td>20.56</td>
<td>19.64</td>
</tr>
<tr>
<td>( σ_{θ₁} )</td>
<td>249.995</td>
<td>249.997</td>
<td>249.943</td>
</tr>
<tr>
<td>( σ_{θ₂} )</td>
<td>249.995</td>
<td>249.997</td>
<td>249.943</td>
</tr>
<tr>
<td>( σ_{θ₃} )</td>
<td>249.995</td>
<td>249.997</td>
<td>249.943</td>
</tr>
<tr>
<td>( δ_{θ₁} )</td>
<td>0.026</td>
<td>0.027</td>
<td>0.038</td>
</tr>
<tr>
<td>( δ_{θ₂} )</td>
<td>0.031</td>
<td>0.030</td>
<td>0.032</td>
</tr>
<tr>
<td>Fₓ (vol)</td>
<td>30938.6</td>
<td>30905.08</td>
<td>31075.24</td>
</tr>
</tbody>
</table>

From the table it is observed that minimum volume is 30905.08mm³ corresponding to values in set 2 where 
\( t₁ = 1.285, t₂ = 1.305, t₃ = 1.325, \)

### Table 2: Data for Modelling in ANSYS for Set2

| t₁ | 1.285 |
| t₂ | 1.305 |
| t₃ | 1.325 |
| d₁ | 128.5 |
| d₂ | 128.446|
| d₃ | 167.7 |
| d₄ | 167.640|
| \( δ_{θ₁} \) | 0.027 |
| \( δ_{θ₂} \) | 0.030 |

### Table 2: Data for Modelling in ANSYS for Set2

Where,
\( d₁, d₂ \) = inner & outer diameters of cylinder 1 respectively.
\( d₃, d₄ \) = inner & outer diameters of cylinder 2 respectively for shrink fit.
\( d₅, d₆ \) = inner & outer diameters of cylinder 3 respectively for shrink fit.

FEM model of three layered compound cylinder is prepared in ANSYS Workbench using values of the diameters from table 2. Results of FEM by ANSYS Workbench are listed in the figures.

### Fig. 10: Maximum Hoop Stress in Cylinder 1

### Fig. 11: Maximum Hoop Stress in Cylinder 2
VII. RESULTS

Analytical results and FEM (ANSYS) results are summarized in Table 3.

<table>
<thead>
<tr>
<th>Set</th>
<th>Results</th>
<th>Contact pressure b/w cy 1, 2, Ps12 (M.pa)</th>
<th>Contact pressure b/w cy 2, 3, Ps23 (M.pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Analytical 249.934</td>
<td>249.934</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ansys 263.26</td>
<td>252.34</td>
</tr>
<tr>
<td></td>
<td></td>
<td>%Diff 5.32</td>
<td>0.958</td>
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<tr>
<td></td>
<td></td>
<td>1.7817</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Comparison of Analytical and ANSYS Results

<table>
<thead>
<tr>
<th>Set</th>
<th>Results</th>
<th>Contact pressure b/w cy 1, 2, Ps12 (M.pa)</th>
<th>Contact pressure b/w cy 2, 3, Ps23 (M.pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Analytical 20.7</td>
<td>17.77</td>
</tr>
<tr>
<td></td>
<td></td>
<td>%Diff 14.14</td>
<td>13.9</td>
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<tr>
<td></td>
<td></td>
<td>Analytical 25.04</td>
<td>20.56</td>
</tr>
<tr>
<td></td>
<td></td>
<td>%Diff 15.58</td>
<td>14.007</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Analytical 29.51</td>
<td>19.64</td>
</tr>
<tr>
<td></td>
<td></td>
<td>%Diff 6.2012</td>
<td>5.04</td>
</tr>
</tbody>
</table>

Table 4: Comparison of Analytical and ANSYS Results

VIII. CONCLUSION

Thus, it is concluded that,

1) Theoretical calculated values by using different formulas are very close to that of the values obtained from ANSYS is suitable for multilayer pressure vessels.

2) Owing to the advantages of the multi layered pressure vessels over the conventional mono block pressure vessels, it is concluded that multi layered pressure vessels are superior for high pressures and high temperature operating conditions.

3) Multi-layering of pressure vessel is very useful in high pressure applications. By comparison it is observed that increasing the number of layers reduces the hoop stresses in pressure vessel.

4) Multi-layering of vessel decreases the hoop stresses at innermost surface of pressure vessel and it can decreases the difference between maximum and minimum hoop stress as compared to mono block (1-layer) vessel.

5) Theoretical calculated values by using different lame’s formula is very close to that of values obtained from ANSYS WORKBENCH 14.5 Finite element code results. This indicates that ANSYS analysis is suitable for multi layered pressure vessels. The difference is due to the numerical techniques.

IX. ACKNOWLEDGMENT

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REFERENCES


