A New S-(a,d) Antimagic Labeling of a Class of Generalized Petersen Graphs

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Abstract— A connected graph G is said to be (a,d)-antimagic, for some positive integers a and d, if its edges admit a labeling with the integers 1,2, ..., $|E(G)|$, such that the induced vertex labels forms and arithmetic progression with the first term a and the common difference d. In this paper we have investigated a new S-(a,d)-antimagic labeling of Generalized Petersen graphs $P(N,2)$, where $N = \frac{3n+7}{2}$, for $n \equiv 1(mod4)$ and $n \geq 5$.

Key words: (a,d) Antimagic Labeling, S-(a,d) Antimagic Labeling, Generlaized Petersen Graph

I. Introduction

A Graph Labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Most graph labeling methods trace their origin to the one introduced by Rosa in 1967, or the one given by Graham and Sloane in 1980 [4].

Labeled graphs are becoming an increasingly useful family of mathematical models for a broad range of applications. Qualitative labeling of graph elements have inspired research in diverse fields of human enquiry such as conflict resolution in social psychology, electrical circuit theory and energy crisis etc. Quantitative labeling of graphs have led to quite intricate fields of application such as coding theory problems, including the design of good Radar location codes, Synch-set codes; Missile guidance codes and convolution codes with optimal auto correlation properties [3].

Hartsfield and Ringel [7] introduced antimagic graphs in 1990. A Graph with q edges can be labeled with 1,2, …, q without repetition, such that the sums of the labels of the edges incident to each vertex are distinct. Hartsfield and Ringel conjecture that every tree other than K_2 is antimagic and more strongly, that every connected graph other than K_2 is antimagic.

The concept of (a,d) - antimagic labelings was introduced by Bodendiek and Walther in 1993. A connected graph G = (V,E) is said to be (a,d)-antimagic, if there exist positive integers a, d and a bijection $f: E \rightarrow \{1,2,\dots,|E|\}$ such that the induced mapping $g_f : V \rightarrow W$, defined by $g_f(v) = \sum\{\frac{f(uv)}{uv} \in E(G)\}$, is injective and $g_f(V) = \{a, a+d, a+2d, \dots \dots a+(|V|-1)d\}$ [4].

The Generalized Petersen Graphs P(n,k), $1 \leq k \leq \frac{n}{2}$, consist of an outer n-cycle $y_1, y_2, y_3, \dots, y_n$, a set of n spokes $x_i y_i$, $(1 \leq i \leq n)$ and n inner edges $x_i x_{i+k}$, $1 \leq i \leq n$ with indices taken modulo n. The Standard Petersen Graph is the instance P(5,2). Generalized Petersen Graphs were first defined by Watkins [6]. Since P(n,k)'s form an important class of 3- regular graphs with 2n vertices and 3n edges, it is desirable to determine which P(n,k)'s are (a,d)-antimagic [8].

Bodendiek and Walther [2] conjectured that P(n,1) is $(\frac{7n+4}{2},1)$ - antimagic for even n and P(n,1) is $(\frac{5n+5}{2},2)$ - antimagic for odd n. These Conjectures were proved in [1], where it was also shown that P(n,1) is $(\frac{3n+6}{2},3)$-antimagic for even n.

Mirka Miller and Martin Bača [6] proved that P(n,2) is $(\frac{3n+6}{2},3)$-antimagic for n≡0 (mod 4), n ≥ 8 and conjectured that P(n,k) is $(\frac{3n+6}{2},3)$ –antimagic for even n and $2 \leq k \leq \frac{n}{2} - 1$. XirongXu, Jun-ming Xu, Min Lü NanCao proved that P(n,3) is$(\frac{3n+6}{2},3)$- antimagic for n≥6. And they also proved the Generalized Petersen Graph P(n,2) is $(\frac{3n+6}{2},3)$- antimagic for n≡2(mod 4), n ≥ 10 [8].

II. S- (a,d) Antimagic Labeling of a Graph

We define S-(a,d)- Antimagic Labeled Graph to be a graph such that there exist positive integers a, d and a bijection $f: E \rightarrow \{1,2,\dots,|E|\}$ such that the induced mapping $g_f: V = V' \cup V'' \rightarrow W$, defined by $g_f(v) = \sum\{f(uv)/uv \in E(G)\}$, is injective where the labeling of the vertices in V' is such that $g_f(V') = \{a_i, a_i + d_i, \dots a_i + (|V'| - 1)d_i\}$ and certain distinct labels are assingned to vertices of V'', where $V'' = V/V'$.

The vertex weight wt(x) of a vertex $x \in V$, under a labeling $\alpha: V \cup E \rightarrow \{1,2,\dots,n+e\}$, is the sum of values $\alpha(xy)$ assigned to all edges incident to a given vertex x together with the value assigned to x itself [5].

III. S – (a,d) Antimagic Labeling of Generalized Odd Petersen Graph

A. Theorem:

The Generalized Petersen Graph, P (N,2) where $N = (\frac{3n+7}{2})$, for $n \equiv 1(mod4)$, $n \geq 5$ is antimagic.

B. Proof:

Let $V = \{y_1, y_2, \dots, y_n, x_1, x_2, \dots, x_n\}$, where $y_1, y_2, \dots, y_n$ are vertices of outercycle and $x_1, x_2, \dots, x_n$ are vertices of inner cycle. Let $V' = \{y_2, y_3, \dots, y_n, x_4, x_5, \dots, x_{n-1}\}$ and $V'' = \{y_1, x_1, x_2, x_3, x_n\}$.

Define the edge labeling f of $P(\frac{3n+7}{2},2)$ $n \equiv 1(mod4)$ as follows: $f(y_i y_{i+1}) = i, 1 \leq i \leq n-1$

$$f(y_i x_i) = n + i, 1 \leq i \leq n-1$$

$$f(x_i x_{i+2}) = \begin{cases} 2n + i, & i = 1 \\ 2n + 1 + i, & 2 \leq i \leq n-1 \\ 2n + 2, & i = n \end{cases}$$

It is easy to verify that the labeling f assigns each integer $1,2,\dots,3n$, exactly once. And this implies that the labeling f is a bijection from the edge set E(P(N,2)) where $N = \frac{3n+7}{2}$, to the set {1,2, …, 3N}.

Let us denote the weights (under the edge labeling f) of vertices x_i and y_i of P(N,2) where $N = \frac{3n+7}{2}$, by $w(y_i) = f(y_i y_{i+1}) + f(y_i x_i) + f(y_i y_{i-1}), 1 \leq i \leq n$ and $w(x_i) = f(x_i y_i) + f(x_i x_{i+2}) + f(x_i x_{n+i-2}), 1 \leq i \leq n$, with the indices taken modulo n.

The weights of vertices of P(N,2), $N = \frac{3n+7}{2}$, under the edge labeling f constitute the sets.

$$W_1 = \{2n + 2\} = w(y_1)$$
$$W_2 = \{(n + i) - 1\,; 2 \leq i \leq n\} = w(y_i)$$
$$W_3 = \{(6n + 2)\} = w(x_1)$$
$$W_4 = \{(5n + 7)\} = w(x_2)$$
$$W_5 = \{(5n + 8)\} = w(x_3)$$
$$W_6 = \{(5n + 3i); 4 \leq i \leq n - 1\} = w(x_i)$$
$$W_7 = \{(7n + 1); i = n\} = w(x_n)$$

We can see that each vertex of P(N,2), where $N = \frac{3n+7}{2}$, receives exactly one label of weight from $W = W_1 \cup W_2 \cup W_3 \cup W_4 \cup W_5 \cup W_6 \cup W_7$ and each number from W is used exactly once as a label of a vertex.

For the vertices y_i, $(2 \leq i \leq n)$, the (a,d) antimagic labeling of P(N,2) is such that $a = (n + 3i) - 1$, and $d = 3$. And for the vertices x_i, $(4 \leq i \leq n - 1)$, the (a,d) antimagic labeling is such that $a = (5n + 3i)$ and $d = 3$.

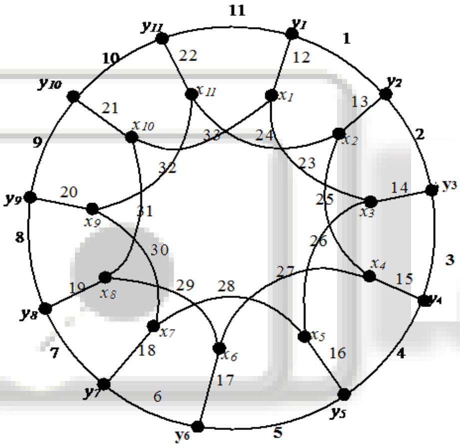

Fig. 1: S-(a,d) Antimagic labeling of Generalized Petersen Graph P(11, 2)

w (y_1) = 24, w (y_2) = 16, w (y_3) = 19 , w (y_4) = 22, w (y_5) = 25, w (y_6) = 28, w (y_7) = 31

w (y_8) = 34, w (y_9) = 37, w (y_{10}) =35, w (y_{11}) =43.

w (x_1) = 68, w (x_2) = 62, w (x_3) = 63,w (x_4) = 67, w (x_5)=70, w(x_6)=73, w(x_7)=76, w(x_8) = 79, w (x_9) = 82, w (x_{10}) =85, w (x_{11}) = 78.

IV. Conclusion

In this paper we have investigated the S – (a,d) Antimagic labeling of a class of Generalised odd Petersen graph. Further we intend to study the problem for another class of Petersen graph.

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