A New S-(a,d) Antimagic Labeling of a Class of Circulant Graphs

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Abstract— A connected graph G is said to be (a,d)-antimagic, for some positive integers a and d, if its edges admit a labeling with the integers 1,2, ..., $|E(G)|$ such that the induced vertex labels forms an arithmetic progression with the first term a and the common difference d. In this paper we have investigated a new S-(a,d)-antimagic labeling of Circulant graphs $G[n,\pm\{1,2\}]$, n odd where $n \geq 5$.

Key words: (a,d) Antimagic Labeling, S-(a,d) Antimagic Labeling, Circulant Graph

I. INTRODUCTION

A Graph Labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Most graph labeling methods trace their origin to the one introduced by Rosa in 1967, or the one given by Graham and Sloane in 1980 [1].

One of the important areas in graph theory is gaph labeling which is used in many applications like Coding theory, radar, astronomy, circuit design, missile guidance, communication network addressing, X-ray crystalography, database management, etc. Further enhancements for the graph labeling should be used in cloud computing, signal processing etc[5].

Hartsfield and Ringel [4] introduced antimagic graphs in 1990. A Graph with q edges can be labeled with 1,2, …, q without repetition, such that the sums of the labels of the edges incident to each vertex are distinct. Hartsfield and Ringel conjecture that every tree other than K_2 is antimagic and more strongly, that every connected graph other than K_2 is antimagic.

The concept of (a,d)-antimagic labelings was introduced by Bodendiek and Walther in 1993. A connected graph G= (V, E) is said to be (a,d)-antimagic if there exist positive integers a, d and a bijection $f: E \rightarrow \{1,2,\dots,|E|\}$ such that the induced mapping $g_f : V \rightarrow W$, defined by $g_f(v) = \sum\{\frac{f(uv)}{uv} \in E(G)\}$, is injective and $g_f(V) = \{a, a+d, a+2d, \dots\dots a+(|V|-1)d\}$ [1].

A Circulant undirected graph, denoted by $G[n,\pm S]$ where $S \subseteq \{1,2,\dots,\left\lfloor\frac{n}{2}\right\rfloor\}$,$n \geq 3$ is defined as an undirected graph consisting of the vertex set $V = \{0,1,\dots,n-1\}$ and the edge set $E = \{ij : \text{there is } s \in S \text{ such that } |j-i| \equiv s (\text{mod } n)\}$ [2].

K.A.Sugeng and N.H.Bong [3] shows how to construct Vertex (a,d)- Antimagic total labeling for circulant graphs $C_n(1,2,3)$, for $d = 0,1,2,3,4,8$.

II. S- (A,D) ANTIMAGIC LABELING OFA GRAPH

We define S-(a,d)- Antimagic Labeled Graph to be a graph such that there exist positive integers a, d and a bijection $f: E \rightarrow \{1,2,\dots,|E|\}$ such that the induced mapping $g_f: V = V' \cup V'' \rightarrow W$, defined by $g_f(v) = \sum\{f(uv)/uv \in E(G)\}$, is injective where the labeling of the vertices in V' is such that $g_f(V') = \{a_i, a_i+d_i, \dots a_i+(|V'|-1)d_i\}$, and certain distinct labels are assingned to vertices of V'', where $V'' = V/V'$.

The vertex weight $wt(x)$ of a vertex $x \in V$, under a labeling $\alpha: V \cup E \rightarrow \{1,2,\dots,n+e\}$,is the sum of values $\alpha(xy)$ assigned to all edges incident to a given vertex x together with the value assigned to x itself.[3]

III. S – (A,D) ANTIMAGIC LABELING OF ODD CIRCULANT GRAPH

A. *Theorem:*

S-(a,d)-Antimagic Labeling of Circulant graphs $G[n,\pm\{1,2\}], n$ odd, $n \geq 5$.

B. *Proof:*

Let $V = \{y_1, y_2, \dots, y_n\}$, where $y_1, y_2, \dots, y_n$ are vertex set of Circulant graph Let $V' = \{y_3, y_4, \dots, y_n,\}$ and $V'' = \{y_1, y_2\}$

Define the edge labeling f of $G[n,\pm\{1,2\}]$ as follows,

$$f(y_i y_{i+1}) = i, (1 \leq i \leq n)$$
$$f(y_i y_{i+2}) = n+i, (1 \leq i \leq n)$$
$$f(y_i\, y_{n+i-2}) = \begin{cases} 2n-1, & i=1 \\ 2n, & i=2, \\ n+i-2, & 3 \leq i \leq n \end{cases}$$

It is easy to verify that the labeling f uses the integer $1,2,3,\dots,2n$ exactly once and this imply that the labeling f is a bijection from the edge set $E(G[n,\pm\{1,2\}])$ to the set$\{1,2,\dots 2n\}$.

Let us denote the weights (under an edge labeling f) of vertices y_i of $G[n,\pm\{1,2\}]$ by

$w(y_i) = f(y_i y_{i+1}) + f(y_i y_{i+2}) + f(y_i y_{i-1}) + f(y_i y_{n+i-2})$ for $1 \leq i \leq n$. The weights of vertices of $G[n,\pm\{1,2\}]$ under the edge labeling f constitute the sets

$$W_1 = \{w(y_i) : i = 1\} = \{4n+1, i = 1\}$$
$$W_2 = \{w(y_i) : i = 2\} = \{3n+5, i = 2\}$$
$$W_3 = \{w(y_i) : 3 \leq i \leq n\} = \{2n+4i-3, 3 \leq i \leq n\}$$

We can see that each vertex of $G[n,\pm\{1,2\}]$ receives exactly one label of weight from $W = W_1 \cup W_2 \cup W_3$, and each number from W is used exactly once as a label of a vertex.

For the vertices y_i $(3 \leq i \leq n)$, the (a,d)-antimagic labeling of Circulant graphs $G[n,\pm\{1,2\}]$, is such that $a = 2n+4i-3$, and $d = 4$.

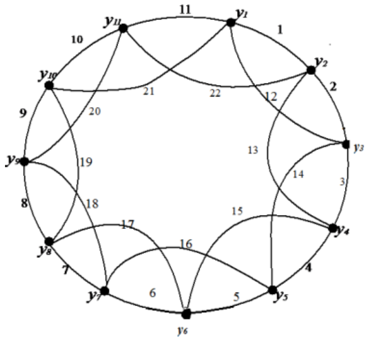

Fig. 1: S-(a,d) Antimagic labeling of Circulant Graph $G[11,\pm\{1,2\}]$

$w(y_1) = 45$, $w(y_2) = 38$, $w(y_3) = 31$, $w(y_4) = 35$, $w(y_5) = 39$, $w(y_6) = 43$, $w(y_7) = 47$, $w(y_8) = 51$, $w(y_9) = 55$, $w(y_{10}) = 59$, $w(y_{11}) = 63$.

IV. Conclusion

In this paper we have investigated the S – (a,d) Antimagic labeling of a class of odd Circulant graph. Further we intend to study the problem for another class of Circulant graph.

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