Abstract— In an automobile or any other such system
transmission unit is used to transfer power from engine
 crankshaft to the drive wheel through reduction unit and
differential. Epicyclic Gear Trains (EGTs) are used to
achieve the desired speed ratios or reduction ratios with
compact arrangement. For all generated non-isomorphic
EGTs of given number of links and Degree of Freedom
(DOF), kinematic analysis is done to compare their gear
ratios, power or torque distribution. In this paper Lever
Analogy method is used to analyse the velocity of EGTs.
Key words: Velocity Analysis, Lever Analogy

I. INTRODUCTION
Analysing EGTs in terms of torque and speed calculations
by analytical method is tedious as the number of links is
increased from lower number to higher numbers. Lever
analogy method of analysis is used to identify the
arrangements of EGTs. Using lever analogy method is more
advantage in the case of connection of two planetary gear
sets [1]. Using the lever analogy for EGTs, the essential
functions of transmissions of EGTs are easily visualized
without actually going in to the complexities of EGTs.

II. METHODOLOGY
The motion of the lever about the reaction point represents
rotational velocities. Using this analogy speed and torques
calculations become easy. In this method each gear set is
represented by a vertical lever. Firstly the EGT normally
represented by a functional diagram, called as stick diagram,
is replaced by lever i.e. each gear in the EGT is replaced by
a lever. Then each lever is rescaled according to
interconnection between pair of gear wheels and based on
the gear pairs the connections are identified. Proportion of
lever is determined by number of teeth on gears like sun,
planet and annulus wheels. Finally the levers are combined
to identify the connections [2].

Tangential velocities are determined at the pairy
point. Torque ratio can be calculated for sun and annulus
gears. Angular speed and torques can also be determined.
Complex mechanical systems become simpler with this
analogy.

The following examples are used to explain the
concept of lever analogy for EGTs. The fundamental
element of an EGT is a set of elementary gear elements,
namely a sun gear (s), a ring gear (R), set of planet gears (p)
and a carrier. The schematic representation of a simple EGT
is shown in Fig: 1 and Fig: 2 is the graphical representation.

Fig. 1: Schematic diagram of an EGT

Fig. 2: Graph of an EGT

Fig. 3: Lever Representation of EGT

If the carrier C is made stationary, the ring gear R is given
an input speed of +1 and taking moments about the carrier
C, then the sun gear speed is \((-\frac{Z_r}{Z_s})\) where Zr and Zs are the
number of teeth on ring and sun gears respectively.

Treating carrier, ring gear and sun gear as three
nodes connected by a line, the distance from C to R is equal
to Zs and the distance from C to S is equal to Zr. This is
something like a force at the distance from C to R is equal to
Zs applied at R and another force at the distance from C to S
is equal to Zr applied at S along a lever RCS [3]. This is a
representation of an EGT elements Sun gear, ring gear and
carrier by a lever with S, R and C as the nodes of a vertical
line and is this line with these nodes is known as a lever as
shown in Fig: 5.23. Thus rotational speeds of sun gear, ring
gear and carrier are computed by treating rotational speeds
like forces acting normal to the lever and moments can be
calculated at the appropriate nodes on the lever.

Let \(\omega_S, \omega_R, \omega_C\) are the angular velocities of the sun,
ring and the carrier and Zs and Zr are the number of teeth on
sun and ring gears respectively. The ratios of speeds and
number of teeth of the meshing gears is given by Eq. (2.1)
\[
\frac{\omega_S - \omega_C}{\omega_R - \omega_C} = \left(\frac{-Z_r}{Z_s}\right) \quad (2.1)
\]

Eq. (5.20) is further simplified to Eq. (2.2)
\[
\omega_C = \frac{(\omega_S Z_s + \omega_R Z_r)}{(Z_s + Z_r)} \quad (2.2)
\]

The geometric form of a lever for an EGT with basic
elements is analytically represented by the Eq. (2.2) and is
applicable to all types of EGTs. Using the Eq. (2.2) the
angular velocity of the carrier can be found knowing values
of the other four variables [4].
\[
\frac{2\pi r_1}{2\pi} = \frac{Z_1}{Z_2} \quad \text{and} \quad v = \omega_1 r_1 = \omega_2 r_2 \quad (2.3)
\]
Also it is known that \( T_1 = F_1 r_1 \) and \( T_2 = F_2 r_2 \).

Now using above three relations one can write

\[
\omega_1 = \omega_2 = \frac{n_2}{n_1} = \frac{r_2}{r_1}
\]

For clear understanding of lever analogy concept the elements (nodes) in lever shown in Fig: 3 can be assigned with some variables like

\[
\omega_s = n, \ Z_s = e, \ \omega_r = l, \ Z_r = 1, \ \omega_c = m
\]

substituting these assigned variables in Eq. (2.2) we get

\[
m = \frac{(n * e) + (1 * l)}{1 + e}
\]

Rearranging further we get another Eq. (2.4)

\[
n = m \left( \frac{1}{1 + e} + \frac{1}{2} \right) - \frac{l}{2} \tag{2.4}
\]

If any two variables in the Eq. (2.4) are specified, the remaining variable can be calculated knowing the value of the parameter ‘e’. Mapping or replacing the parameter value in different ways can induce different possibilities of nodes. Three nodes on the lever represent a planetary gear set and possible arrangement of these three nodes result in six levers or EGTs. They are the permutations like \((l, m, n), (l, n, m), (m, n, l), (m, l, n), (n, l, m)\) and \((n, m, l)\). All these lever arrangements are obtained by mapping the e value. For example, mapping ‘e’ value with \((1/e)\) the Eq. (2.4) becomes Eq. (2.5).

\[
n = m \left( \frac{1}{1 + e} + \frac{1}{2} \right) - \frac{l}{2} \tag{2.5}
\]

By rearranging Eq. (2.5) we get\( l = m \left( 1 + \frac{1}{e} \right) - n \left( \frac{1}{e} \right) \) which is the lever analogy representation with nodes \((n, m, l)\). With the similar replacements of ‘e’ values one can get six possibilities of lever representation. Values of this parameter in certain range can be used in high speed gear train applications and this range depends on gear train packaging, pinion gear sizes, pinion speeds. Thus single analysis is enough instead of several individual analyses of various graphs.

Consider a system of two EGTs which consists of two planets, two sun gears and two ring gears along with two carriers. Input speed comes from the driving source like a motor for a gear train shown in Fig: 4. Motion is transferred from driving source to a member of the gear train in the form of fixed or clutched interconnection [5]. Some of the linear equations can be eliminated when the clutch is not engaged. Let the teeth on ring gear for the first and second gear set be \( r_1 = 50, r_2 = 40 \), and teeth on sun gears be \( s_1 = 30, s_2 = 20 \) respectively.

Fig. 4: Two planetary gear set

![Fig. 4: Two planetary gear set](image)

Fig. 5: Lever

A linear equation that relates the speeds of the various elements in lever graph is identified. Fixed connection to the casing is a constraint to the system. To analyse two planetary gear pair system with one fixed interconnection, it is necessary to draw all the possible arrangements of nodes. Then the required algebraic equations are identified.

The two EGSs are individually shown as levers in Fig: 5 along with the number of teeth on the gears and then the interconnections between gear sets are replaced by links normal to the lever at appropriate places. Levers connected by a pair of parallel lines normal to the lever can be replaced by a single lever with the same dimensions between the points. These normal lines are used for scaling the levers.

![Fig. 5: Lever](image)

Fig. 6: Combined Lever Diagram

The combination of two levers with fixed interconnections is shown in Fig: 6 and this can be further developed after scaling to make the combined levers into a single lever with two gear sets which is shown in Fig: 5.7 and Fig: 5.8.

![Fig. 6: Combined Lever Diagram](image)

Fig. 7: Scaling of two levers

![Fig. 7: Scaling of two levers](image)
EGTs are readily adapted to automatic control. Some EGTs are designed to change velocity ratios simply by using electrically or hydraulically operated band brakes to keep one or more of the gears stationary [6]. Other EGTs operating with fixed velocity ratios are selected for their compact design and high efficiency. Lever analogy translates a planetary gear set to a lever of certain amount of nodes. The generalized lever is a useful extension of traditional concept of an EGT. It is possible to determine the angular velocities and different possible arrangements of the EGTs using this method.

REFERENCES