“Application of Applied Element Method for Dynamic Analysis”

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Abstract— The objective of present study is to understand application of AEM for dynamic analysis of structure. In applied element method structure is assumed to be divided into number of rigid elements connected by spring. Springs represents forces displacement relationship of structural elements. So, derivation of stiffness matrix in AEM is different than that of FEM. The present study deals with derivation of stiffness matrix and mass matrix for one dimensional and two dimensional elements. The application of Applied Element Method is illustrated through dynamic analysis of cantilever, beam is divided into four number of elements. The elements are connected through the springs, and natural frequency is also presented.

Key words: Dynamic Analysis, FEM, AEM

I. INTRODUCTION

Damages to building during natural and manmade disasters have clearly shown behavior of the structure under static or dynamic loading in vertical or lateral direction. Deformation capacities of the individual components of the structure decide the strength of a structure. In order to determine capacity of any structure beyond the elastic limits some form of nonlinear analysis such as the pushover procedure are performed. Usually seismic demands are computed by nonlinear static analysis of the structure by applying monotonically increasing lateral forces with constant distribution of the forces throughout height until a target displacement is reached. However, clear understanding about the performance of structure under critical dynamic loading is difficult to understand by following nonlinear static procedure, for this purpose, highly efficient numerical modeling procedures are required. Currently available numerical method for structural analysis can be classified into two categories as Continuum Method and Discrete Element Method. Finite Element Method (FEM) is Continuum method while Rigid Body and Spring Model (RBSM) and Extended Distinct Element Method (EDEM) are Discrete Element techniques.

Recently, a new displacement based method has been developed known as Applied Element Method (AEM) [1]. In AEM structural member is divided into virtual elements connected through normal and shear spring representing stresses and strains within the structure. AEM has the capability of simulating behavior of structure from zero loading to collapse. FEM assumes the material as continuous and is able to indicate highly stressed region of structure but it is difficult to model separation of element unless crack location is known. The main advantage of Applied Element Method is that it can track the structural collapse behavior passing through all stages of the application of load loads, elastic stage, crack initiation and propagation in tension-weak material, reinforcement yielding, element separation, element collision (contact), and collision with the ground and with adjacent structures. It can also be used for modeling large displacement and separations of structural element [2].

In this paper formulation of stiffness matrix for generalized two dimensional elements is presented. The factor affecting the accuracy of AEM like size of element and number of connecting springs are studied.

II. APPLIED ELEMENT METHOD

In AEM, a structure is modeled by virtually dividing in into an assembly of small elements are connected through series of normal and shear springs located at contact points that are distributed over the surface of each element. At each contact point, there is one normal spring and one shear springs for two dimensional problems. While one normal spring and two shear spring in orthogonal directions are considered for three dimensional problem. Fig.1 illustrates the division of a structure into elements and shows the connection of elements through springs.

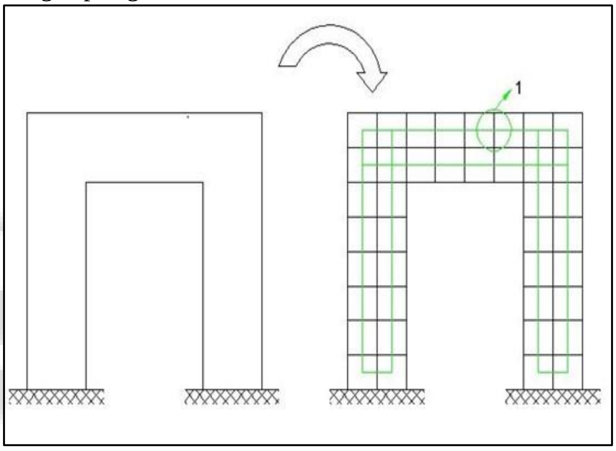
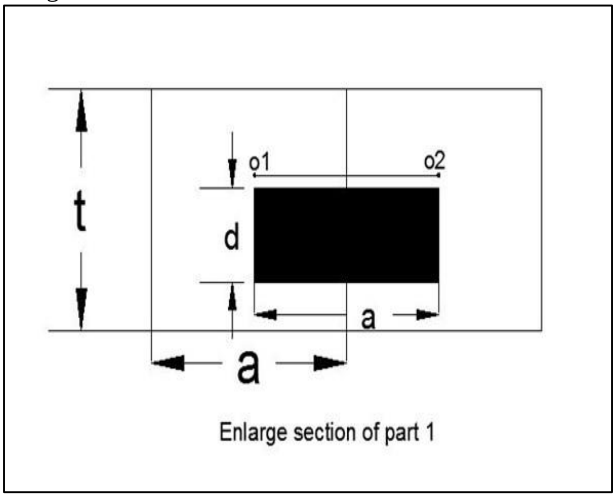


Fig. 1: Division of structure into various elements [1]



The area of influence on each element for a set of springs is also highlighted in fig.2. The stiffness in normal and tangential direction is given by:

$$K_{normal} = \frac{E x d x t}{a} \quad K_{shear} = \frac{G x d x t}{a} \tag{1.1}$$

Where, E and G are Young's Modulus and Shear Modulus respectively, 'd' is the distance between spring, 't' is the thickness, and 'a' is the

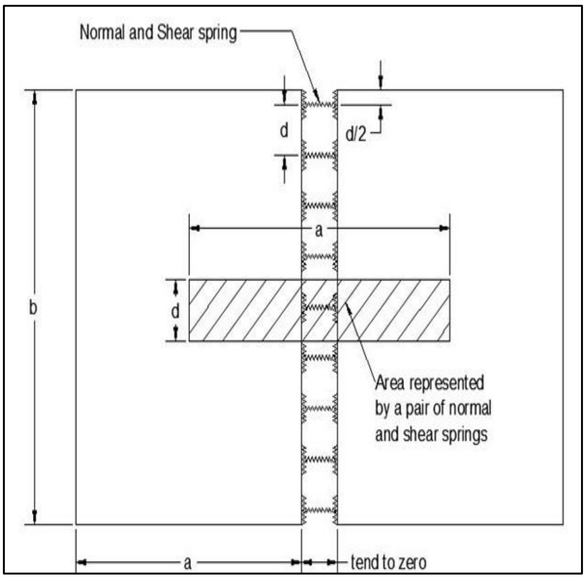


Fig. 2: Distribution of springs and area of influence of each spring [1]

Length of the representative area. Equation 1.1 simply represents the axial stiffness of spring. Each element has three degree of freedom at its centroid representing the rigid body motion of the element. Deformation in each set of spring on an elements surface can be geometrically related to degree of freedom at centroid, thus creating a stiffness matrix for that set of springs. The element stiffness matrix is then created by summing up the stiffness matrices of each individual set of springs. The model can then be analyzed by utilizing the following equation:

$$[F] = [\Delta] \qquad (1.2)$$

Where, [F] is the applied load vector, [Δ] is the displacement vector, and [KG] is the global stiffness matrix.

The advantage of the AEM arises from the use of springs to connect adjacent elements. For each set of normal and shear springs, stress and the corresponding strain is calculated throughout the loading. Considering material properties, the maximum force that can be resisted by spring can be determined. Once the maximum force is reached, springs are cut or it will not be considered in further analysis. This can occur anywhere within the model, therefore no pre-conceived location of cracks is necessary. Crack propagation follows the same principles. If all of the springs connecting an element are cut, the element is allowed to separate from the structure. In a dynamic analysis, the element has an assigned mass and generates inertial forces. The ability to cover this vast range of structural behavior in a single model is the main difference of the AEM from other analysis methods.

III. FORMULATION OF STIFFNESS MATRIX OF 2-D ELEMENT

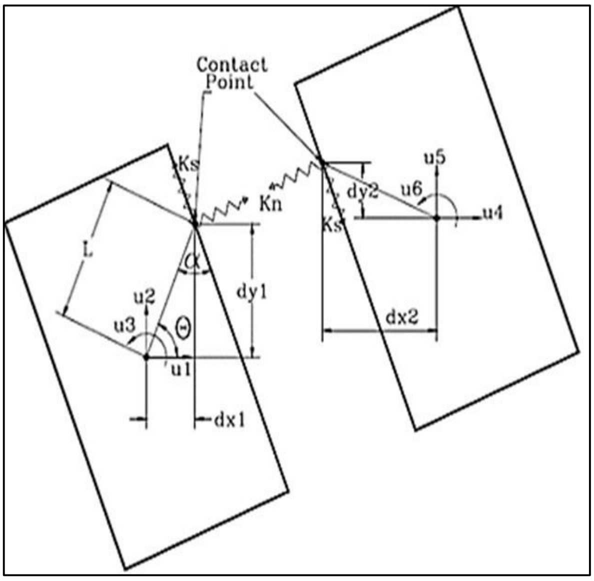


Fig. 3: General position of deformed element [1]

Fig. 3 shows the general position of an element. The two elements are assumed to be connected with pair of normal and shear springs at each contact points distributed along the faces of each element. Coordinates (dx, dy) of each contact point are obtained with respect to centroid of each element. Two transverse and one rotational degree of freedom are considered at the centroid of an element. The stiffness matrix components corresponding to each degree of freedom are determined by applying a unit displacement in the respective direction and by determining forces at the centroid of each element. The element stiffness matrix size is 6 × 6. Equation 3 shows the components of upper left quarter of the stiffness matrix.

$$\begin{bmatrix} K_n\sin(\theta+\alpha)\sin(\theta+\alpha) & -K_n\sin(\theta+\alpha)\cos(\theta+\alpha) & K_s\cos(\theta+\alpha)L\sin(\alpha) \\ +K_s\cos(\theta+\alpha)\cos(\theta+\alpha) & +K_s\cos(\theta+\alpha)\sin(\theta+\alpha) & -K_n\sin(\theta+\alpha)L\cos(\alpha) \\ -K_n\sin(\theta+\alpha)\cos(\theta+\alpha) & K_n\sin(\theta+\alpha)\sin(\theta+\alpha) & K_s\sin(\theta+\alpha)L\sin(\alpha) \\ +K_s\cos(\theta+\alpha)\sin(\theta+\alpha) & +K_s\cos(\theta+\alpha)\cos(\theta+\alpha) & +K_n\cos(\theta+\alpha)L\cos(\alpha) \\ K_s\cos(\theta+\alpha)L\sin(\alpha) & K_s\sin(\theta+\alpha)L\sin(\alpha) & K_sL\sin(\alpha)L\sin(\alpha) \\ -K_n\sin(\theta+\alpha)L\cos(\alpha) & +K_n\cos(\theta+\alpha)L\cos(\alpha) & +K_nL\cos(\alpha)L\cos(\alpha) \end{bmatrix}$$

IV. FORMULATION OF MASS MATRIX OF 2-D ELEMENT

The beam element with associated nodal degree of freedom (transverse displacement and rotation) is shown in fig 4.

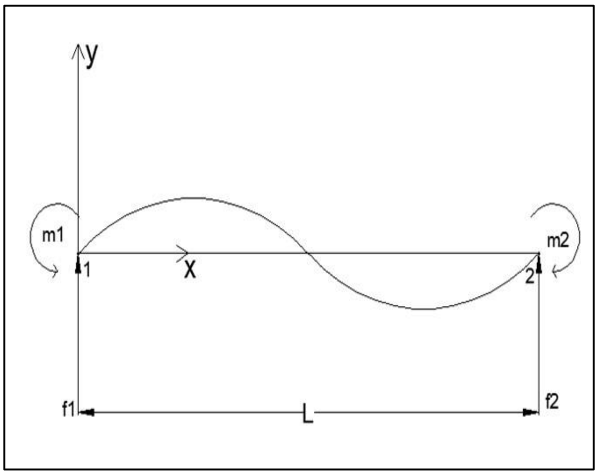


Fig. 4: Beam element with positive nodal displacements, rotations, forces and moments. [1]

The consistent mass matrix can be obtained by applying the equation 1.3 for the beam element, where the shape function are now given below.

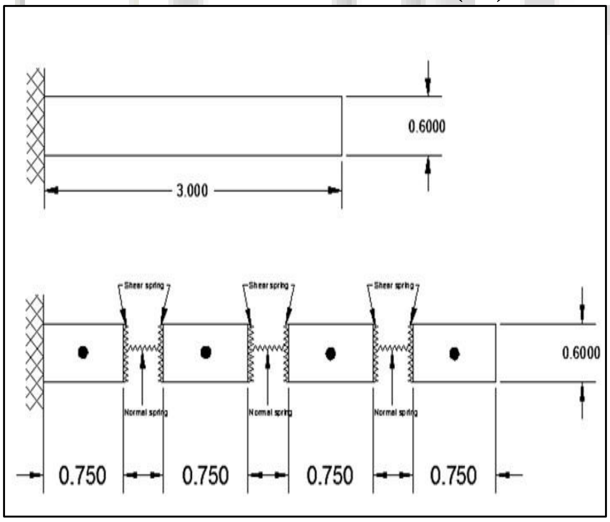
$$[m] = \iiint \rho [N]^T [N] dv \quad ^{[*1]} \qquad (1.3)$$

Where N is shape function,

Substituting the shape function in equation 1.3 and performing the integration, the consistent mass matrix becomes

[M]=

$$\frac{\rho AL}{420}\begin{pmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{pmatrix} \qquad (1.4)$$



The beam is divided into 4 equal size of elements, as dynamic analysis is carried out in present study, the reinforcement in the beam is neglected. In this paper only least nodal frequency is presented. From the nodal degree of freedom stiffness and mass matrix is obtained. Fundamental natural frequency is obtained by Jacobi iteration.

After Applying boundary condition Fundamental frequency are obtained as follows.

$\Omega 4 = 0.138$ rad/sec

V. Conclusion

- In AEM fundamental natural frequency in element are represented by frequency in spring connecting the elements. Two types of springs i.e. normal and shear, if considered in various directions model one-dimensional, two dimensional and three dimensional problems.Elements connected by one shear and one axial spring represents two dimensional problem.
- Stiffness matrix in AEM is derived by adding the forces in spring between the elements. Mass matrix is derived by using equation 1.3

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LIST OF USEFUL BOOK

*1 a first course in the Finite Element Method by Daryl L. Logan

