Application of Queuing Model on Hospital OPD

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Abstract— Waiting in line is a very hectic situation for everybody. At present It is general scenario that waiting in queue for getting service is a part of life I everybody life. Wherever we go like shopping mall, railway reservation counter, petrol pump, hospital we have seen that the people are standing in a queue to get services. Queuing is a big challenge for health care organization all over the world. This survey probes the application of queuing theory and modeling to the queuing problem at the outpatient department at arogyam hospital Rajkot. In this paper we study what is the required number of servers (doctors) so patients have not to wait in queue and get medical treatment immediately. The analysis of the study with the help of queuing theory shows that they need 5 servers instead of 3 at present.

Key words: Queuing Model, OPD

I. INTRODUCTION

A common situation that occurs in everyday life is that of queuing or waiting in line. Queue generate when demand for service when overstep its supply. Waiting time in a queue is depend on how many customers(patients) in a queue, the number of servers of serving line, and the proportion of service time for each individual customers. Customer has not satisfied with the services and as a result health care institution suffers a loss economically. Customer wasted his time on queue which can utilize it elsewhere. The goal of queuing is therefore to decrease the waiting time in queue.

For this purpose we use queuing theory which is a mathematical study of waiting lines or queues. In queuing theory a model is constructed so that queue length and waiting times can be predicted. Queuing theory is a branch of operation research. Queuing theory started with research by A. K. Erlang in the reference of telephone facilities. Queuing theory has been studied in health care sector since 1952. This vital tool is unfortunately minimally used in most health care system in world. The use of queuing theory and modeling is improving different services in various hospital settings. It has also been used in reducing the unwanted expenditure in hospitals. Long waiting lines have shown the inefficiency of hospital services. Unfortunately this is the case in many hospitals in India. Capacity management decisions in this hospital are based on experience rather than with the help of strategic research model based analysis. Arogyam hospital in Rajkot receives numerous patients every day and this generally results in long patient waiting times. In response to this challenge this paper analyzes the queuing system of the outpatient department of Arogyam hospital in order to develop a model that can help reduce the waiting time of patients. In most health care setting, appointment system is in place; the queue discipline is either first-in-first-out or divided in different classes as patients need treatment. Mcquarrie (183) analyze that it is possible to decrease the waiting times by giving priority to patients who require less service times. One of the major elements in improving efficiency in the delivery of health care services is patient flow. Good patient flow means that patient queuing is less and poor patient flow means that patients suffer a queuing delays a lot. Thus queuing theory and modeling can be useful modern tools for decision making on cases of capacity and resourcing.

II. RESEARCH METHODOLOGY

The study assumes a descriptive and observational study approach. Data were gathered for 30 days in the OPD department of hospital. Primary Data were collected by direct observation, from the appointment records and asking questions to the patients by researcher. Secondary data were collected through interviews with hospital management, doctors and administrative staff. The method used in this research is a quantitative method. The method used in this study were to investigate queuing systems and techniques and also the development of queuing model for the analysis of queuing method establish a method will solve the problem of customer waiting time. The model will find that the actual time it takes to serve the customer as at when due and estimate the actual working servers are necessary in the organization. The model developed was used to analysis the queuing system against the number of servers and customers arrival rate of the establishment.

A. Analysis of the Data

Fig. 1: Analysis of Data

From the above chart it can be seen that total arrival patients for 30 days are 3468. Hence, average arrival rate per day is 3468/30=115 patients per day and per hour c=29 patients are arriving. From the interview with doctors, one doctor can take 5 to 7 min to complete the care the patient. This implies that a doctor can see 12 patients per hour. Some patients have already consulted with doctor but they come again to see their laboratory reports to doctor who are not taking any appointments so they are also included in this service rate. Hence, on an average μ=8 patients per hour will be taken complete care from the doctor who are newly registered patients.

In this hospital there are serving 3 doctors who consult with opd. Doctors are coming in two session morning and evening. So they are almost 8 hours in the hospital but when they come into the hospital first they check their ward patients who are already admitted in the hospital and 1 hour for medical representative and some time they busy for emergency cases if there any. So after, all they can give 4 to 5 hours in the opd section.
1) Using Queuing Model M/M/S for Calculation
The model used in this study is the M/M/S (FIFO) Multi server queuing Model. For this queuing system it is assumed that the arrival of patients follows a Poisson probability distribution at an average customer (patient) per unit of time. It is also assumed that they are served on a first in first out basis on any of the servers (doctors). The service time are distributed exponentially with an average of customer (patient) per unit of time and the number of servers S.

From the model the probability of having n customers in the system is given by

\[ P_n = \left( \frac{\lambda^n}{n!} \right) P_0, \quad n \leq s \]

\[ P_n = \left( \frac{\lambda^n}{n!} \right) P_0, \quad n > s, \quad \rho = \frac{\lambda}{n \mu} \]

\[ P_s = \left[ \sum_{n=0}^{s} \frac{\lambda^n}{n!} \right] P_0 - \frac{\lambda^s}{s!} \rho^s \]

\[ \rho = \frac{\lambda}{n \mu} \]

\[ \omega = \frac{1}{n \mu} \]

We now proceed to compute the performance of the queuing system.

The expected number of customer (patients) waiting on the queue (length of line) is given as:

\[ L_q = \left[ \frac{1}{1-\rho} \right] \frac{\rho^s}{s^2} \] \[ \sum_{n=0}^{s} \frac{\lambda^n}{n!} \]

Equation (2)

\[ L_q = \frac{1}{\rho} \left( \frac{\lambda}{\mu} \right)^2 \]

\[ \mu = \text{the service rate per unit time} \]

\[ \rho = \text{arrival rate of patients per unit time} \]

\[ s = \text{the number of servers} \]

The utilization factor i.e. the fraction of time servers (doctors) are busy

\[ \rho = \frac{\lambda}{s \mu} \]

Where \( \lambda \) is the arrival rate of patients per unit time \( \mu \) is the service rate per unit time \( s \) is the number of servers \( P_0 \) is the probability that there are no customers (patients) in the system

\[ L_q = \text{Expected number of customers in the queue} \]

\[ L_s = \text{Expected number of customers in the system} \]

\[ W_q = \text{Expected time a customer (patient) spends in the queue} \]

\[ W_s = \text{Expected time a customer (patient) spend in the system} \]

From the above equation we calculate \( L_q \), \( L_s \), \( W_q \) and \( W_s \) for 3 servers.

We have \( \lambda = 29, \mu = 8, \rho = 29/8 = 3.625 > 1 \)

First we calculate \( P_0 \) from eqn (1) for \( s = 3 \)

\[ P_0 = \left[ \frac{\sum_{n=0}^{29} \frac{\lambda^n}{n!}}{s!} \right] \frac{1}{\rho} \left( \frac{\lambda}{\mu} \right)^2 \]

\[ = \left[ \frac{\sum_{n=0}^{29} \frac{1}{n!}}{3!} \right] \frac{1}{3.625} \left( \frac{3.625}{8} \right)^2 \]

\[ = -0.391 \]

From equ (2)

\[ L_q = \left[ \frac{1}{1-\rho} \right] \frac{\rho^s}{s^2} \]

\[ L_s = L_q + \frac{1}{s \mu} \]

\[ W_q = \frac{L_q}{\lambda} \]

\[ W_s = \frac{L_s}{s \mu} \]

Similarly we can take case for \( S = 4, 5, 6, 7 \) and calculate all above variables which are shown in the following table.

### Table 1: Calculations

<table>
<thead>
<tr>
<th>Case</th>
<th>No of servers</th>
<th>Arrival rate (( \lambda ))</th>
<th>Service rate (( \mu ))</th>
<th>Prob. that no cust in syst. (Po)</th>
<th>No. of cust. in the que. (Lq)</th>
<th>No. of cust. in the system. (Ls)</th>
<th>Avg time in queue (Ws)</th>
<th>Avg time in syst. (Ws)</th>
<th>Syst. Utilization (( \rho ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>29</td>
<td>8</td>
<td>-3.7%</td>
<td>-8.17</td>
<td>-4.55</td>
<td>-0.281</td>
<td>-0.15</td>
<td>120.83%</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>29</td>
<td>8</td>
<td>1.04%</td>
<td>7.74</td>
<td>11.36</td>
<td>0.266</td>
<td>0.391</td>
<td>90.63%</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>29</td>
<td>8</td>
<td>2.21%</td>
<td>1.10</td>
<td>4.73</td>
<td>0.038</td>
<td>0.163</td>
<td>72.50%</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>29</td>
<td>8</td>
<td>2.53%</td>
<td>0.30</td>
<td>3.94</td>
<td>0.010</td>
<td>0.135</td>
<td>60.42%</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>29</td>
<td>8</td>
<td>2.63%</td>
<td>0.09</td>
<td>3.72</td>
<td>0.003</td>
<td>0.128</td>
<td>51.79%</td>
</tr>
</tbody>
</table>

### III. Discussion of Results

From the above table server utilization is 120.83% in case 1 with 3 doctors and 90.63% in case 2 with 4 doctors i.e. patients has spent less time in the queue and system utilization is good. Same when we compare the case 2 and case 3 server utilization is very good as case 2. Also, from above table we can analyze that waiting time in queue is also decrease. From the case 2 waiting time is \( W_q = 0.038 \) hour and in case 3, \( W_q = 0.010 \). So it can be observed that the number of servers necessary to better serve is 5. This is the appropriate number of servers that can serve the customers without waiting long in queue. From the above table it can be seen that if the number of server is 6 then patient can get service without waiting in queue but if we consider the cost effectiveness 5 servers are considerable for better service to the patient.

### IV. Conclusion

Providing appropriate medical care to patients within time limits is an important factor of healthcare organization to increase patient satisfaction. Patients are generally dissatisfied with long waiting times and it leaves negative effects on patients. Furthermore, queuing theory and modeling is an effective tool that can be provided to make decisions on requirements of staffing for good performance with regards to queuing challenges in hospitals. This analysis should be useful in other hospitals in India and other countries regarding on the usefulness of queuing theory and modeling as a tool for improved decision making with regard to the queuing challenges that are faced by hospitals.

### References

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