A Survey on simulation methods for wave propagation in fiber bragg grating
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Abstract— In recent years, many research and development projects have focused on the study of fiber Bragg gratings. Fiber Bragg gratings have been used in the field of sensors, lasers and communications systems. Commercial products that use fiber Bragg gratings are available. On the other hand, in the field of software development, object-oriented programming techniques are also becoming very popular and powerful. For fiber Bragg grating problems, widely used theories and numerical methods such as the coupled-mode theory and the transfer matrix method will be applied in the analysis, modeling and simulation. The coupled-mode theory is a suitable tool for analysis and for obtaining quantitative information about the spectrum of a fiber Bragg grating. The transfer matrix can be used to solve non-uniform fiber Bragg gratings. Two coupled-fmode equations can be obtained and simplified by using the weak waveguide approximation. The spectrum characteristics can be obtained by solving these coupled-mode equations. Uniform, chirped, apodized, discrete phase shifted and sampled Bragg gratings have already been simulated by using the direct numerical integration method and the transfer matrix method. The reflected and transmitted spectra, time delay and dispersion of fiber Bragg gratings can be obtained by using this simulation program. At the same time, the maximum reflectivity, 3dB-bandwidth and centre wavelength can also be obtained.

Key words: Simulation, Wave propagation, Fiber, bragg grating, Apodization, Couple mode theory, Transfer matrix method, Analytical.

I. INTRODUCTION

In 1978, at the Canadian Communications Research Center (CRC), Ottawa, Ontario, Canada, K.O. Hill et al first demonstrated refractive index changes in a germanosilica optical fibre by launching a beam of intense light into a fiber. In 1989, a new writing technology for fibre Bragg gratings, the ultraviolet (UV) light side-written technology, was demonstrated by Meltzet et al. Fibre Bragg grating technology developed rapidly after UV light side-written technology was developed. Since then, much research has been done to improve the quality and durability of fibre Bragg gratings. Fibre gratings are the keys to modern optical fibre communications and sensor systems. The commercial products of fibre Bragg gratings have been available since early 1995.

II. GRATING FABRICATION TECHNIQUES

The historical beginnings of photosensitivity and fiber Bragg grating (FBG) technology are recounted. The basic techniques for fiber grating fabrication, their characteristics, and the fundamental properties of fiber gratings are described. The many applications of fiber grating technology are tabulated, and some selected applications are briefly described.

There are three type of Fabrication Techniques of Fiber Bragg Grating

(1) Holographic Techniques
(2) Phase Mask Techniques
(3) Point By Point Techniques

Out of these three, the phase mask is the most common technique due to its simple manufacturing process, great flexibility and high performance.

A. Phase Mask Techniques:

Historically, Bragg gratings were first fabricated using the internal writing [1] and the holographic technique. Both these methods, which have been described already, have been largely superseded by the phase mask technique which is illustrated in Fig. 1. The phase mask is made from flat slab of silica glass which is transparent to ultraviolet light. On one of the flat surfaces, a one dimensional periodic surface relief structure is etched using photolithographic techniques. The shape of the periodic pattern approximates a square wave in profile. The optical fiber is placed almost in contact with the corrugations of the phase as shown in Fig. 1. Ultraviolet light which is incident normal to the phase mask passes through and is diffracted by the periodic corrugations of the phase mask.

Fig. 1: Bragg grating fabrication apparatus based on a zero-order nulling diffraction phase mask [1]

III. COUPLE MODE THEORY

Coupled Mode Theory is a method to analyze the light propagation in perturbed or weakly coupled waveguides. The basic idea of the Coupled Mode Theory method is that the modes of the unperturbed or uncoupled structures are defined and solved first. Then, a linear combination of these modes is used as a trial solution to Maxwell’s equations for
complicated perturbed or coupled structures. After that, the derived coupled mode equations can be solved analytically or by numerical methods. The theory assumes that the field of the coupled structures may be sufficiently represented by a linear superposition of the modes of the unperturbed structures. In many practical cases, this assumption is valid and does give an insightful and often accurate mathematical description of electromagnetic wave propagation.

Assuming the electric field is a linear combination of the ideal modes (with no grating perturbation), such that

\[ E(z) = \sum_{i} (a_i^{(+)} \exp(-\beta_i z) + a_i^{(-)} \exp(\beta_i z)) \]

(1)

Where \(a_i^{(+)}\) and \(a_i^{(-)}\) are the slowly varying amplitudes of the \(i\)th mode traveling in the +z and -z directions. \(\beta_i\) is the propagation constant and modal field of the \(i\)th mode. The above electric field is used as a trial solution in the Maxwell’s equation. The following Coupled mode Equations (CMEs) can be derived by using the properties of waveguide modes,

\[ \frac{d a_i^{(+)}(z)}{dz} = -j \sum_{k} \{ a_k^{(+)} K_{ki} \exp[-j(\beta_k - \beta_i)z] + a_k^{(-)} K_{ki} \exp[j(\beta_k + \beta_i)z] \} \]

(2)

\[ \frac{d a_i^{(-)}(z)}{dz} = j \sum_{k} \{ a_k^{(+)} K_{ki} \exp[-j(\beta_k + \beta_i)z] + a_k^{(-)} K_{ki} \exp[j(\beta_k - \beta_i)z] \} \]

(3)

The coupling coefficient between modes \(k\) and \(i\) is given by:

\[ K_{ki} = \frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sqrt{n_e(x,y,z)} \sqrt{n_o(x,y,z)} \sin(\pi n_o(x,y,z) - n_o(x,y,z)) \sin(\pi n_e(x,y,z)) \sin(\pi n_o(x,y,z) + n_o(x,y,z)) \sin(\pi n_e(x,y,z)) \]

(4)

\[ n_e(x,y,z) = n_e^2(x,y,z) - n_o^2(x,y,z) \]

(5)

Where \(n_e(x,y,z)\) is the periodic refractive index perturbation of the grating, and \(n_o(x,y,z)\) is the index profile of waveguide. \(n(x, y, z)\) is the grating index profile.

The well-known transfer matrix method is applied to solve the coupled mode equations and to obtain the spectral response of the fiber grating. In this approach, the grating is divided into uniform sections, each section is represented by a 2x2 matrix. By multiplying these matrices, a global matrix that describes the whole grating is obtained.

The refractive index, inside an \(i\)th uniform section from a Bragg grating, can be described by

\[ n(x, y, z) = n_{eff} + \Delta n \cos \left( \frac{2 \pi x}{\Lambda} \right) \]

(6)

Where \(n_{eff}\) is the effective index of fiber core, \(\Delta n\) is the refractive index amplitude modulation, and \(\Lambda\) is the section grating period.

A non-uniform fiber grating of length \(L\) is divided into \(M\) uniform gratings, i.e., section, as illustrated in Fig.2. The propagation through each uniform section \(i\) is described by a matrix \(F_i\) defined such that

\[ \begin{bmatrix} R_i \\ S_i \end{bmatrix} = F_i \begin{bmatrix} R_{i-1} \\ S_{i-1} \end{bmatrix} \]

(7)

The matrix \(F_i\) for one section is defined by

\[ F_i = \begin{bmatrix} \cos(\gamma_i \Lambda_i) - j \gamma_i \sin(\gamma_i \Lambda_i) & -j \frac{\gamma_i}{2} \sin(\gamma_i \Lambda_i) \\ j \frac{\gamma_i}{2} \sin(\gamma_i \Lambda_i) & \cos(\gamma_i \Lambda_i) + j \gamma_i \sin(\gamma_i \Lambda_i) \end{bmatrix} \]

(8)

Fig. 2: The transfer matrix method applied to obtain the spectral characteristics of a fiber grating[2]

IV. MODELING

In Figure 3, the fiber contains a Bragg grating, of length \(L\) and uniform pitch length \(\Lambda\). The electric fields of the propagating waves can then be expressed as

\[ E_a(z, t) = A(z) e^{i(\omega t - \beta z)} \]

(9)

\[ E_b(z, t) = B(z) e^{i(\omega t + \beta z)} \]

(10)

For the backward and forward propagating waves, respectively

The reflected wave, \(a(0)\), and the transmitted wave, \(b(L)\) can be expressed by means of the scattering matrix

\[ \begin{bmatrix} a(L) \\ b(L) \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a(0) \\ b(0) \end{bmatrix} \]

(13)

Substituting \(a(L)\) and \(b(0)\) from equation (12) into equation (13) we get

\[ S_{11} = S_{22} = -\Delta \beta \sinh(\gamma L) + i \cosh(\gamma L) \]

(14)

Based on equations (13) and (14), the scattering matrix, we can obtain the transfer-matrix, or T-matrix equation
\[
\begin{align*}
\begin{bmatrix}
a(0) \\
b(0)
\end{bmatrix} &=
\begin{bmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{bmatrix}
\begin{bmatrix}
a(L) \\
b(L)
\end{bmatrix}
\end{align*}
\] (15)

Where,
\[
T_{11} = T^{*}_{22} = \frac{\Delta \beta \sinh(SL) + i \cos \theta_1 \cos \theta_2}{i S} e^{-i \phi_0 L}
\]
\[
T_{12} = T^{*}_{21} = \frac{k \sinh(SL)}{i S} e^{-i \phi_0 L}
\] (16)

This matrix approach is effective at treating a single grating as a series of separate gratings each having reduced overall lengths and different pitch lengths, and describing each with its own T-matrix. Combining all the matrices yields the properties of the initial non-uniform grating. The resultant system of matrices is treated as an individual matrix
\[
[T_z] = [T_1] [T_2] \cdots [T_M]
\] (17)

Light passing through successive optical elements can be calculated by series of matrices, as such
\[
\begin{align*}
\begin{bmatrix}
a(0) \\
b(0)
\end{bmatrix} &=
\begin{bmatrix}
T_{M-1} & T_{M-2} \\
& \vdots \\
& T_1
\end{bmatrix}
\begin{bmatrix}
a(L) \\
b(L)
\end{bmatrix}
\end{align*}
\] (18)

V. ANALYTICAL MODEL

The effect of number of grids and grating length which will be useful in designing the wavelength splitter with the help of FBG. The Analytical model has been proposed for the reflectivity of grating which is given by Equation (19).

\[
R = \frac{\sin^2 \left( \frac{\lambda}{2} \right) \left( 1 - \left( \frac{\delta}{\delta} \right)^2 \right)}{\cos^2 \left( \frac{\lambda}{2} \right) \left( 1 - \left( \frac{\delta}{\delta} \right)^2 \right)}
\] (19)

Where ‘l’ is the length of the grating, ‘k’ is the coupling coefficient; δ is detuning factor and \( \frac{\delta}{\delta} \) is the detuning ratio. The detuning parameter for FBG of period is \( \delta = \varepsilon - \frac{\pi}{\lambda} \Delta \) is known as pitch or grating period as in Equation (20).

\[
\Lambda = \frac{1}{N}
\] (20)

N is number of grids or number of grating periods. Pitch of grating also depends upon the value of effective refractive index and Brag wavelength as shown in Equation (21).

\[
\Lambda = \frac{\lambda_B}{\Lambda_{Eff}}
\] (21)

For sinusoidal variation in index perturbation, the coupling co-efficient for 1st order Bragg grating is \( K = \frac{\eta}{\lambda_B} \) where \( \eta \) is overlap integral between forward and reverse propagating mode. \( V \) is normalized frequency as given in Equation (22).

\[
V = Z \frac{n_{Eff} \pi \varepsilon}{\lambda_B}
\] (22)

Where
\( n_{Eff} \) is effective refractive index
\( \varepsilon \) is radius of core
\( \Delta \) is index difference

VI. TWO-MODE COUPLING IN UNIFORM GRATING

A fiber grating is simply an optical diffraction grating, and thus its effect upon a light wave incident on the grating at an angle \( \theta_1 \) can be described by the familiar grating equation.

\[
n \sin \theta_2 = n \sin \theta_1 + \frac{\lambda}{\Lambda}
\] (23)

Where \( \theta_2 \) is the angle of the diffracted wave and the integer \( m \) determines the diffraction order (see Fig. 4). This equation predicts only the directions \( \theta_2 \) into which constructive interference occurs, but it is nevertheless capable of determining the wavelength at which a fiber grating most efficiently couples light between two modes.

Fig. 4. The diffraction of light wave by a grating.

Fig. 5(a) illustrates reflection by a Bragg grating of a mode with a bounce angle of \( \theta_1 \) into the same mode traveling in the opposite direction with a bounce angle of \( \theta_2 = -\theta_1 \). Since the mode propagation constant \( \beta \) is simply \( \beta = \left( \frac{2\pi}{\lambda} \right) n_{Eff} \) where \( n_{Eff} = n_{co} \sin \theta \), we may rewrite (23) for guided modes as

\[
\beta_2 = \beta_1 + m \frac{2\pi}{\Lambda}
\] (24)

The solid circles represent bound core modes \( n_{cl} < n_{Eff} < n_{co} \), the open circles represent cladding modes \( 1 < n_{Eff} < n_{cl} \), and the hatched regions represent the continuum of radiation modes. Negative \( \beta \) values describe modes that propagate in the -z direction. By using (24) and recognizing \( \beta_2 < 0 \), the resonant wavelength for reflection of a mode of index \( n_{Eff} \), 1 into a mode of index \( n_{Eff} \), 2 is

\[
\lambda = ( n_{Eff} \), 1 + n_{Eff} \), 2 \) \( \Lambda \)
\] (25)

If the two modes are identical, the familiar result for Bragg reflection: \( \lambda = 2n_{Eff} \Lambda \).

Diffraction by a transmission grating of a mode with a bounce angle of \( \theta_1 \) into a co-propagating mode with a bounce angle of \( \theta_2 \) is illustrated in Fig. 5(b). In this illustration the first mode is a core mode while the second is a cladding mode. Since here \( \beta_2 > 0 \), predicts the resonant wavelength for a transmission grating as

\[
\lambda = ( n_{Eff} \), 1 - n_{Eff} \), 2 \) \( \Lambda \)
\] (26)

The \( \beta \) axes below each diagram demonstrate the grating condition in (6) for \( m = -1 \)
Fig 5. Ray-optic illustration of (a) core-mode Bragg reflection by a fiber Bragg grating and (b) cladding-mode coupling by a fiber transmission grating.

VII. CONCLUSION

The fibre Bragg grating can be viewed as an ideal fibre (as reference) plus a certain index variation (as perturbations). Fibre Bragg gratings have already been commercialized in recent years. It has become popular to use fibre Bragg gratings in sensor systems for their high sensitivity and potentially low cost. Fibre Bragg gratings have been used in many applications, such as wavelength division multiplexing communication systems, lasers, strain and temperature sensing, and fibre lasers.

We can study about fabrication method, couple mode theory, modeling, analytical model, Two-mode coupling in uniform grating. The coupled-mode equations can be solved by two different approaches for calculating the reflection and transmission spectra under the two-mode approximation. The first analysis involves the use of the transfer matrix method.

REFERENCES