Review on Image Denoising using DWT Algorithm
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Abstract—The prime focus of this thesis is related to the pre processing of an image. The pre processing being worked upon is the de noising of images. In order to achieve this in terms of the concerned work, wavelet transforms have been applied: Discrete wavelet transform and EEMD. In this thesis, a new technique which is combination of Enhanced Empirical Mode Decomposition (EEMD), has been presented along with the standard wavelet thresholding techniques like soft and hard thresholding. And a comparative analysis of different combinations of the suggested threshold values and thresholding techniques has been carried out very efficiently. A new constraint, of either thresholding the low pass components or keeping them as such before applying the inverse DWT has also been added. This has been done in order to find more possible combinations that can lead to the best denoising technique.

Key words: EEMD, Wavelet transform, DWT, Decomposition

I. INTRODUCTION

Image processing is a field that continues to grow, with new applications being developed at an ever increasing pace. It is a fascinating and exciting area to be involved in today with application areas ranging from the entertainment industry to the space program. One of the most interesting aspect of this information revolution is the ability to send and receive complex data that transcends ordinary written text. Visual information, transmitted in the form of digital images, has become a major method of communication for the 21st century. Image processing is any form of signal processing for which the input is an image, such as photographs or frames of video and the output of image processing can be either an image or a set of characteristics or parameters related to the image. Most image processing techniques involve treating the image as a two-dimensional signal and applying standard signal-processing techniques to it. There are applications in image processing that require the analysis to be localized in the spatial domain. The classical way of doing this is through what is called Windowed Fourier Transform. Central idea of windowing is reflected in Short Time Fourier Transform (STFT). The STFT conveys the localized frequency component present in the signal during the short window of time. The same concept can be extended to a two-dimensional spatial image where the localized frequency components may be determined from the windowed transform. This is one of the basis of the conceptual understanding of wavelet transforms. Hence, wavelet transforms have been kept as the main consideration in this thesis. It is well known that while receiving the input image some aberrations get introduced along with it and hence a noisy image is what we are left with for future processing. The image de-noising naturally corrupted by noise is a classical problem in the field of signal or image processing. Additive random noise can easily be removed using simple threshold methods. De-noising of natural images corrupted by noise using wavelet techniques is very effective because of its ability to capture the energy of a signal in few energy transform values. The wavelet denoising scheme thresholds the wavelet coefficients arising from the wavelet transform. The wavelet transform yields a large number of small coefficients and a small number of large coefficients. Simple de-noising algorithms that use the wavelet transform consist of three steps. Calculate the wavelet transform of the noisy signal. Modify the noisy wavelet coefficients according to some rule. Compute the inverse transform using the modified coefficients.

The problem of Image de-noising can be summarized as follows, Let \( A(i,j) \) be the noise-free image and \( B(i,j) \) the image corrupted with noise \( Z(i,j) \),

\[
B(i,j) = A(i,j) + sZ(i,j) \ldots (1.1)
\]

The problem is to estimate the desired signal as accurately as possible according to some criteria. In the wavelet domain, the problem can be formulated as

\[
Y(i,j) = W(i,j) + N(i,j) \ldots (1.2)
\]

where \( Y(i,j) \) is noisy wavelet coefficient; \( W(i,j) \) is true coefficient and \( N(i,j) \) noise. In this thesis work, the algorithm has been carried out by using variety of inputs.

A. Introduction to Wavelet

The concept of wavelet was hidden in the works of mathematicians even more than a century ago. In 1873, Karl Weirstrass mathematically described how a family of functions can be constructed by superimposing scaled versions of a given basis function. The term wavelet was originally used in the field of seismology to describe the disturbances that emanate and proceed outward from a sharp seismic impulse [22]. Wavelet means a “small wave”. The smallness refers to the condition that the window function is of finite length (compactly supported) [23]. A wave is an oscillating function of time or space and is periodic. In contrast, wavelets are localized waves. They have their energy concentrated in time and are suited to analysis of transient signals. While Fourier Transform and STFT use waves to analyze signals, the Wavelet Transform uses wavelets of finite energy [22].

![Fig.1: Difference between Wave and Wavelet (a) wave (b) wavelet](image)

In wavelet analysis the signal to be analyzed is multiplied with a wavelet function and then the transform is computed for each segment generated. The Wavelet Transform, at high frequencies, gives good time resolution...
and poor frequency resolution, while at low frequencies, the Wavelet Transform gives good frequency resolution and poor time resolution.

An arbitrary signal can be analyzed in terms of scaling and translation of a single mother wavelet function (basis). Wavelets allow both time and frequency analysis of signals simultaneously because of the fact that the energy of wavelets is concentrated in time and still possesses the wave-like (periodic) characteristics. As a result, wavelet representation provides a versatile mathematical tool to analyze transient, time-variant (non stationary) signals that are not statistically predictable especially at the region of discontinuities—a feature that is typical of images having discontinuities at the edges.

B. Mathematical Representation of Wavelet

Wavelets are functions generated from one single function (basis function) called the prototype or mother wavelet by dilations (scalings) and translations (shifts) in time (frequency) domain. If the mother wavelet is denoted by \( y(t) \), the other wavelets \( \varphi_{a,b}(t) \) can be represented as

\[
\varphi_{a,b}(t) = (1 \ast \varphi\left(\frac{t-b}{a}\right) / \sqrt{a})
\]

where \( a \) and \( b \) are two arbitrary real numbers. The variables ‘\( a \)’ and ‘\( b \)' represent the parameters for dilations and translations respectively in the time axis.

The mother wavelet can be essentially represented as

\[
\varphi_{a,b}(t) = \varphi_{1,0}(t)
\]

For any arbitrary \( a \neq 1 \) and \( b = 0 \), we can derive that

\[
\varphi_{a,0}(t) = (1 \ast \varphi\left(\frac{t}{a}\right) / \sqrt{a})
\]

As shown above, \( \varphi_{a,0}(t) \) is nothing but a time-scaled (by \( a \)) and amplitude-scaled (by \( a \)) version of the mother wavelet function \( y(t) \).

C. Properties of Wavelet

- ‘Regularity’ defined as: if \( r \) is an integer and a function is \( r \)-time continuously differentiable at \( x_0 \), then the regularity is \( r \). If \( r \) is not an integer, let \( n \) be the integer such that \( n < r < n+1 \), then function has a regularity of \( r \) in \( x_0 \) if its derivative of order \( n \) resembles \( (x-x_0)^r \)-n locally around \( x_0 \). This property is useful for getting nice features, such as smoothness, of the reconstructed signals.
- The support of a function is the smallest space-set (or time-set) outside of which function is identically zero.
- The number of vanishing moments of wavelets determines the order of the polynomial that can be approximated and is useful for compression purposes.
- The wavelet symmetry relates to the symmetry of the filters and helps to avoid dephasing in image processing. Among the orthogonal families, the Haar wavelet is the only symmetric wavelet. For biorthogonal wavelets it is possible to synthesize wavelet functions and scaling functions that are symmetric or antisymmetric.

D. Types of Wavelet Transforms

There are mainly two types of Wavelet Transforms—
- Continuous Wavelet Transformation (CWT)
- Discrete Wavelet Transformation (DWT)

Since our algorithm is to be based on discrete wavelet transform, so we will discuss only the concepts of DWT (leaving CWT as such) in the following paragraphs. Two commonly used abbreviations are DWT and IDWT.

DWT stands for Discrete Wavelet Transformation. It is the Transformation of sampled data, e.g. transformation of values in an array, into wavelet coefficients.

IDWT is Inverse Discrete Wavelet Transformation: procedure converts wavelet coefficients into the original sampled data.

Here the case of square images has been considered. Let us take an \( N \) by \( N \) image.

1) Decomposition Process

To start with, the image is high and low-pass filtered along the rows and the results of each filter are down-sampled by two. Those two sub-signals correspond to the high and low frequency components along the rows and are each of size \( N/2 \) by \( N/2 \). Then each of these sub-signals is again high and low-pass filtered, along the column data. The results are again down-sampled by two.

As a result the original data is split into four sub-images each of size \( N/2 \) by \( N/2 \) containing information from different frequency components. Figure 1.2 shows the level one decomposition step of the two dimensional image.

Fig. 2: One decomposition step of the two dimensional image

Fig. 3: One DWT decomposition step

The LL subband is the result of low-pass filtering both the rows and columns and it contains a rough description of the image as such. Hence, the LL subband is also called the approximation subband. The HH subband is high-pass filtered in both directions and contains the high-frequency components along the diagonals as well. The HL and LH images are the result of low-pass filtering in one direction and high-pass filtering in another direction. LH contains mostly the vertical detail information that corresponds to horizontal edges. HL represents the horizontal detail information from the vertical edges. All three subbands HL, LH and HH are called the detail subbands, because they add the high-frequency detail to the approximation image.
2) Composition Process
The inverse process is shown in Figure 1.4. The information from the four sub-images is up-sampled and then filtered with the corresponding inverse filters along the columns. The two results that belong together are added and then again up-sampled and filtered with the corresponding inverse filters. The result of the last step is added together in order to get the original image again. Note that there is no loss of information when the image is decomposed and then composed again at full precision.

![Figure 4: One composition step of the four sub images](image)

With DWT we can decompose an image more than once. Decomposition can be continued until the signal has been entirely decomposed or can be stopped before by the application at hand. Mostly two ways of decomposition are used. They are:

1) Pyramidal decomposition
2) Packet decomposition

Pyramidal decomposition is the simplest and most common form of decomposition used. For the pyramidal decomposition we only apply further decompositions to the LL subband. Figure 1.5 shows a systematic diagram of three decomposition steps. At each level the detail subbands are the final results and only the approximation subband is further decomposed.

![Figure 5: Three decomposition steps of an image using Pyramidal Decomposition](image)

Figure 1.6 shows the pyramidal structure that result from this decomposition. At the lowest level there is one approximation subband and there are a total of nine detail subbands at the different levels. After L decompositions, a total of D (L) = 3 * L + 1 subbands are obtained.

![Figure 6: Pyramid after three decomposition steps](image)

Fig. 6: Pyramid after three decomposition steps

Figure 1.7 is an example of this decomposition process. It shows the “Lena” image after one, two and three pyramidal decomposition steps.

![Figure 7: Pyramidal decomposition of Lena image (1, 2 and 3 times)](image)

b) Wavelet Packet Decomposition
For the wavelet packet decomposition, the decomposition is not limited to the approximation subband only but a further wavelet decomposition of all subbands on all levels is considered. In figure 1.8, the system diagram for a complete two level wavelet packet decomposition has been shown.

![Figure 8: Two complete decomposition steps using wavelet packet decomposition](image)

In figure 1.9, the resulting subband structure is on display. Again the simple decomposition step is used as a basic building block. The composition step is equivalent to the pyramidal case. All four subbands on one level are used as input for the inverse transformation and a resultant in the subband on the higher level is obtained. This process is repeated again and again until the original image is reproduced.
The discrete wavelet transform is very efficient from the computational point of view. Its only drawback is that it is not translation invariant. Translations of the original signal lead to different wavelet coefficients. In order to overcome this and to get more complete characteristic of the analyzed signal the undecimated wavelet transform was proposed. The general idea behind it is that it doesn’t decimate the signal. Thus it produces more precise information for the frequency localization. From the computational point of view the undecimated wavelet transform has larger storage space requirements and involves more computations.

**E. Wavelet Families**

There are a number of basis functions that can be used as the mother wavelet for Wavelet Transformation. Since the mother wavelet produces all wavelet functions

![Wavelet Families](image)

**Fig. 10:** Several different families of wavelets used in the transformation through translation and scaling, it determines the characteristics of the resulting Wavelet Transform. Therefore, the details of the particular application should be taken into account and the appropriate mother wavelet should be chosen in order to use the Wavelet Transform effectively.

Figure 1.10 illustrates some of the commonly used wavelet functions. Haar wavelet is one of the oldest and simplest wavelet. Therefore, any discussion of wavelets starts with the Haar wavelet. Daubechies wavelets are the most popular wavelets. They represent the foundations of wavelet signal processing and are used in numerous applications. These are also called Maxflat wavelets as their frequency responses have maximum flatness at frequencies 0 and R. This is a very desirable property in some applications. The Haar, Daubechies, Symlets and Coiflets are compactly supported orthogonal wavelets. These wavelets along with Meyer wavelets are capable of perfect reconstruction. The Meyer, Morlet and Mexican Hat wavelets are symmetric in shape. The wavelets are chosen based on their shape and their ability to analyze the signal in a particular application.

**F. Wavelet Domain Advantages**

Why we prefer denoising in wavelet domain, because it has many advantages. Like:

- Wavelet-based denoising provides multi-resolution hierarchical characteristics. Hence an image can be denoised at different levels of resolution and can be sequentially processed from low resolution to high resolution.
- High robustness as compared to common signal processing.

**II. CONCLUSION**

This thesis presents a comparative analysis of various image denoising techniques using wavelet transforms. A lot of combinations have been applied in order to find the best method that can be followed for denoising intensity images. The image formats that have been used in this work are JPG, BMP, TIF and PNG, but all has to be converted into BMP.

The analysis, of all the obtained experimental results, demonstrates that EEMD-DWT outperforms DWT for denoising all of the above mentioned images (whether the low pass components are thresholded or are kept as such).

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