Analysis of Quarter Car Passive Suspension System with Non Linear Suspension Parameters
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Abstract— Modeling the dynamic performance of an automobile car system is a complex task and forms an important step in the design of suspension system of an automobile. In this paper the stationary response of a quarter car vehicle model moving with a constant velocity over a rough road is considered for the performance study. Passive suspension system comprises spring and damper which actually behaves non-linearly. For proper designing of suspension system, nonlinearities in suspension parameters must be considered. For this a simplified model by considering nonlinearity of suspension spring and suspension damper is developed. The results are obtained by using MATLAB Model and compared with linear quarter car passive suspension system.

Key words: Passive Suspension System, Nonlinearity, Suspension Spring, Suspension Damper, Quarter Car Model, MATLAB Simulink.

I. INTRODUCTION

Suspension system is one of the important part of the vehicle. Therefore, it is quite necessary to design finer suspension system in order to improve the quality of vehicles. Since the disturbances from the road may include uncomfortable shake and noise in the vehicle body, it is important to study the vibrations of the vehicle [1]. An automobile is a nonlinear system in practical terms because it consists of suspensions, tires and other components having nonlinear properties. Therefore, the chaotic response may appear as the vehicle moves over a road [2]. Suspension is subjected to various road conditions like a single step road profile, brake and release maneuver, sinusoidal road profile with pitching, heaving and mixed model excitation, broad band road profile etc. at constant or variable speed [3]. The measurement of road surface qualities is one of the important opportunities of vehicle manufactures all over the world. The operations of the measuring devices depend mainly on the use of displacement transducers [4].

This paper deals with the analysis of the vibrational effect when the vehicle is subjected to harmonic road excitation by the road profile. For this purpose quarter car vehicle model with linear, nonlinear suspension spring and nonlinear damper parameters are developed with MATLAB Simulink. For analysis Hyundai Elantra 1992, quarter car model with front suspension is simulated in MATLAB Simulink and linear and Nonlinear Simulation results are compared.

II. ROAD PROFILE AND WHEEL TRAVEL

Road is considered as an infinite cam with wavy profile of harmonic waves and wheel of quarter car model is considered as follower. The road is considered as cam which will give harmonic road excitation to suspension system.

The road profile is approximated by a sine wave represented by the equation [5].

\[ q = Y \sin(\omega t) \]

Where,

- \( q \) = Road surface excitation at time \( t \) (m)
- \( Y \) = Amplitude of sine wave = 0.02 m
- \( \lambda \) = Wavelength of road surface = 6 m

Fig. 1: Harmonic Road Surface Profile.

Vehicle is assumed to be traveling over a road with velocity of 10 km/hr to 120 km/hr, during this travel the excitation frequency is calculated as

\[ F = \frac{2\pi V}{\lambda} \]

For 10 km/hr \( F \) is calculated as

\[ F = \frac{(2\pi \times 10 \times 1000)}{(6 \times 3600)} = 2.90 \text{ rad/sec} = 0.46 \text{ Hz} \]

Similarly other values of excitation frequencies are calculated upto 120 km/hr i.e. 5.55 Hz.

III. MODELING OF SYSTEM

To analyze the effect of the non-linearities in suspension spring on vehicle dynamic system in this passive quarter car model shown in Fig. 2 is taken for study.

Fig. 2: TDOF Passive Quarter Car Model.

The parameters of Hyundai Elantra 1992 model considered for the analyses are given in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sprung Mass (m_s)</td>
<td>236.12 Kg</td>
</tr>
<tr>
<td>Unsprung Mass (m_u)</td>
<td>23.61 Kg</td>
</tr>
<tr>
<td>Suspension Stiffness (k_s)</td>
<td>12394 N/m</td>
</tr>
<tr>
<td>Passive Suspension Damping Coefficient (c_s)</td>
<td>1385.4 N-sec/m</td>
</tr>
<tr>
<td>Tire Stiffness (195/65R15 Tire) (k_t)</td>
<td>181818.88</td>
</tr>
</tbody>
</table>
The equations of motion for the linear model is

\[ \ddot{x}_s = -\frac{1}{m_s} \left[ k_s (x_s - x_u) + c_s (\dot{x}_s - \dot{x}_u) \right] \]

\[ \ddot{x}_u = -\frac{1}{m_u} \left[ k_s (x_s - x_u) + c_s (\dot{x}_s - \dot{x}_u) - k_s (q - x_u) + c_s (q - \dot{x}_u) \right] \]

From these equations of motion nonlinear equations of motion are formulated by considering nonlinearity in spring force and damping force.

### IV. SUSPENSION NONLINEARITIES.

#### A. Spring Nonlinearity:

The non-linear effects included in the spring force \( f_s \) are due to two parts. One is bump stop, which restricts the wheel travel within the given range and prevents the tire from contacting the vehicle body. The other is strut bushing which connects the strut with the body structure and reduces the harshness from the road input. These non-linear effects can be included in spring force \( f_s \) with non-linear characteristic versus suspension rattle space \((x_s - x_u)\) from measured data (SPMD) [6] shown in Fig. 3.

Fig. 3. Non-linear Spring Force Property of Hyundai Elantra 1992 Model Suspension Spring

The spring force \( f_s \) is modeled as third order polynomial function as

\[ f_s = k_0 + k_1 \Delta x + k_2 \Delta x^2 + k_3 \Delta x^3 \]

Where the co-efficients are obtained by fitting the experimental data, which resulted in \( k_1 = 3170400 \) N/m, \( k_2 = -73696 \) N/m\(^2\), \( k_3 = 12394 \) N/m and \( k_0 = -1385.4 \) N-s/m. Therefore, the equations of motion for non-linear suspension damper passive system are:

\[ \ddot{x}_s = -\frac{1}{m_s} \left[ k_s (x_s - x_u) + c_s (\dot{x}_s - \dot{x}_u) + c_s (\dot{x}_s - \dot{x}_u)^2 \right] \]

\[ \ddot{x}_u = -\frac{1}{m_u} \left[ k_s (x_s - x_u) + c_s (\dot{x}_s - \dot{x}_u) - k_s (q - x_u) + c_s (q - \dot{x}_u) \right] \]

#### B. Damper Nonlinearity:

Generally, the damping force is asymmetric with respect to speed of the rattle space, damping force during bump is bigger than that during rebound in order to reduce the harshness from the road during bump while dissipating sufficient energy of oscillation during rebound at the same time [6]. Measured data for the damping force versus relative velocity of upper and lower struts, shows the asymmetric property which is shown in Fig. 4.

Fig. 4: Non-linear Damping Force Property of Hyundai Elantra 1992 Model Suspension Damper

From the measured data the damping force \( f_d \) is modeled as second order polynomial function as

\[ f_d = c_1 \Delta \dot{x} + c_2 \Delta \dot{x}^2 \]

Where the co-efficients are obtained from fitting the experimental data, which resulted in \( c_1 = 524.28 \) N-s/m\(^2\) and \( c_1 = 1385.4 \) N-s/m. 

The MATLAB Simulink models for linear and nonlinear passive suspension systems are prepared and the sprung mass acceleration for different excitation frequencies were obtained in time domain and from these results values of sprung mass acceleration responses were obtained at different excitation frequencies.

The sprung mass acceleration for linear passive and non-linear suspension spring and damper passive suspension system for Harmonic road excitation \((q)\) are as shown in Fig. 5.

### V. RESULT ANALYSIS

The results are tabulated in Table 2.

<table>
<thead>
<tr>
<th>Excitation Frequency in Hz</th>
<th>Max Sprung Mass Acceleration for Linear Passive Controller in ( \text{m/s}^2 )</th>
<th>Max Sprung Mass Acceleration for Nonlinear Spring Passive Controller in ( \text{m/s}^2 )</th>
<th>Max Sprung Mass Acceleration for Nonlinear Damper Passive Controller in ( \text{m/s}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.46</td>
<td>0.23</td>
<td>2.40</td>
<td>0.23</td>
</tr>
<tr>
<td>0.92</td>
<td>0.58</td>
<td>2.61</td>
<td>0.57</td>
</tr>
</tbody>
</table>
Table 2: Max. Sprung Mass Acceleration for Linear Passive and Nonlinear Spring and Damper Passive Controller for Different Excitation Frequencies.

From the results it is observed that there is considerable difference in nonlinear suspension spring passive controller compare with linear passive controller but there is no much difference in nonlinear suspension damper passive controller and linear passive controller.

VI. CONCLUSION

The simulation result shows considerable difference in linear and non-linear passive sprung mass acceleration. It is found that the chaotic response exists in nonlinear suspension. From the analysis of results it is observed that nonlinearities in the suspension damper do not affect much on ride comfort of vehicle. But where the nonlinearities in the suspension spring have considerable difference when compared with linear, hence nonlinearities in suspension spring must be considered while analyzing the vehicle dynamic system because it is observed that the suspension spring nonlinearities are making more impact on sprung mass acceleration than nonlinear suspension damper.

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REFERENCES