Soft Computing Technique to solve MHD Falkner Skan Equation

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Abstract— This paper examines the boundary value problem (BVP) of the MHD Falkner–Skan equation over a semi-infinite interval. The free boundary formulation has changed this problem from semi infinite into finite interval. This problem was solved with the help of genetic algorithm based shooting technique. The boundary value problem was converted into the initial value problem which was solved with the help of Runge- Kutta Fehlberg method. The results obtained are in good agreement with the existent literature.

I. Introduction

Falkner Skan equation is a classic boundary layer flow problem where the two dimensional steady laminar flow over a fixed wedge was studied by Falkner Skan[1]. This was later extended by Hartree [2] who used similarity transformation and provided numerical results for different values of the wedge angle for wall shear stress. Different researchers have contributed in this area by applying different techniques to solve this equation in an effective way. Asaithambi [3] investigated the Falkner-Skan equation with the help of finite difference scheme. Abbasbandy and Hayat [4, 5] studied the Magnetohydrodynamics effects on the Falkner Skan wedge flow. Cebeci and Keller [6] Na [8], Elgazery [9] have reported the solution of this equation using shooting method. Shi- Jun Liao [7] applied the homotopy analysis method to solve the Falkner-Skan equation. Classical Fourier series method was used recently to get the solutions of magneto-hydro-dynamic (MHD) Falkner-Skan equation by Eswara [8].

Due to the enormous application of boundary layer flows of incompressible fluids it has attracted many researchers to it [9]. A recent novel and more simpler Genetic algorithm based shooting technique was given to solve the Falkner-Skan equation by Singh et. al [9]. John Holland[10] developed the Genetic algorithm technique. Genetic algorithms are general search and optimisation algorithms employing genetics as its model of problem solving. Due to its extensive applications in recent years in almost every field whether it is physical sciences, life sciences, computer and social sciences as well as in engineering field it has attracted many researchers to it. Genetic Algorithm totally relies on the concept of evolution put forward by Darwin's theory as the "survival of the fittest". From nature one can find numerous instances where a competition takes place among individuals for survival and the end result is domination of fittest individual over the weak ones. The principle of Genetic algorithm is quite straightforward is to imitate genetics and natural selection by the computer programming. It includes selection, crossover and mutation of genes to find out the best and optimal solution.

II. Formulation of Problem

The Falkner-Skan equation constitutes a third order nonlinear two point boundary value problem. The Falkner-Skan equation governing MHD boundary layer flow past a stationary wedge can be written as -

$$\frac{d^3f}{d\eta^3}+\beta_0 f\frac{d^2f}{d\eta^2}+\beta\left[1-\left(\frac{df}{d\eta}\right)^2\right]+M\left[1-\left(\frac{df}{d\eta}\right)\right]=0,\quad 0<\eta<\infty \qquad (1)$$

Along with the boundary conditions given as under-

$$\left.\begin{array}{l} f=0 \quad at \quad \eta=0 \\ \dfrac{df}{d\eta}=0 \;\; at \;\; \eta=0 \\ \dfrac{df}{d\eta}=1 \;\; at \;\; \eta\to\infty \end{array}\right\}$$

where dimensionless stream function f is of η variable, β is a constant and the parameter of the stream wise pressure gradient, β_0 is a constant and *M* is dimensionless parameter representing the transverse magnetic field, applied normal to the wedge surface.

The non-linear differential equations (1) subject to the boundary conditions (2) constitute a two-point boundary value problem. Genetic algorithm based shooting technique is used to solve these equations numerically and then Runge–Kutta Fehlberg integration scheme is used to obtain the solution.

The Falkner–Skan equation has two coefficients β 0 and β. The solutions corresponding to β > 0 have being known as accelerating flows, those corresponding to β = 0 are called constant flows, and those corresponding to β < 0 are known as decelerating flows. The range $-0.1988<\beta\leq 2$ provides the physically feasible solutions.

When β = 0 , $\beta_0=0.5$ then the flow becomes Blasius flow. The equation (1) is written as under-

$$\frac{d^3f}{d\eta^3}+0.5f\frac{d^2f}{d\eta^2}+M\left[1-\left(\frac{df}{d\eta}\right)\right]=0$$

When β = 1/2 , $\beta_0=1/2$ then the flow becomes Homann flow. The equation (1) is written as under-

$$\frac{d^3f}{d\eta^3}+f\frac{d^2f}{d\eta^2}+0.5\left[1-\left(\frac{df}{d\eta}\right)^2\right]+M\left[1-\left(\frac{df}{d\eta}\right)\right]=0,$$

When β = 1 , $\beta_0 = 1$, then the flow becomes Hiemenz flow. The equation (1) is written as under-

$$\frac{d^3f}{d\eta^3} + f\frac{d^2f}{d\eta^2} + \left[1 - \left(\frac{df}{d\eta}\right)^2\right] + M\left[1 - \left(\frac{df}{d\eta}\right)\right] = 0,$$

When β = 1 , $\beta_0 = 0$, then the flow becomes Pohlhausen flow. The equation (1) is written as under-

$$\frac{d^3f}{d\eta^3} + \left[1 - \left(\frac{df}{d\eta}\right)^2\right] + M\left[1 - \left(\frac{df}{d\eta}\right)\right] = 0,$$

III. RESULT AND DISCUSSION-

To verify, different values of the parameter b have been taken to check the accuracy of our present method, values of velocity gradient at the wall, are compared with the values obtained numerically in literature.

Table1: Comparison of f'' with variation of different values of β , M=0 and comparison with the other authors.

Values of β	Zhang and Chen[10]	Zhu et al.[11]	A.T.Eswara [12]	Present Results
2	1.687218	1.687218	1.68721	1.687014
1	1.232587	1.232588	1.23258	1.232465
0.5	0.927680	0.927680	0.92768	0.927570
0	0.469600	0.469600	0.46960	0.469593
-0.1	0.319270	0.319270	0.31927	0.319572
-0.15	0.216362	0.216362	-	0.217352
-0.18	0.128636	0.128637	0.12863	0.129087
-0.1988	0.005222	0.005225	0.00521	0.006019

Table 2: Comparison of f'' with variation of different values of β , M=1 and comparison with the other authors.

Values of β	A.T.Eswara [12]	Present Results
2	1.95923	1.960598
1	1.58356	1.585365
0.5	1.35923	1.3600789
0	1.08845	1.0910401
-0.1	1.02756	1.0294207
-0.15	-	0.997326
-0.18	0.97528	0.9776218
-0.1988	0.96256	0.965093

Table 2: Comparison of f'' with variation of different values of β , M=1 and comparison with the other authors

Table 3: Values of f' with variation of different values of M

Magnetic Field	BLASIUS	HOMANN	HIEMENZ
M=0	0.40454969	0.92914950	1.23281167
M=1	1.04662496	1.36007896	1.58536562
M=2	1.44498683	1.68662281	1.87352900
M=3	1.75672483	1.96010291	1.45734077

Table 3: Values of f' with variation of different values of M

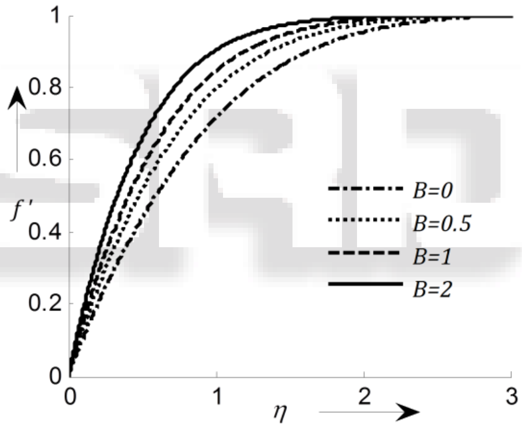

Figure 1 shows variations of the velocity function $f'(\eta)$ with the similarity variable η for different positive values of β in the presence of magnetic field ($M = 1.0$)

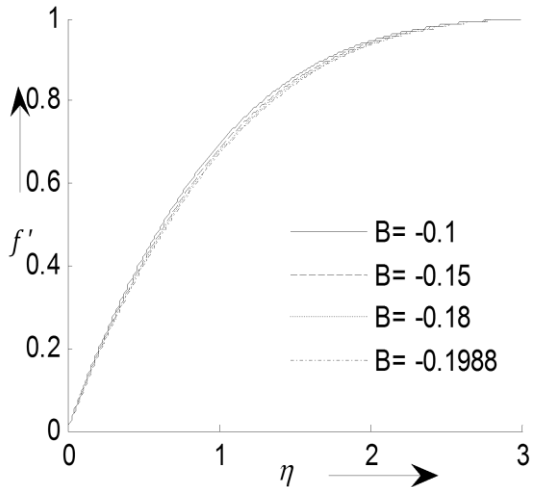

Figure 2 shows variations of the velocity function $f'(\eta)$ with the similarity variable η for different negative values of β in the presence of magnetic field ($M = 1.0$),

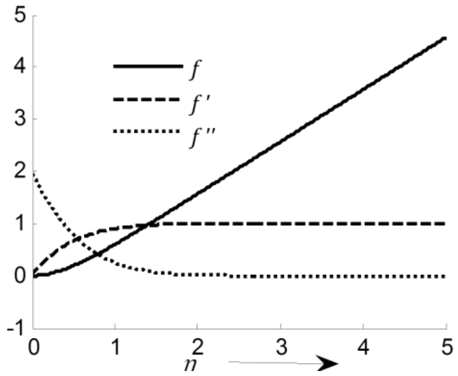

Figure 3 Solution of the problem of Pohlhausen flow

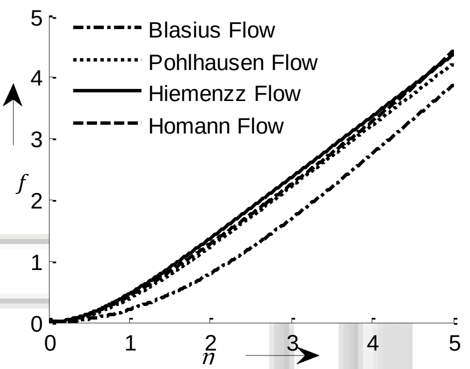

Figure 4 shows variations of the velocity function $f'(\eta)$ with the similarity variable η for different types of flows

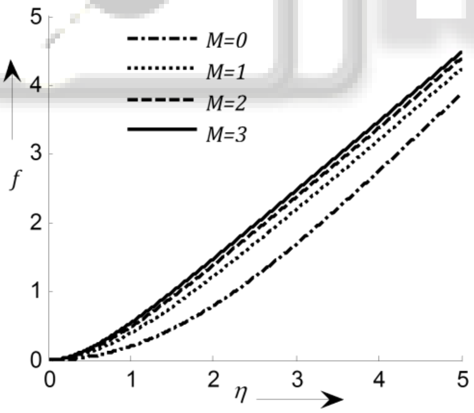

Figure 5 shows variations of the velocity function $f'(\eta)$ with the similarity variable η for Blasius flow

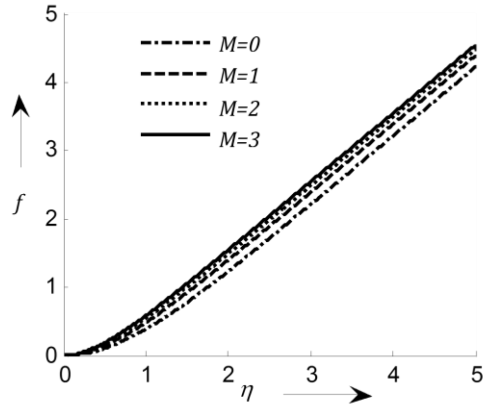

Figure 6 shows variations of the velocity function $f'(\eta)$ with the similarity variable η for Homann flow

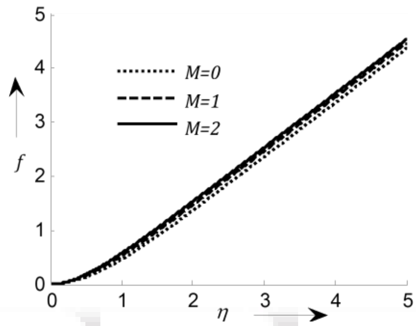

Figure 7 shows variations of the velocity function $f'(\eta)$ with the similarity variable η for Hiemenz flow

IV. CONCLUSIONS

The genetic algorithm based shooting technique employed in this article to solve the third order nonlinear MHD Falkner Skan equation is quite effective The Runge Kutta Fehlberg method is used to solve the initial value problem and the results obtained in the paper are in well agreement with the existing literature.

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