

# Adaptive Variable Structure Control for Trajectory Tracking of a Two-Link Robotic Manipulator System with Linear Sliding Manifolds

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**Abstract**— Adaptive control proves to be a useful solution for problems involving system uncertainties and modelled inaccuracies. In the presence of uncertainties in the manipulator dynamics and model, due to incorrect parameter values and changes while the manipulator is being operated, fixed controllers cannot be implemented accurately. Adaptive laws in manipulator control (by altering the control signal) accounts for changes in manipulator dynamics and for this reason adaptive control of manipulators has become one of the most active research areas during the last few years. This paper presents a novel yet simple technique that combines the adaptive laws with conventionally robust Sliding Mode Control technique. The stability and convergence of the robotic manipulator control system are guaranteed by applying the Lyapunov stability theorem.

**Keywords:** Variable Structure control, Sliding Variable, Adaptive robotic control, Lyapunov Stability.

## I. INTRODUCTION

Variable structure control (VSC) of robotic manipulators has been receiving more and more attention lately. The variable structure system (VSS) has numerous interesting and important properties that cannot be easily obtained by other approaches. When a system is in a sliding mode, it emulates a prescribed reduced-order system and is insensitive to parameter variations and disturbance. Precise dynamic models are not required and the control algorithms are easy to implement. All these properties make the VSC an ideal candidate for robot manipulator control. Non linear control methodologies are more general because they can be used in linear and non linear systems. These controllers can solve different problems such as, invariance to system uncertainties and resistance to the external disturbance. The most common non linear methodologies that have been proposed to solve the control problem consist of the following methodologies: feedback linearization control methodology, passivity-based control methodology, sliding mode control methodology, robust Lyapunov based control methodology, adaptive control methodology and artificial intelligence-based methodology.

An adaptive sliding mode controller for uncertain systems is projected in this paper. The actual control law is obtained by equivalent control approach and torque computing method. The sliding mode control is based on the design of a high-speed switching control law that drives the system's trajectory onto a user-chosen hyper plane in the state space, also known as sliding surface. The main feature of sliding mode control are the following: (i) Quick response and good transient routine; (ii) Robustness against a large class of perturbations or model uncertainties; and (iii)

Possibility of stabilizing some complex nonlinear systems which are difficult to stabilize by state feedback control laws. Chattering phenomenon can cause some problems such as saturation and heat for mechanical parts of robot manipulators or drivers. Chattering can occur in the system due to unmodelled dynamics or imperfections in the system. The unmodelled dynamics can be excited by the discontinuities in the control, therefore they must be carefully modeled into the system.

However, without the knowledge of these dynamics, designing the system using boundary layer control can be used, but with the trade of disturbance rejection properties. In the presence of imperfection in the system, such as time delays, chattering can also appear. Delay "is the most relevant to any electronic implementation of the switching device, including both analog and digital circuits, and microprocessor code executions. The chattering due to delays can be suppressed using discrete-time control design techniques. In the scope of this paper proper techniques have been employed to reduce chattering. The requirement of prior knowledge about the disturbance bands for designing adaptive sliding mode controllers is not a necessary requirement in the proposed controller.

## II. SYSTEM DESCRIPTION

The dynamic model of an n-link robot manipulator may be expressed in the following Lagrange form:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + F(\dot{q}) + \tau d = \tau \quad (1)$$

Where;

$$q, \dot{q}, \ddot{q} \in R^n;$$

are the joints position, velocity, and acceleration vectors,

$$M(q) \in R^{n \times n};$$

denotes the inertia matrix,

$$C(q, \dot{q})\dot{q} \in R^{n \times n};$$

expresses the coriolis and centrifugal torques,

$$G(q) \in R^n; \text{ is the gravity vector,}$$

$$F(\dot{q}) \in R^n; \text{ is the friction,}$$

$$\tau d \in R^n; \text{ is the external disturbances,}$$

$$\tau \in R^{n \times 1}; \text{ is the control input.}$$

The robot model (1) is regarded by the subsequent structural properties, which are of importance to our stability analysis.

Property 1 :  $M(q)$  is a positive definite symmetric matrix for all  $q$ .

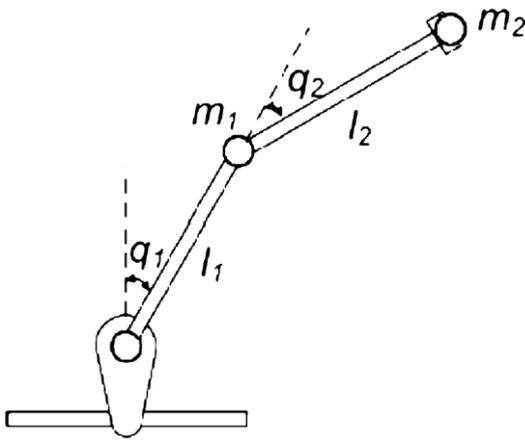


Fig. 1: Configuration of a two-link robotic manipulator [2]

Property 2 :  $M(q) - 2 C(q, \dot{q})$  is a skew symmetric matrix, that is,

$$\psi^T (M(q) - 2 C(q, \dot{q})) \psi; \quad (2)$$

where  $\psi$  is an  $n \times 1$  nonzero vector.

The dynamics of a two link manipulator involves the following matrices:

$$M(q) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

where  $M_{11} = m_1 l_{c1}^2 + m_2 (2l_1 l_{c2} c_2 + l_1^2 + l_{c2}^2) + I_1 + I_2$

$M_{12} = l_{c2}^2 m_2 + l_1 l_{c2} m_2 c_2 + I_2$

$M_{21} = l_{c2}^2 m_2 + l_1 l_{c2} m_2 c_2 + I_2$

$M_{22} = l_{c2}^2 m_2 + I_2$

$$C(q, \dot{q})^T = \begin{bmatrix} -m_2 l_1 l_{c2} s_2 \dot{q}_2^2 - 2m_2 l_1 l_{c2} (q_1) \dot{q}_2 & m_2 l_1 l_{c2} s_2 \dot{q}_1^2 \end{bmatrix}$$

where  $c_i = \cos q_i$  and  $s_i = \sin q_i$

### III. SLIDING MANIFOLD

Sliding mode control design approach consists of two phases: (i) selection of a sliding surface so as to achieve the desired system behavior, when the control system reaches the sliding surface; and (ii) selection of a control law such that the existence of sliding mode can be guaranteed. This part of control algorithm focuses on stabilizing the state  $y(t)$  of an uncertain system whose only bounds of uncertainty are known. Some suitable manipulation is done in order to make the sign of the derivative of the state, i.e.  $\dot{y}(t)$  opposite to that of the state itself that is to be stabilized. If  $y(t) > 0$ , then  $\dot{y}(t) < 0$  and vice versa. Thus, if the initial condition  $y(0) > 0$ , then  $\dot{y}(0) < 0$  and  $y(t)$  tends to decrease and reach  $y(t) = 0$ . Similarly for  $y(t) < 0$ , its derivative  $\dot{y}(t) > 0$  and approaches  $y(t) = 0$ . In either case, the state is moving towards  $y(t)$  irrespective of any initial condition. Due to this nature, any moment the trajectory crosses  $y(t) = 0$ , it is reinforced towards it. This requires an essentially very high switching (infinite frequency switching) to consistently maintain the state there. The motion of the system while confined to the switching line or a hyperplane is referred to as sliding. A sliding mode will exist in the vicinity of the switching surface if the state velocity vectors are directed toward the surface. In such a case, the switching surface attracts trajectories when they are in its vicinity; and once a trajectory intersects the switching surface, it will stay on it thereafter.

Implementing the sliding mode control scheme to control a robotic manipulator generally involves two steps. An appropriate sliding surface must be selected first, capable of ensuring the stability of the equivalent dynamics in the sliding mode such that the error dynamics can converge to zero. A sliding mode control must then be determined to ensure not only the reaching of the sliding surface in finite time, but also that the state trajectory can remain on the sliding mode thereafter even when undergoing the system uncertainties. As mentioned earlier, a proper sliding surface must be designed to ensure the system stability in the sliding mode. The next step involves designing an adaptive sliding mode control scheme to drive the extended error system trajectories onto the sliding surface.

The main motive behind designing a control for the proposed system is to drive the joint position  $q$  to the desired position  $q_d$ . The tracking error, velocity error,  $\dot{q}_r$  and  $\ddot{q}_r$  are also defined as:

$$e = q_d - q; \quad \dot{e} = \dot{q}_d - \dot{q},$$

$$\dot{q}_r = \dot{q}_r + \lambda e; \quad \ddot{q}_r = \ddot{q}_d + \lambda \dot{e} \quad (3)$$

The Linear Sliding surface is selected as:

$$s = \dot{e} + \lambda e \quad (4)$$

Where,  $\lambda = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_n)$  and  $\lambda_i > 0$ , is a positive constant.

### IV. ADAPTIVE VSC

Most of the adaptive control schemes suggested ensure asymptotic tracking of a desired reference trajectory for the robot manipulator dynamics; however, in reality its certain that there will always be disturbances in any electromechanical system. A simplistic way to take into account some sort of disturbance effect is to add a bounded disturbance term to the manipulator dynamic equation and same is employed here.

The main step in designing the variable structure control is to ensure that state trajectory will be induced into the sliding surface. The value of the sliding variable  $s$  can be thought of as a distance between the state trajectory and sliding surface. As a result, the absolute value of  $s$  is needed to be reduced over time in order for state trajectory to reach sliding surface. Using a Lyapunov function  $v$ , the origin of the state plane is asymptotically stable (i.e.,  $s$  will always decrease) if derivative of Lyapunov function is negative definite.

The proposed adaptive variable structure control law for the manipulator system is represented as:

$$\tau = u_a + u_b + u_c \quad (5)$$

where:

$$u_a = \hat{M}(q) (\ddot{q}_d + \lambda \dot{e}) + \hat{C}(q, \dot{q}) + \hat{G}(q) \quad (6)$$

$$u_b = -K \text{Sign}(s) \quad (7)$$

$$u_d = \begin{cases} \varphi \hat{\mu} \frac{s}{\|s\|}, & \text{if } \|s\| > \varepsilon \\ \varphi \hat{\mu} \frac{s}{\varepsilon}, & \text{otherwise} \end{cases} \quad (8)$$

$$\hat{\mu} = \begin{cases} -\alpha \hat{\mu} + \beta \varphi \|s\| & \text{if } \|s\| > \varepsilon \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

Since the parameters of robot manipulators are exactly not known because they are time varying, therefore  $\hat{M}$ ,  $\hat{C}$  and  $\hat{G}$  represent the nominal or assumed values.  $u_d$  is the computed torque vector,  $u_b$  is the feedback error vector and  $u_c$  is the sliding mode torque vector with adjustable gain.

A generalized Lyapunov function, that characterizes the motion of the state trajectory to the sliding surface, can be defined in terms of the surface. For each chosen switched control structure, corresponding “gains” are chosen so that the derivative of this Lyapunov function is negative definite, thus guaranteeing motion of the state trajectory to the surface. After proper design of the surface, a switched controller is constructed so that the tangent vectors of the state trajectory point towards the surface such that the state is driven to and maintained on the sliding surface. Such controllers result in discontinuous closed-loop systems. A Lyapunov function candidate defined as  $v = 1/2(s)^2$  can be selected for stability analysis of the proposed law and using the control law it is easy to find that manifold  $s$  can be shown to possess finite time reachability to zero which ensures that the tracking error of the robotic manipulator converges to zero in finite time. The gain in the second control term is also designed to be adaptable with somewhat similar method as followed in the third control term. By doing this the system becomes more robust to external disturbances and system uncertainties.

### V. SIMULATION & RESULTS

Trajectory tracking of a 2-link robotic manipulator which is a non-linear system with mismatched uncertainties is considered in our simulation study. Simulation results demonstrate that the proposed control strategy is successful in eliminating the undesired chattering in the control input while ensuring satisfactory stabilization as well as tracking performances. Hence the proposed controller is suitable for practical applications.

The parameter values of the robotic model are chosen as follows:

$l_1=1$  m,  $l_2=0.8$  m;  $J_1=5$  kgm<sup>2</sup>,  $J_2=5$  kgm<sup>2</sup>;  $m_1=0.5$  kg,  $m_2=6.25$  kg.

The desired trajectory had the form:

$$q_{d1} = 1 + 0.2 \sin(\pi t) \quad \& \quad q_{d2} = 1 - 0.2 \cos(\pi t) \quad (10)$$

The disturbance values were defined as:

$$\tau_{d1} = 5 \sin(4\pi t) \quad \& \quad \tau_{d2} = 5 \sin(4\pi t) \quad (11)$$

Initial states were assumed to be :

$$q(0)_1=1; q(0)_2=0.8; q(0)_3=0; q(0)_4=0.$$

Adaptive law coefficients are taken to be as:

$$\beta=250; \alpha=-1; \varepsilon=0.005; k=3; \lambda=5I_2.$$

The simulations are carried out in MATLAB – Script platform by using ODE solver suite with a fixed step size of 0.005sec. The results obtained are satisfactory and robust over a certain range of disturbances that affect the system performance.

The Adaptive scheme can be implemented as shown in Figure 2:

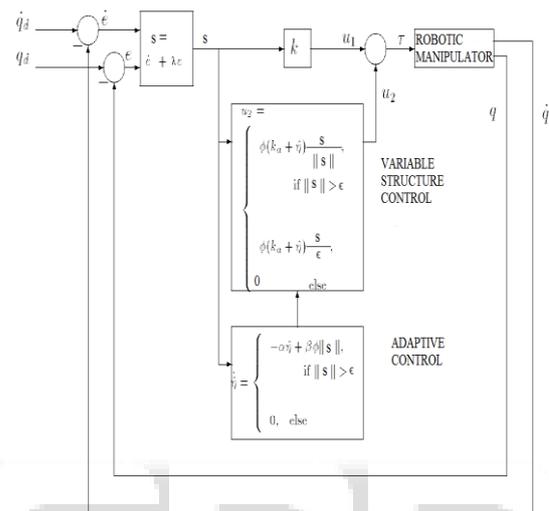
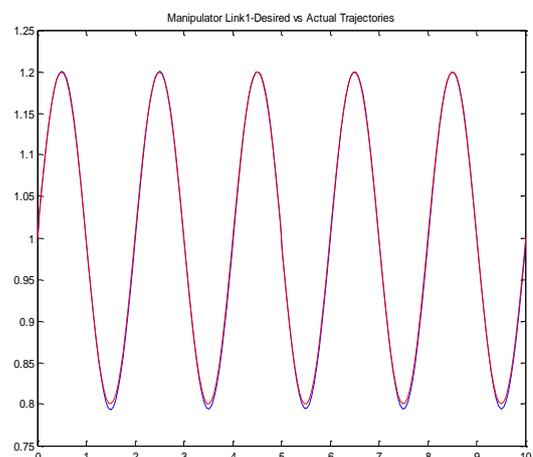


Fig. 2: Block representation of Adaptive Scheme.[13]

The output and input performances of the proposed adaptive VSC controller as well as the controllers designed by Jae-Sam et al for the two-link robotic manipulator are studied and compared. It is noted that the proposed adaptive VSC controller offers comparable tracking performance by applying an adaptive control input having minimal variation as compared to the controllers proposed previously . Moreover, the overall control energy spent in the case of the proposed adaptive VSC controller is not more than those in the other methods. The proposed algorithms seldom uses linear approximation for the defining of the adaptation law or in the stability proof and eradicates the requirement for inversion of the inertial matrix for measurement of joint acceleration or for calculation of a regressor.



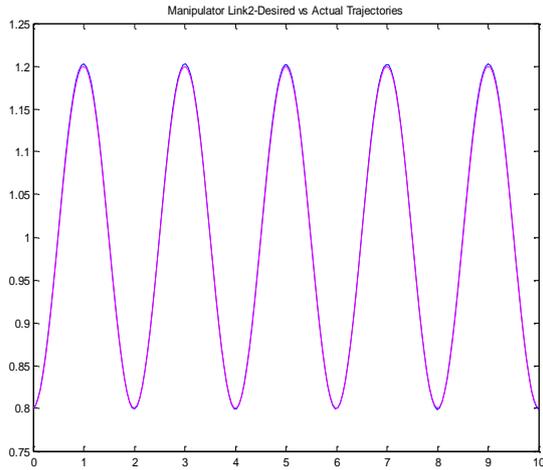


Fig. 3: Trajectory Tracking Response of Link1 and Link2. Figure. 3. represents the position tracking of actual system output trajectory with respect to desired trajectory. Tracking trajectories of both the links is depicted.

The variation in the gains of the adaptive controllers with changing positions of manipulator system can be seen in Figure.4 and Figure 5. After considerable increase in the system disturbance the gains reach a saturation value which in turn inhibits the systems ability to track the desired trajectory. However, systems robustness is very much satisfactory.

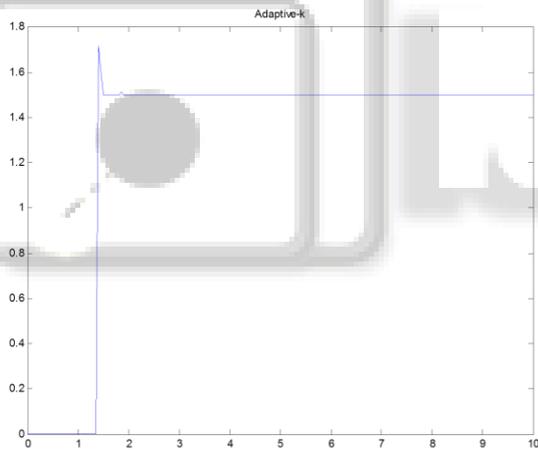


Fig. 4: Adaptive Gain of second control term.

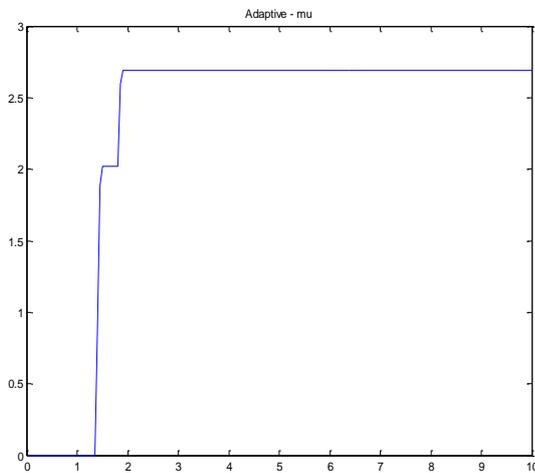


Fig. 5: Adaptive Gain of third control term.

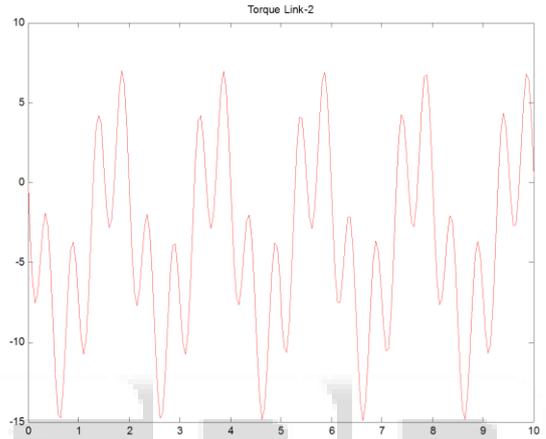
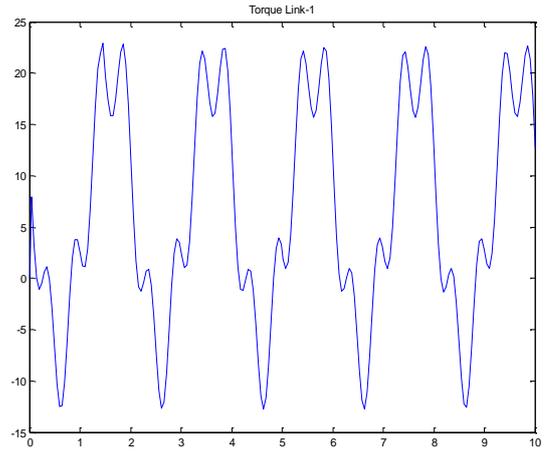


Fig. 6: Input Torque Responses of Link1 and Link2

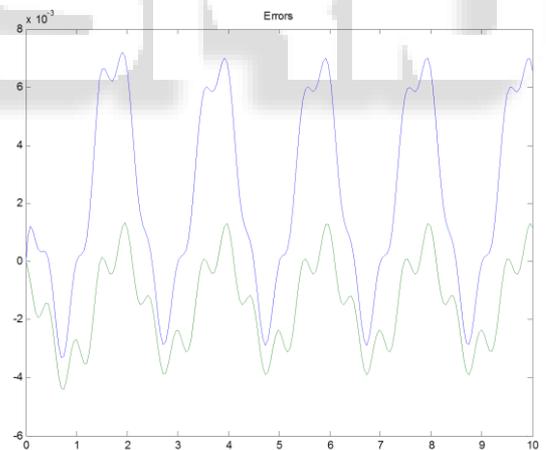


Fig. 7: Tracking errors for Link-1 & Link-2.

## VI. CONCLUSIONS

The projected technique do not use linear approximation for deriving the adaptation law or in the lyapunov stability analysis and lessens the requirement for inversion of the inertial manipulator matrix for measurement of joint acceleration or for calculation purposes and hence the computational load needed is roughly the same as in any conventional PID controller. Its noteworthy that the simulation results achieved in the presence of the disturbance as depicted in (11) over the interval  $t = (0,10)$  indicates that the proposed algorithm worked effectively for both the given parameter uncertainties and the disturbances.

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