A Review on Stress Analysis of an Infinite Plate with Cutouts in Composite Materials

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Abstract---In this paper an effort is made to review the investigations that have been made on the “Stress Analysis of an Infinite Plate with Two Square or Rectangular Cutouts in Composite Materials”. To reduce the weight of the system, different cutout shapes are required to make in structural elements. These discontinuities are weakening agent for the structure or machine. The presence of a cutout complicates the stress distribution in the plate so stress field around such hole must be known under different loading conditions. A number of analytical and experimental techniques are available for stress analysis around the different types of cut-outs for different condition in an infinite plate, made up of different materials under different loading condition has been reported in this article. The methods compared are tabulated with their findings. Singularities of square and rectangular hole in rectangular plate are considered in present study.

I. INTRODUCTION
Thin plates are commonly used in marine, aerospace structures. Different types of holes or openings are usually made in the plates for practical reasons such as to reduce the weight of the system and to provide access to system equipment. When a plate is subjected to tension or shear loading, high stresses are produced around holes or openings due to the presence of holes or openings. The stress concentration problem of a plate with a hole is needed to be considered in structure design and to always be a concern for many researchers. Many researchers have paid more attention to the problem of stress distribution around holes in an infinite elastic plate, and one of the most powerful methods for the problem is complex variable method. This hole/opening works as stress raisers and may lead to the failure of the structure/machine component. Hence it is an important aspect of stress analysis to predict stress concentration for regular or irregular holes. The irregularity in the hole shape may be because of chemical degradation. Under the effect of external loading and chemical process some irregular shapes may evolved. It is necessary to know stress distribution around such irregular shaped hole which may be useful to know hole shape evolution.

A. Stress Concentration:
Stress concentration is localization of high stresses mainly due to discontinuities in continuum, abrupt changes in cross section and due to contact stresses. To study the effect of stress concentration and magnitude of localized stresses, a dimensionless factor called Stress Concentration Factor (SCF), Kt as defined by Eq. (1) is used.

\[ K_t = \frac{\sigma_{\text{max}}}{\sigma_{\text{nom}}} \quad (1) \]

Where, \( \sigma_{\text{max}} \) is maximum stress at the discontinuity and \( \sigma_{\text{nom}} \) is nominal or background stress.

The stress concentration factor can be determined analytically by applying elasticity theory. For a large thin plate with a small circular hole at the center, that is subjected to uni-axial far-field tension, \( \sigma \), acting along the x-axis, the stresses (radial, circumferential and tangential ) around the vicinity of the hole are given in polar coordinates (r,θ) which shown in figure 1:

![Fig. 1: Plate with single circular hole subjected to uni-axial stress](image)

B. Measurement of stress:
Stress analysis of the critical elements under various loading conditions is carried out by the researchers for safe design of the element. Stress is measured by experimental methods or analytical/numerical method.

1) Experimental Methods:
From the various experimental methods available for stress analysis, the following methods are long established.

a) Photo Elasticity:
Photo elastic stress analysis is a full field technique for measuring the magnitude and direction of principal stresses. When polarized light is passed through a stressed transparent model, interference patterns or fringes are formed. These patterns provide immediate qualitative information about the general distribution of stress, positions of stress concentrations and of areas of low stress using the principals of stress optic law, Eq. (5) [2].

\[ \sigma_1 - \sigma_2 = N \frac{f_x}{t} \quad (2) \]

where, \( \sigma_1 \) and \( \sigma_2 \) are the values of the maximum and minimum principal stresses at the point under consideration, N is the fringe number or fringe order at the point, \( f_x \) is the material fringe value and t is the model thickness.

b) Brittle Coating:
The brittle-lacquer technique of experimental stress analysis relies on the failure by cracking of a layer of a brittle coating which has been applied to the surface under investigation. Specially prepared lacquers are usually applied by spraying on the actual part. Pattern of small cracks appear on the surface of this coating where the strain is high indicating the presence of stress concentration. The cracks also indicate the directions of maximum strain at these points since they are
always aligned at right angles to the direction of the maximum principal tensile strain. These crack data could be used to locate strain gauges for precise measurement of the stress. The method is however, sensitive to temperature and humidity [2].

c) Electrical Strain Gauges:
The method is one of the most popular and widely accepted for strain measurements and stress analysis. The strain gauge consists of a grid of strain-sensitive metal foil bonded to a plastic backing material. Any change in length will result in a change of resistance. Thus measurement of this resistance change with suitably calibrated equipment enables a direct reading of linear strain to be obtained [3].

Change of resistance and strain may be expressed as follows:

\[ \Delta R / R = K \times \Delta L / L \]

\[ \varepsilon = (\Delta R / R) / K \]  

(3)

Where, \( \Delta R \) and \( \Delta L \) are the changes in resistance and length respectively, \( K \) is termed as the gauge factor and \( \varepsilon \) is the strain. Thus measurement of this resistance change with suitably calibrated equipment enables a direct reading of linear strain to be obtained.

2) Analytical/Numerical Methods:
Distribution of stresses in a structure with boundary conditions, i.e. displacements and/or forces on the boundary can be determined by using either the closed form analytical methods or by approximate numerical methods. Boundary value problems can be solved analytically by using constitutive equations based on the elastic or plastic behavior of the material under load. Analytical or close-form solutions can be obtained for simple geometries, constitutive relations and boundary conditions. Approximate solutions for boundary-value problems can be obtained through the use of numerical methods such as finite element method, finite difference method, boundary element method and finite volume method.

a) Finite Element Method:
The structural model to be analyzed is divided into many small pieces of simple shapes called elements. Finite Element Analysis (FEA) program writes the equations governing the behavior of each element taking into consideration its connectivity to other elements through nodes. These equations relate the unknowns, for example displacements in stress analysis, to known material properties, restraints and loads. The program assembles the equations into a large set of simultaneous algebraic equations - thousands or even millions. These equations are then solved by the program to obtain the stress distribution for the entire model.

In recent years, with the advent of advanced software’s, the FEA based software ANSYS, COSMOL, DIANA, ABACUS and NASTRAN have been very useful for stress analysis. These software’s are preferred by users according to the type of stress analysis, the type of elements to be analyzed and the depth of accuracy required.

b) Boundary Element Method:
In this method, the governing differential equation is converted into an integral form, often involving only integrals over the boundary of the domain. Consequently, only the boundary has to be discretized in order to carry out the integrations. The dimensionality of the problem is thereby effectively reduced by one. A three dimensional volume problem becomes a two dimensional surface one, while a two-dimensional plane problem involves only one-dimensional line integrations. Also, because the interior of a solution domain is not discretized, there is much less approximation involved in representing the solution variables and rapid variations of, for example, stresses and displacements can be resolved very accurately. Stresses are accurate as there are no approximations imposed on the solution in interior domain points. The method is suitable for modeling problems of rapidly changing stresses. Boundary Element Method (BEM) uses less number of nodes and elements for the same level of accuracy as other methods. The Boundary Element Method is unsuitable if information is required at a large number of internal points [3].

c) Complex Variable Approach:
Complex analysis, traditionally known as the theory of functions of a complex variable, is the branch of mathematics investigating functions of complex numbers. The difference between real functions and complex function is that complex functions can handle areas while real functions can handle only direction. So it can use easily for two dimensional problems of physics. Complex analysis is one of the classical branches in mathematics with its roots in the 19th century and some even before. Important names are Euler, Gauss, Riemann, Cauchy, Weierstrass, and many more in the 20th century. In 1909 G V Kolossoff wrote his doctoral degree dissertation of Dorpat University, USSR on the subject on “One application of the theory of functions of a complex variable to a plane problem in the mathematical theory of elasticity.” In which systematic use of the complex variable theory was first proposed, making use of Goustat’s, a French Mathematician’s and two complex stress functions to represent the biharmonic equations. Then in next 40 years theory come to successful conclusion. In 1933, N I Muskelishvili wrote a book on the basis of this theory titled “Some basic problems of the mathematical theory of elasticity,” in Russian language which was translated in English by J M Radok in 1953.

In case of complex variable approach complex functions are used in which independent as well as dependent variables all are complex numbers. Let us consider one complex function \( f(z) \) where, \( z = x + iy \) and the relation \( w = f (z) \) will give representation of complex variable function. \( w = u(x, y) + iv(x, y) \) represents the function in form of two real valued functions \( u \) and \( v \) [5]. This template, modified in MS Word 2007 and saved as a “Word 97-2003 Document” for the PC, provides authors with most of the formatting specifications needed for preparing electronic versions of their papers. All standard paper components have been specified for three reasons: (1) ease of use when formatting individual papers, (2) automatic compliance to electronic requirements that facilitate the concurrent or later production of electronic products, and (3) conformity of style throughout a conference proceedings. Margins, column widths, line spacing, and type styles are built-in; examples of the type styles are provided throughout this document and are identified in italic type, within parentheses, following the example. Some components, such as multi-leveled equations, graphics, and tables are not prescribed, although the various table text styles are provided. The formatter will need to
create these components, incorporating the applicable criteria that follow.

II. LITERATURE REVIEW

In order to satisfy certain service requirements holes/openings are made in structures and machine components. These holes are of different shape and size. Also, the chemical erosion may lead to irregularity in the hole shape. These holes are the stress raisers and it is essential to know the stress pattern around such holes. An attempt is made to review few of the important contribution for stress analysis in the infinite plate with the different types of cut-out in the present work.

A. Stress analysis of plate with hole:

K R Y Simha and S S Mohapatra [5] used complex variable method to find the Stress concentration around irregular hole. Conformal mapping method has been used for evaluation of stresses. The method is an operation in complex mathematics which maps a set of points in one coordinate system to a corresponding set in another, keeping the angle of intersection between two curves constant, and is widely used in solving elasticity problem. They used complex variable in conjunction with conformal mapping to obtain tangential stress variation along the boundary of isolated irregular holes in large plates under uniform loading. Nine hole shapes with same area and different perimeter are studied. Irregular holes may change their shape if not their size by exchanging surface energy with strain energy. They considered that such irregular holes will form due to the combined action of loads and chemical attacks. A linear elastic analysis followed in this paper may not support the physics of change in shapes but can be extended to a linear viscoelastic material.

Savin [6] obtain the formulation which gives the stresses around holes in anisotropic plates under in plane loading, a general solution is obtained to consider an arbitrary shape of hole and arbitrarily oriented uniaxial, biaxial, and shear stresses at infinity as well as uniform tangential force, and uniform pressure around the hole.

The stress in anisotropic plates under in plane loading had been analyzed by V G Ukadgaonker and D K N Rao [7]. They achieved this by introducing a general form of mapping function and an arbitrary biaxial loading condition in the boundary condition and extended this formula for the multilayered plates. To obtain stress around hole, they introduced computer program in computer in which, the arbitrary biaxial loading factor, the orientation angle and complex parameter for the laminate had to be given. By selecting appropriate values of the complex parameters of an isotropic for the laminate, different laminate geometry are considered. This solution can be degenerated to the isotropic case by taking the complex parameters s1 = 1.0005i and s2 = 0.9995i. Finally they found that the stress distribution around a given shape hole depends on the combined effect of hole shape, laminates geometry, types of loading. The failure strength is also calculated using different failure theories to get the allowable applied stress.

Milan Batista [8] proposed a method, represents an improvement over known method, by which one may efficiently calculate the stress distribution around holes of relatively complex shapes in infinite plates subjected to a uniform load at infinity. He used Schwartz – Christoffel mapping function in the calculation of stress distribution around polygonal holes. He had done same example on his proposed method and came out with some result after this, this results are compared with savin’s result for the same loading condition. And this results of improved method shows that any huge stresses at convex corners may noticeably alter the overall stress distribution around the hole. So for accurate stress distribution around a hole this method may be alternative.

Dharmendra S Sharma [9] studied the effect of cut-out shape, corner radius, load angle and hole orientation on stress pattern on triangular, square, pentagonal, hexagonal, heptagonal and octagonal cut-out shapes in infinite plate subjected to different types of loading condition. He used complex variable method to obtain general solution for finding stress distribution around holes in an infinite plate. Stress functions are obtained by evaluating Cauchy’s integral for the given boundary condition. The results are obtained for the uniaxial tension at infinite, hydrostatic tension at infinite, shear loading at infinite.

Zuxing Pan, Yuansheng Cheng and Jun Liu [10] studies complex variable method together with proposed stress functions to obtain the solution for stress distribution around rectangular hole in finite plate subjected to uniaxial tension. This stress function is generated by superposition of the stress function for an infinite plate with a rectangular hole and ones for a finite plate without a hole. The finite area external to the rectangular hole and the rectangular hole and in z-plane are mapped into the finite area outside a unit circle and a unit circle in ω-plane with the help of mapping function. They analyzed effect of hole sizes, hole orientation and plate’s aspect ratio on stress distribution. The results of infinite plates obtained by the modified stress function are compared with the ones in literature and the results of finite plates obtained by the modified stress function are compared with the obtained by ANSYS.

D K Nageswara Rao, M Ramesh Babu, K Raja Narender Reddy and D Sunil [11] presented the solution is very elegant to get the stress distribution around holes in Symmetric laminates and the failure strength of the laminate on first ply failure basis by Tsai-Hill theory, Hashin-Rotem criterion and Tsai-Wu criterion. Square and rectangular holes in symmetric laminates of graphite/epoxy and glass/epoxy are studied. Savin’s basic formulation is used to derive the stress function for symmetric laminates with a general shape of hole under arbitrary biaxial loading. The stress results are also obtained by ANSYS for comparison.

Ukadgaonkar and Vyasaraj [12] have analyzed the stress around the irregular cut-outs in an orthotropic plate for different in-plane loading. They find the solution for isotropic as well as for orthotropic cases for in plane loading. They found that angle ply laminates are inducing more stress concentration for shear loads and are not suitable. They are produce many results with different shape of hole for uni-axial, (loading in x- or y-direction), as well as for biaxial loading conditions and also a pure shear loading case is considered. In solution, the general form of mapping function is introduced, which allow to consider solution of large variety of holes shapes and loading condition. These results are supported with finite element solutions. The solution is capable of considering single and multi-layered plates of arbitrary fiber orientation and lay-
ups. Equations are given to determine the corresponding complex parameters of anisotropy.

J R-ezaeeapazhand and M Jafari [13] presented accurate and simple method for analytical solution for stress analysis of plates with central cut-outs. Lekhnitskii’s solution for circular and elliptical cut-outs is extended to special cut-out shape using complex variable mapping. They included isotropic plates with triangular, square and pentagonal cut-outs in their studies. The effects of cut-out shape and load directions on the stress distribution and SCF in the perforated plates are studied. They found that stress concentration factor of perforated plates can be significantly changed by using proper cut-out shape, bluntness and orientation.

D S Sharma, Khushbu Panchal and Patel Nirav [14,15] obtain the general solution for determining the stress field around circular hole and triangular hole in infinite orthotropic plate subjected to internal pressure by use of the Muskhelisvili’s complex variable method and numerical results are obtain using MATLAB 7.6. They studied the effect of fiber orientation and material parameter on stress pattern around pressurized circular hole. They use the ANSYS for preparation of model and results are compared with the Muskhelisvilli’s complex variable method. The formulation which they find is a good tool for the designer to predict stress pattern around internally pressurized hole and to predict failure pattern of mechanical component and structures.

\[ \sigma_x = 2 \text{Re} \left[ s_1^2 \phi'(z_1) + s_2^2 \psi'(z_2) \right] \]
\[ \sigma_y = 2 \text{Re} \left[ \phi'(z_1) + \psi'(z_2) \right] \]
\[ \tau_{xy} = -2 \text{Re} \left[ s_1^2 \phi'(z_1) + s_2^2 \psi'(z_2) \right] \]

Yi Yang, Jike Liu and Chengwu Cai [16] derive the analytical solution to the stress concentration problem in plates with a rectangular hole subjected to biaxial loading. They use the U-transformation technique and the finite element method to drive series form of the analytical displacement solution of the finite element equations. A finite element governing equation with cyclic periodicity is established, and the U-transformation is applied to decouple the FEM equation and the nodal displacement and stress expressions are derived.

![Plate with circular hole](image)

**Fig. 2: Plate with circular hole**

They find the stress components for plane stress conditions in terms of Muskhelisvili’s complex function \( \phi (z_1) \) and \( \Psi (z_2) \) as follows:

<table>
<thead>
<tr>
<th>Material</th>
<th>Stress</th>
<th>Present Method Result</th>
<th>ANSYS Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isotropic</td>
<td>Max ( \sigma_x ) = 1.00</td>
<td>1.00</td>
<td>1.001</td>
</tr>
<tr>
<td></td>
<td>Max ( \sigma_y ) = 1.00</td>
<td>1.00</td>
<td>1.001</td>
</tr>
<tr>
<td></td>
<td>Max ( \tau_{xy} ) = 1.00</td>
<td>1.00</td>
<td>1.001</td>
</tr>
<tr>
<td>Glass/Epoxy - Fiber</td>
<td>Max ( \sigma_x ) = 1.6521</td>
<td>1.555</td>
<td></td>
</tr>
<tr>
<td>orientation</td>
<td>Max ( \sigma_y ) = 1.3085</td>
<td>1.209</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Max ( \tau_{xy} ) = 0.8537</td>
<td>0.8554</td>
<td></td>
</tr>
<tr>
<td>Glass/Epoxy - Fiber</td>
<td>Max ( \sigma_x ) = 1.7916</td>
<td>1.611</td>
<td></td>
</tr>
<tr>
<td>orientation</td>
<td>Max ( \sigma_y ) = 1.7916</td>
<td>1.599</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Max ( \tau_{xy} ) = 0.7164</td>
<td>0.7942</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Comparison of results of present method with ANSYS results for circular hole [14]

V. Anil, C.S. Upadhyay, N.G.R. Iyengar [17] used finite element method to study buckling behavior of composite laminated plates was carried out with both uniaxial and biaxial loads. The change in critical buckling response of both thick and thin rectangular symmetric laminates with respect to the fiber orientation angle has been studied. The change in critical buckling response of both thick and thin rectangular symmetric laminates with respect to the fiber orientation angle has been studied. The critical fiber orientation angle reduces as the length/thickness ratio of the laminate increases. The critical value of the fiber orientation angle remains almost same for shear buckling load with change in aspect ratio.

Chen Puhui, Shen Zhen [18] extended the classical Lekhnitskii’s complex potential approach for the stress and deformation analysis of unsymmetrical composite laminates with elliptical hole. The stresses and deformations of an infinite unsymmetrical composite laminate containing an elliptic hole under arbitrary Loadings can be derived through these general solutions.

V.G. Ukadgaonker, D K N Rao [19] gave a general solution for bending of symmetric laminates with holes considering any shape of hole in symmetric laminates subjected to remotely apply bending or twisting moments. Moments around circular, elliptical, triangular, square, rectangular and several irregular shaped holes in cross-ply and angle ply symmetric laminates are obtained. The study on stacking sequence revealed that the moments around the hole will decrease for laminates with large number of ply groups and vice versa. The results also revealed that the magnitude of moments and symmetry of distribution depend on the combined effect of all the parameters, viz., hole geometry, loading, Young’s moduli and Poisson’s ratios of the material and flexural moduli of the laminates.

L.Q. Zhang, A.Z. Lu, Z.Q. Yue, Z.F. Yang [20] presented an efficient and accurate stress solution for an infinite elastic plate around two elliptic holes, subjected to uniform loads on the hole boundaries and at infinity with the help of Schwarz’s alternating method and the Muskhelishvili’s complex variable function techniques. They used the iterative algorithm to obtain accurate stress
solutions for the double elliptic hole problems. The present algorithm can be used to compute the stress concentration factors (SCF).

III. SUMMARY
In this paper an effort is made to review the investigations that have been made on the stress analysis of infinite plate with cut-out. An attempt has been made in the article to present an overview of various techniques developed for stress analysis of infinite plate. Singularities of circular hole, elliptical hole, triangular hole, square hole and rectangular hole and multiple hole like circular hole and elliptical hole in infinite plate are considered.

REFERENCES