

# Review Paper on Comparative Analysis of Denoising Technique Such As Curvelet and Wavelet Transform

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*Abstract*--The images usually bring different kinds of noises in process of receiving, coding & transmission. Image denoising has been a fundamental problem in field of image processing. The growth of media communication industry and demand of high quality of visual information has opened the doors for researchers to develop various method of denoising. The curvelet transform is one kind of new multi-scale transform after 1999 that is based on wavelet transform, whose structural elements include the parameters of dimension and location, and orientation parameter more. Therefore, curvelet transform is superior to wavelet in the expression of image edge, such as geometric characteristic of curve and beeline.

## I. INTRODUCTION

Every image contains some or the other type of noise, to remove that noises from the image many techniques have been developed, and many filters are also used such as averaging filter werner filter etc.. The aim of removing noise is achieved by inverse transformation, like wavelet transform. To overcome some of the disadvantage in wavelet transform such as edge preserving and smoothing, new multiscale transform based on wavelet transform-curvelet transform has been developed. The structural elements of curvelet transform include the parameters of dimension and location, and orientation parameter more. As a result curvelet transform is more superior to wavelet transform in terms of image edge, geometry characteristic of curve and beeline, which has obtained a good result in image denoising.[8][4]

## II. WAVELET TRANSFORM

Short time Fourier transform is used for nonstationary signal analysis but due to its limitation in analyzing time localized events wavelet transform came into existence. We now review the Fourier and short time Fourier transform (STFT) and will discuss properties that STFT does not possess and Wavelet do possess. [5]

### A. The Fourier and Short-Time Fourier Transforms

For any function  $f$  with finite energy, the *Fourier transform* off is defined to be the integral

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t) e^{-\omega t} dt \quad (1)$$

$\omega$  being the angular rate, equal to  $2\pi$  times frequency. A Fourier transform is often represented by its power spectrum-the square of the modulus of  $\hat{f}(\omega)$   $V5 \omega$ . For example, the power spectrum of an impulse function has a constant value of unity and is independent of the time at which the impulse occurs. Time of occurrence affects only the phase of each frequency component. The Fourier transform is best suited to analyze stationary periodic functions-those that exactly repeat themselves once per period, without modification.[5]

The short-time Fourier transform (or STFT) of a function at some time  $t$  is the Fourier transform of that function as examined through some time-limited window centered on  $t$ . A different Fourier transform exists for each position  $t$  of the window. These transforms, produced by sliding the examination window along in time, constitute the STFT. If the examination window simply omits the signal outside the window, two problems are encountered. One is the sudden change in the power spectrum as a discontinuity enters or leaves the window, compounded by a lack of sensitivity to the position of the discontinuity within the window. The other problem is spectral leakage: if some component of the signal has a cycle time which is not an integral divisor of the window width, the transform exhibits spurious response at many frequencies. These problems are ameliorated by attenuating samples away from the center of the window, by a "windowing function,"  $g$ .

### B. Wavelet Transform

Wavelet analysis adopts a wavelet prototype function known as the mother wavelet given in eqn. 2. This mother wavelet in turns generates a set of basic functions known as child wavelets through recursive scaling and translation. The variable  $s$  reflects the scale or width of a basis function and the variable  $t$  is the translation that specifies its translated position on the time axis.

$$\Psi(\tau, s) = \frac{1}{\sqrt{s}} \Psi\left(\frac{t-\tau}{s}\right) \quad (2)$$

In eqn. 2.  $\Psi\left(\frac{t-\tau}{s}\right)$  Is the mother wavelet and the factor  $\frac{1}{\sqrt{s}}$  is normalized factor used to ensure energy across different scale remains same.

### C. Continuous Wavelet Transform

Continuous wavelet transform off ( $t$ ), with respect to the wavelet  $\Psi(t)$  is defined as

$$CWT(\tau, s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} f(t) \Psi\left(\frac{t-\tau}{s}\right) dt \quad (3)$$

Where  $it$  is the translation coefficient and  $s$  is the scaling coefficient. CWT analyses the signal through the continuous shifts of a scalable function over a time plane. This technique results in redundancy and it is numerically impossible to analysis at infinite number of wavelet sets.

### D. Discrete Wavelet Transform

Discrete wavelet transform is one of the most promising multiresolution approaches used in curvelet base image denoising. The advantage of a time frequency representation of signals where Fourier transform is only frequency localized. The location, at which a frequency component of an image exists, is important as it draws the discrimination line between images. An image  $f(x, y)$ , its continuous wavelet transform is given by

$$WT_{\Psi}(a_1, a_2, b_1, b_2) = \int_R \int_R f(x, y) \overline{\Psi_{a_1, a_2, b_1, b_2}(x, y)} dx dy \quad (4)$$

Where a wavelet with scale parameter  $a_1, a_2$  and position parameter  $b_1, b_2$  can be described as follows

$$\Psi_{a_1, a_2, b_1, b_2}(x, y) = a_1^{-\frac{1}{2}} a_2^{-\frac{1}{2}} \Psi\left(\frac{x-b_1}{a_1}\right) \Psi\left(\frac{y-b_2}{a_2}\right) \quad (5)$$

Unlike the FT and STFT, the window size varies at each resolution level when the wavelet transform is applied to an image. In discrete wavelet transform, the original image is highpass filtered yielding three detail images, describing the local changes in horizontal, vertical and diagonal direction of the original image. The image is then low pass filtered yielding an approximation image which is again filtered in the same manner to generate high and low frequency subbands at the next lower resolution level (Fig. 1.). This process is continued until the whole image is processed or a level is determined as the lowest to stop decomposition. This continuing decomposition process is known as down sampling and shown in Fig. 1.

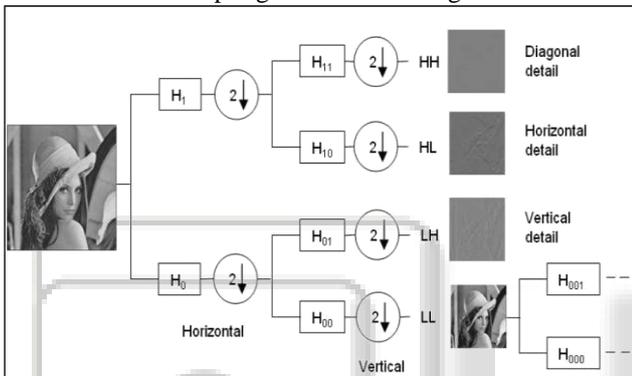


Fig. 1: DWT decomposition tree

These coefficients can then be used to analyse the texture patterns of an image. Wavelet subbands obtained from the Lena image using 4 decomposition levels are shown in Fig. 1. Due to its good image texture representation ability, wavelet transform has been used in many image processing applications, e.g., texture study curvelet base image denoising methods, texture classification and image deconvolution. The wavelet transform in yielded a much higher texture classification rate and retrieval accuracy than discrete cosine transform or spatial partition.

### E. Curvelet transform.

Curvelet transform has been developed to overcome the limitations of wavelet and Gabor filters. Though wavelet transform has been explored widely in various branches of image processing, it fails to represent objects containing randomly oriented edges and curves as it is not good at representing line singularities. Gabor filters are found to perform better than wavelet transform in representing textures and retrieving images due to its multiple orientation approach. Due to the loss of spectral information in Gabor filters they cannot effectively represent images. This affects the curvelet base image denoising performance. To achieve a complete coverage of the spectral domain and to capture more orientation information, curvelet transform has been developed. The initial approach of curvelet transform implements the concept of discrete ridge let transform [3]. Since its creation in 1999 [3], ridgelet based curvelet transform has been successfully used as an effective tool in

image denoising [3], image decomposition [3] image deconvolution [3], astronomical imaging [3] and contrast enhancement [3], etc.

This block ridgelet-based transform, which is named curvelet transform, was first proposed by Candes and Donoho in 2000. Apart from the blocking effect, however, the application of this so-called first generation curvelet transform is limited because the geometry of ridgelets is itself unclear, as they are not true ridge functions in digital images. Later, a considerably simpler second-generation curvelet transform based on frequency partition technique was proposed. The second-generation curvelet transform has been shown to be a very efficient tool for many different applications in image processing. The overview of the curvelet transform is shown below for four step: [book]

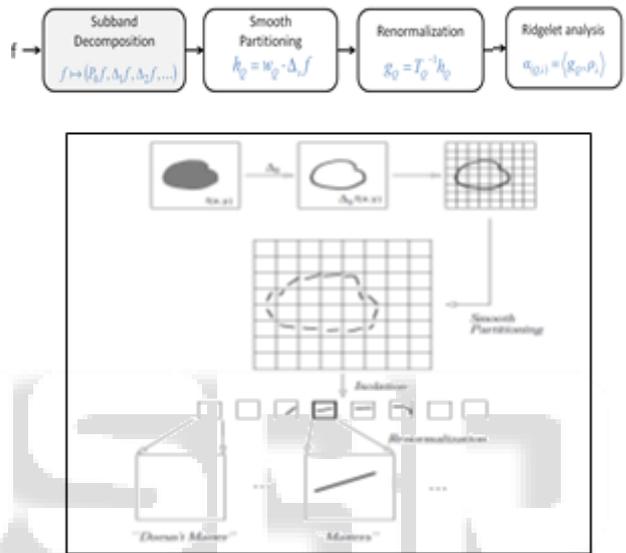


Fig. 2: subband decomposition

### F. Subband Decomposition

We define a bank of subband filter  $P_0, (\Delta_s, s \geq 0)$ . The object  $f$  is filter into subbands:

$$f \mapsto (P_0 f, \Delta_1 f, \Delta_2 f, \dots) \quad (6)$$

This step divid the image into sevel resolution layers. Each layer contains details of different frequencies:

- $P_0 \rightarrow$  Lowpass filter
- $\Delta_1, \Delta_2, \dots$  - Band-pass (high-pass) filters.

So the original image can be reconstructed from the sub-bands:

$$f = P_0 (P_0 f) + \sum_s \Delta_s (\Delta_s f) \quad (7)$$

The paper of curvelet build some simbol to denote the filter:  $\Phi_0$  : A lowpass filter. The filter deal with low frequency near  $|\xi| \leq 1$   $\Psi_{2^s}$  : The band pass filters. The filter deal with frequencies near domain  $|\xi| \in [2^{2s}, 2^{2s+2}]$ . Besides, there is Recursive construction :  $\Psi_{2^s}(x) = 2^{4s} \Psi(2^{2s}x)$ . The sub-band decomposition is simply applying a convolution operator:

$$P_0 f = \Phi_0 * f \quad \Delta_s f = \Psi_{2^s} * f \quad (8)$$

There are some connection between Curvelet and Wavelet this part. The sub-band decomposition can be approximated using the well known wavelet transform:

- Using wavelet transform,  $f$  is decomposed into  $S_0, D_1, D_2, D_3$ , etc.
- $P_0 f$  is partially constructed from  $S_0$  and  $D_1$ , and may include also  $D_2$  and  $D_3$ .
- $\Delta_s f$  is constructed from  $D_{2^s}$  and  $D_{2^{s+1}}$ .

$P_0 f$  is “smooth” (low-pass), and can be efficiently represented using wavelet base.

But it is confuse that the discontinuity curves effect the high-pass layers  $\Delta_s f$ . Can they be represented efficiently?

Ans: Looking at a small fragment of the curve, it appears as a relatively straight ridge.

Besides, we will dissect the layer into small partitions to avoid mistake.

Example of Subband Decomposition: [1]

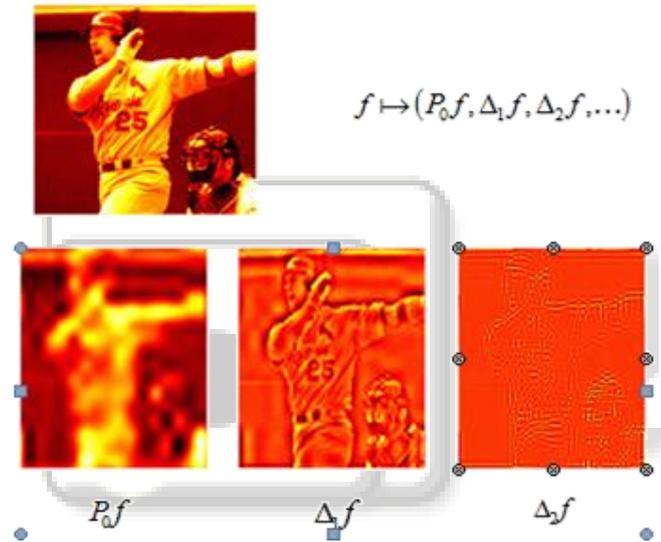


Fig. 3: subband decomposition with each step

### G. Smooth Partitioning

It is define a collection of smooth window  $w_Q(x_1, x_2)$  localized around dyadic squares:

$$Q_{(s,k_1,k_2)} = \left[ \frac{k_1}{2^s}, \frac{k_1+1}{2^s} \right] \times \left[ \frac{k_2}{2^s}, \frac{k_2+1}{2^s} \right] \in Q_s \quad (9)$$

Let  $w$  be a smooth windowing function with ‘main’ support of size  $2^{-s} \times 2^{-s}$ . Multiplying a function by the corresponding window function  $w_Q$  produces a result localized near  $Q$  ( $\forall Q \in Q_s$ ). Doing this for all  $Q$  at a certain scale, i.e. all  $Q=Q(s, k_1, k_2)$  with  $k_1$  and  $k_2$  varying but  $s$  fixed, procedure, we apply this windowing dissection to

$$h_Q = w_Q \cdot \Delta_s f \quad (10)$$

each of the subbands isolated in the previous stage of the algorithm. And this step produces a smooth dissection of the function into ‘squares’.

Example of Smooth Partitioning: [1]

Take the  $\Delta_1 f$  part of last example to apply Smooth Partitioning

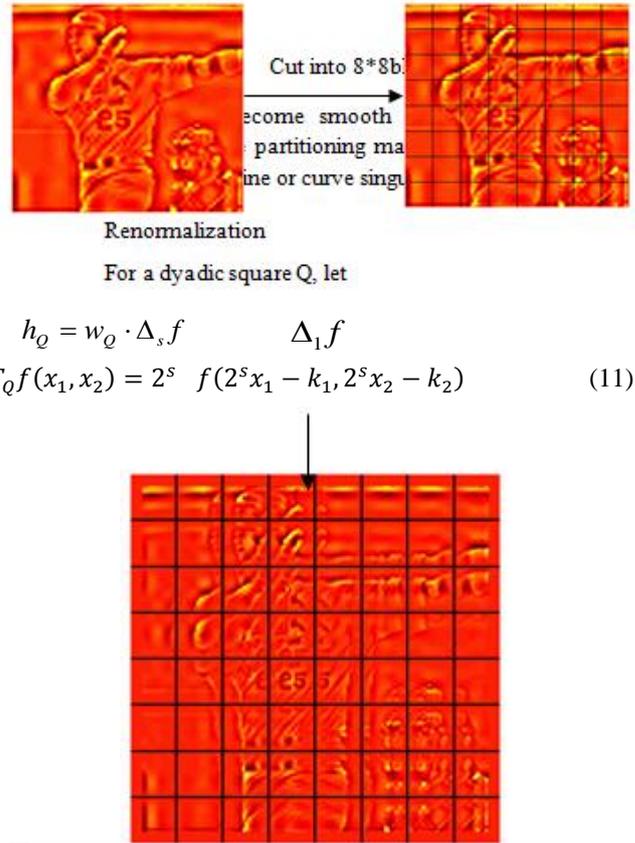


Fig. 4:smooth partitioning

denote the operator which transports and renormalizes  $f$  so that the part of the input supported near  $Q$  becomes the part of the output supported near  $[0,1] \times [0,1]$ . In this stage of the procedure, each ‘square’ resulting in the previous stage is renormalized to unit scale:

$$g_Q = T_Q^{-1} h_Q \quad (12)$$

### III. CONCLUSION

The comparison of wavelet transform and curvelet transform technique is rather a new approach, and it has a big advantage over the other techniques that it less distorts spectral characteristics of the image denoising. The primary goal of noise reduction is to remove the noise without losing much detail contained in an image. In wavelet transform the curves are not taken into consideration so the image obtained after wavelet transform may suffer from some loss or poor quality of transformation to overcome this disadvantage of wavelet we are using curvelet transform to obtain a loss free and better quality image.

### REFERENCES

[1] Research And Application Of Image Denoising Method Based On Curvelet Transform Jiang Taoa, Zhao Xinb A. Jiang Tao, Shandong University Of Science And Technology, Qingdao, 266510, China,tjiang@126.com b. ZHAO Xin, Shandong University of Science and Technology, Taian, 271000, China, ztx\_zhaoxin@126.com.

- [2] A Comparative Study of Wavelet and Curvelet Transform for Image Denoising Miss Monika shukla<sup>1</sup>, Dr.Soni changlani<sup>2</sup> <sup>1</sup>(Department.of Electronics & telecommunication;Lakshmi Narain College of Technology & Science,BHOPAL) <sup>2</sup>(Head of Department.of Electronics & telecommunication; Lakshmi Narain College of Technology & Science ,BHOPAL) IOSR Journal of Electronics and Communication Engineering (IOSR-JECE) e-ISSN: 2278-2834,p- ISSN: 2278-8735. Volume 7, Issue 4 (Sep. - Oct. 2013), PP 63-68 www.iosrjournals.org
- [3] .Denoising Performance Of Lena Image Between Filtering Techniques, Wavelet And Curvelet Transforms At Different Noise Level R.N.Patel<sup>1</sup>, J.V.Dave<sup>2</sup>, Hardik Patel<sup>3</sup>, Hitesh Patel<sup>4</sup> Address For Correspondence <sup>1</sup>, <sup>3</sup> Department Of Electronics And Communication Engineering, S.P.B.Patel Engineering College, Linch, Mehsana, Gujarat: 384435, India <sup>2</sup>\*Department Of Electronics And Communication Engineering, Government Engg. College, Sector-28, Gandhinagar. <sup>4</sup>V. T. Patel Department Of Electronics And Communication Engineering Charotar University Of Science And Technology, International Journal Of Advanced Engineering Technology E-ISSN 0976-3945
- [4] A Review of Curvelets and Recent Applications Jianwei Ma<sup>1</sup>;<sup>2</sup> and Gerlind Plonka<sup>3</sup> <sup>1</sup> School of Aerospace, Tsinghua University, Beijing 100084, China <sup>2</sup> Centre de Geosciences, Ecole des Mines de Paris, 77305 Fontainebleau Cedex, France <sup>3</sup> Department of Mathematics, University of Duisburg-Essen, 47048 Duisburg, Germany.
- [5] An Introduction To Wavelets Lee A. Barford, R. Shane Fazio, David R. Smith Instruments And Photonics Laboratory HPL-92-124 September, 1992 Curvelet.Org
- [6] Talwandi Sabo Bathinda , Suman Thapar, Amit Kamra, "A Novel Method Of Denoising Natural Images Using Curvelets Combining With Cycle Spinning" International Journal Of Computing & Business Research ISSN (Online): 2229-6166 Proceedings Of 'I-Society 2012' At GKU
- [7] Usha Rani, Charu Narula ,Pardeep "Image Denoising Techniques A Comparative Study" India International Journal Of Electronics, Communication & Instrumentation Engineering Research And Development (IJEIERD) ISSN 2249-684X Vol.2, Issue 3 Sep 2012 64-74
- [8] Cédric Vonesch And Michael Unser, "A Fast Multilevel Algorithm For Wavelet-Regularized Image Restoration" Ieee Transactions On Image Processing, Vol. 18, No. 3, March 2009