A Review Comparison of different Spreading Codes for DS CDMA
Menakshi1 Paras Chawla2
1, 2 JMIT Radaur, Kurukshetra University Kurukshetra, India

Abstract---In CDMA based systems; the selection of the spreading codes used for multiplexing multiple users’ data places a pivotal role. Some of the known classes of spreading codes used in CDMA-based cellular system are Walsh-Hadamard, Gold codes and Kasami codes. In this paper, presents a comparative study of different properties of these codes with the multi-level codes based on gray and inverse-gray codes In a cellular system, due to multi-path propagation channel, the orthogonality among the codes in the received CDMA signal is destroyed. To characterize this we define metrics based on all delayed auto-correlation and cross-correlation of the codes in a family. Mean Square Aperiodic Auto-Correlation (MSAAC), Mean Square Aperiodic Cross-Correlation (MSACC) measures and the Merit Factor (MF). These metrics for the GIG codes as designed and proposed in [1] were also evaluated and found to be better than other code family.

Keywords: DS-CDMA (Direct Sequence Code Division Multiple Access), MSAAC (Mean Square Aperiodic Auto-Correlation), MSACC. Mean Square Aperiodic Cross-Correlation, FOM (Figure of Merit), GIG (Gray Inverse Gray) codes

I. INTRODUCTION
Various forms of spread-spectrum (SS) techniques have been used in cellular communication. DS-CDMA systems like WCDMA and CDMA2000. The orthogonal codes are used to multiplexing data to multiple users. A communication technique in which a signal is transmitted on a larger bandwidth than the bandwidth of the original information is called Spread Spectrum Technique. In Spread Spectrum technology several techniques including frequency hopping (FHSS), direct sequence (DSSS), Time hopping (THSS) and/or hybrid of these are available, which can be used for multiple access (like CDMA) and/or multiple functions. Main advantages of SS includes: Anti-jamming of signal, low interference to signal, lower probability of signal intercepting, CDMA, information privacy and less interference and more privacy. [2]

SS technology normally uses a noise-like signal to spread the narrowband information signal over a relatively wideband signal (radio frequency or a band of radio frequencies). At the receiving side, the original information signal is recovered from the received spread signal using the same procedure in reverse. Actually, from [3, 4, 5], a spread-spectrum system uses a process to expand/spread the bandwidth of the signal instead of sending the information signal directly. The codes used for spreading [6] have low cross-correlation values and are unique to each user. That is the reason that a receiver which has knowledge about the spreading code of the particular transmitter, is capable of selecting the desired signal.

In this Paper, an analytical approach is adopted to show the comparison of some important PN [7] [8] sequences in terms of auto-correlation and cross-correlation properties and figure of merit. The rest of the paper is organized as follows. In Section 2, we deal with different kind of Pseudo Noise (PN) sequence [9]. Section 3 describes the techniques to measure the correlation properties and figure of merit. Results and discussions are presented in section 4. Finally, section 5 draws some conclusions

II. SPREADING CODES
A pseudo-noise (PN) sequence is a sequence of 1’s and 0’s. PN sequences are periodic sequences that have noise-like behavior. Sequences referred to as pseudorandom numbers or pseudo noise sequences.[10] They can be conveniently generated using shift registers, modulo-2 adders (i.e. XOR gates) and feedback logic circuitry or by an algorithm using initial seed. The maximum length of a PN sequence is determined by the length of the register and the configuration of the feedback network. An N-bit register can take up to 2N different combination of zeros and ones. Sequence isn’t statistically random but will pass many test of Randomness. Unless algorithm and seed are known, the sequence is impractical to Predict. So there is more privacy than other modulation technique the spreading codes increase bandwidth of signal to be transmitted. But there are several reasons for apparent waste of spectrum. That can be used for hiding and encrypting signals; Immunity from various kinds of noise and multipath distortion, several users can independently use the same higher bandwidth with very little interference.

III. CLASSIFICATION OF SPREADING CODES
Codes that can be found in practical DS-systems are: Walsh-Hadamard codes, Gold codes, Kasami-codes, M-sequences and GIG codes, [11]

A. Walsh-Hadamard code
The Walsh-Hadamard code is an example of a linear code over a binary alphabet that maps messages of length n to code words of length 2n. The Walsh-Hadamard code is unique in that each non-zero codeword has hamming weight of exactly 2n-1. The Walsh-Hadamard code is a locally decodable code. That’s why only looking at a small fraction of the received word, it provides a way to recover the original message with high probability. This gives rise to applications of Walsh codes in complexity theory. Using list decoding, the original message can be recovered as long as less than 1/2 of the bits in the received word have been corrupted.

A code indexing method for orthogonal variable spreading factor (OVSF) codes introduces a single number mapped to each code as shown in fig 1. The new code number itself provides the code signature, as well as it is also used for the OVSF code generation. In addition of this,
it provides easy and fast generation of the available code list without the help of look-up table.

Fig. 1: OVSF Code Tree

The Walsh-Hadamard code is an example of a linear code over a binary alphabet that maps messages of length \( n \) to code words of length \( 2^n \). The Walsh-Hadamard code is unique in that each non-zero codeword has Hamming weight of exactly \( 2^n - 1 \). The Walsh-Hadamard code is a locally decodable code. That’s why only looking at a small fraction of the received word, it provides a way to recover the original message with high probability. This gives rise to applications of Walsh codes in complexity theory. Using list decoding, the original message can be recovered as long as less than 1/2 of the bits in the received word have been corrupted.

Walsh-Hadamard codes are mathematically mathematically orthogonal codes. If the two Walsh-Hadamard codes are correlated, the result of correlation is intelligible only if these two codes are the same. Hence, a Walsh-encoded signal appears as random noise to a CDMA mobile user, unless that terminal uses the same code as the one used to encode the incoming signal [12]. The Walsh-Hadamard code is also used to uniquely define the individual communication channels.

B. Gold codes

Gold codes have bounded small cross-correlations within a set, when multiple devices are broadcasting in the same frequency range then it is useful. A set of Gold code sequences consists of \( 2n - 1 \) sequences each one with a period of \( 2n - 1 \).

\[
c_n = \sum_{i=1}^{r} C_{a_i} t_{m_{n-i}} + \sum_{i=1}^{r} C_{b_i} b_{m_{n-i}}.
\]

A set of Gold codes can be generated with the following different steps. Firstly, pick two maximum length sequences of the same length \( 2^n - 1 \) such that their absolute cross-correlation is less than or equal to \( 2^{n/2} \), where \( n \) is the size of the LFSR used to generate the maximum length sequence. The set of the \( 2^n - 1 \) exclusive-ores of the two sequences in their various phases (i.e. translated into all relative positions) is a set of Gold codes which is used as spreading codes. The highest absolute cross-correlation in this set of codes is \( 2^{(n+1)/2} + 1 \) for even \( n \) and \( 2^{(n+1)/2} + 1 \) for odd \( n \).

The exclusive or of two different Gold codes from the same set is another Gold code in some phase. Gold codes have bounded small correlations. Within a set of Gold codes about half of the codes are balanced – the number of ones and zeros differs by only one.[13] Gold code, also known as Gold sequence used in telecommunication (CDMA) and satellite navigation (GPS). Gold codes are named after Robert Gold.[14]

C. Kasami codes

Kasami sequences are binary sequences of length \( 2^N - 1 \) where \( N \) is an even integer. The Kasami sequences have good cross-correlation values. There are two classes of Kasami sequences - the small set and the large set.

The process of generating a Kasami sequence is initiated by generating a maximum length sequence \( a(n) \), where \( n = 1, 2, ..., 2^N - 1 \). Maximum length sequences are periodic sequences with a period of exactly \( 2^N - 1 \). Next, a secondary sequence is derived from the initial sequence via cyclic decimation sampling as \( b(n) = a(q^n) \), where \( q = 2^{N/2} + 1 \). Modified sequences are then formed by adding a \( n \) and cyclically time shifted versions of \( b(n) \) using modulo-two arithmetic, which is also termed the exclusive or (xor) operation. Computing modified sequences from all \( 2^{N/2} \) unique time shifts of \( b(n) \) forms the Kasami set of code sequences. [15]

D. M sequence

A pseudo-noise (PN) sequence is a sequence of 1’s and 0’s. PN sequences are periodic sequences that have noise-like behavior. They can be conveniently generated using shift registers, modulo-2 adders (i.e. XOR gates) and feedback logic circuitry. [16] The maximum length of a PN sequence is determined by the length of the register and the configuration of the feedback network.

An N-bit register can take up to \( 2^N \) different combination of zeros and ones. The maximum length of any PN sequence is \( 2^N - 1 \) and sequences of that length are called maximum-length sequences or m-sequences.

E. Barker codes

The Barker code is a sequence of \( n \) values (code-symbols) of +1 and −1. Barker code has good auto-correlation properties and with some pairs, the low cross-correlation so that they can be used in multi-user environment [17]. The Barker code gives code with different lengths and similar autocorrelation properties as the m-sequence. Mostly, Barker codes are used maximum length of 13 and have low correlation side lobes. The correlation side lobe of a codeword is defined as the correlation with a time-shifted
version of itself. So for a k-symbol shift of an N-bit code sequence, \( [X_j] \), the correlation side lobe, \( C_k \), is given by

\[
\sum_{j=1}^{N-k} C_k = \sum X[j] + k
\]

<table>
<thead>
<tr>
<th>Length</th>
<th>Barker codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>+1-1</td>
</tr>
<tr>
<td>3</td>
<td>+1+1-1</td>
</tr>
<tr>
<td>4</td>
<td>+1-1+1+1</td>
</tr>
<tr>
<td>5</td>
<td>+1+1+1+1+1</td>
</tr>
<tr>
<td>7</td>
<td>+1+1+1+1+1+1</td>
</tr>
<tr>
<td>11</td>
<td>+1+1+1+1+1+1+1+1</td>
</tr>
<tr>
<td>13</td>
<td>+1+1+1+1+1+1+1+1+1+1+1+1</td>
</tr>
</tbody>
</table>

Table 1: 13 bit barker codes

Where \( X_j \) is an individual code symbol taking values +1 or -1 for \( j = 1, 2, 3, N \), and the adjacent symbols are assumed to be zero.

PN sequence may also be a periodic. Such sequences are known as Barker sequences. Barker sequences are too short to be of practical use for spectrum spreading. Barker codes, which are subsets of PN sequences, are mostly used for frame synchronization in digital communication systems.

**F. GIG codes**

GIG codes are the Binary Orthogonal Codes which is consist of using Gray and Inverse Gray codes. An n-bit Cyclic Gray Code consists of a circular list of \( 2^n \) bit strings such that successive code words differ in only one bit position. An "n" bit Inverse Gray Code , is defined exactly opposite to Gray code, which is a circular list of all 2n bit Strings of length ‘n’ each, such that successive code words differ in (n-1) bit positions. An algorithm is used for generation of GIG codes. Firstly, Generate n-bit Gray code using the algorithm [18] with any permutation. Then using the same permutation generate n-bit Inverse Gray Code using the algorithm [19]

In this algorithm, as a generalization, a digt is referred as a git (generalized unit). Let an n-git radix ‘r’ Gray code be needed. Let (P1, P2………Pn) be a permutation of (1, 2, 3, ……n). the \( r^i \) integers (0, 1, 2, (m-1)) can be arranged in the following doubly indexed indicial sets.

\[
Q_{j,k} = \{k, k+r, ………k+rm\}
\]

j = 0, 1, 2, …… n

k = 1, 2, …., r-1

Where ‘m’ is the largest positive integer such that \( m \leq r^{i+1}-1/kr \).

Over the integers (0, 1, (r-1), 0) a new succession order (0, s1, s2, sr-1, 0) is defined where s1, s2, sr-1 is a permutation of (1, 2, 3, (r-1)). Then, starting with the row of all zeros as a zeroeth row, the i\(^{th} \) row is obtained from the (i-1)\(^{th} \) row by replacing the \( P_i \)th git by its successor bit, if it is in \( Q_{j,k} \).

After that, Append Inverse Gray Code to the Gray Code to result in 2n-length gig code (Gray Inverse Gray). Each row of this GIG Code is a Binary Orthogonal Code word of length 2n. codes can be of different level according to our requirement. GIG codes can be generated with any permutation using the algorithm which is given in [1].

| 3 0 3 1 0 3 2 |
| 3 0 0 1 1 0 0 3 |
| 3 0 0 2 2 1 1 3 |
| 3 0 0 3 3 2 2 3 |

Table 2: 4 level 8 length GIG codes

Auto correlation of this GIG codes is good while cross correlation[20] performance is somewhat similar to the Walsh codes.

**IV. EVALUATION OF DIFFERENT SPREADING CODES**

A. **Mean Square Correlation**

In DS-CDMA, performance measures for correlation properties are the Mean Square Aperiodic Auto-Correlation (MSAAC) and Mean Square Aperiodic Cross-Correlation (MSACC). These measures are widely accepted of sequences applied. Let \( c_i \) presents a sequence of length \( N \), \( c_i(n) \) denotes nondelayed version of \( c_i(t) \), and \( c_i(n+\tau) \) denotes the delayed version of \( c_i(t) \) by ‘\( \tau \)’ units. The discrete aperiodic correlation function is given by:

\[
r_{ij}(\tau) = \frac{1}{N} \sum_{r=1-N}^{r=1-N} c_i(n)c_j(n+\tau)
\]

The \( R_{AC} \) mean square aperiodic auto-correlation (MSAAC) value \( R_{AC} \) for a given code set containing M sequences is defined as:

\[
R_{AC} = \frac{1}{M} \sum_{i=1}^{M} \sum_{j=1-M,j\neq0}^{j=1-M} |r_{ij}(\tau)|^2
\]

And an analogous measure for the \( R_{CC} \) mean square aperiodic cross-correlation (MSACC) value is given by:

\[
R_{CC} = \frac{1}{M(M-1)} \sum_{i=1}^{M} \sum_{j=1-M}^{M} \sum_{j=1-M, j\neq0}^{j=1-M} \sum_{r=1-N}^{r=1-N} |r_{ij}(\tau)|^2
\]

Auto-correlation relates to the level of correspondence between a sequence and its phase-shifted replica version, whereas cross-correlation is the measure of agreement between two different codes. The sequences which have less MSACC values removes the correlation between the bits with in a sample, and those which have less MSACC values removes the sample to sample correlation, and make the information signal less intelligible [22].

B. **Figure of Merit**

Figure of merit(FOM) is defined over autocorrelation function spreading code. Ideally the autocorrelation of every spreading code should be like Impulse-response (Requirement for Frequency selective channel) . Hence to have a good SET of spreading codes, average FOM is one such metric i.e.

\[
MF = \frac{1}{M} * \sum_{i=1}^{M} (FOM_i)
\]

Ideal Cross-correlation is to have Impulse response it minimizes interference (termed as inter-code-interference) or interference from other user’s data. In studying the properties of aperiodic auto-correlation of a family of codes, the price for being able to select good cross-correlation
properties signifies degradation in the autocorrelation properties of the set of sequences. Note also that a degradation of the auto-correlation properties has a direct relation on the frequency spectrum of the sequences in the set, which is means if the RAC values are less, the sequence spectrum will not be wide-band and flat. so in addition to correlation measures (RAC), and to determine quantitatively how significant this degradation for a given set of sequences, there is another criterion called Merit Factor (MF) which provides a measure of all the side lobes of the RAC compared to the main peak. Sequences with low MF has narrow flat spectrum and they are neither suitable for CDMA. The Merit Factor for a sequence, $c_i(n)$ which is of length $N$ having the auto-correlation function $r_i(\tau)$ is defined as follows:

$$F_x = \frac{r_{ij}^2(0)}{\sum_{\tau=0}^{N-1} |r_{ij}(\tau)|^2} = \frac{N^2}{2 \sum_{\tau=1}^{N-1} |r_{ij}(\tau)|^2}$$

V. RESULTS AND DISCUSSION

In order to meet the continually rising demand of the users in a CDMA system, spreading code designers are always in search of code families that contain significantly more members. But as we make any attempt to increase the number of distinct spreading codes, the multi-user interference between the codes affects the performance of the system badly. While selecting spreading code from a number of code families, our objective will be to choose that particular code set which is capable to serve a large number of users efficiently in multi-user environment. As well as, we have to ensure that the overall performance of the system must not deteriorate extensively with the increase in the number of users. 

CDMA technology [22] is a fast growing technology in the field of Wireless Communication system. The ultimate aim of service providers is to accommodate the maximum number of users in the available bandwidth as allotted by ITU-T. The desirable characteristics of CDMA codes for next generation wireless CDMA systems include (i) availability of large number of codes (ii) impulsive auto-correlation function (iii) zero cross-correlation value (iv) low Peak to Average Power Ratio (PAPR) value and (v) support for variable data rates. Here the comparison is done is of 8 length Walsh and GIG codes. The comparison is done in terms of Mean Square Aperiodic Auto-Correlation (MSAAC), Mean Square Aperiodic Cross-Correlation (MSACC) measures and the Merit Factor (MF) are implemented in MATLAB. The auto correlation and merit factor of GIG codes is better than Walsh codes. The plot of input sequence of GIG codes is shown in figure 4. The auto-correlation and cross-correlation of GIG codes is shown in figure 5 and figure 6 respectively.

<table>
<thead>
<tr>
<th>Sequence</th>
<th>MSAAC</th>
<th>MSACC</th>
<th>MF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walsh</td>
<td>0.0852</td>
<td>2.8949</td>
<td>1.4297</td>
</tr>
<tr>
<td>GIG codes</td>
<td>29.1250</td>
<td>24.8779</td>
<td>1.4416</td>
</tr>
</tbody>
</table>

Table 3: Aperiodic Correlation measures and Merit factor for 8 length Walsh and GIG codes
as an efficient spreading code in synchronous downlink CDMA system. Walsh code is applied for increasing number of active users with the assumption that all users have equal power. GIG codes are orthogonal codes. Auto correlation of this GIG codes is good while cross correlation performance is somewhat similar to the Walsh codes

VI. CONCLUSION

In the paper we review the different spreading codes for DS CDMA. The correlation properties of Gold codes, Walsh-Hadamard, Kasami, Barker, M-sequence, and GIG codes have been evaluated and compared for application to CDMA based wireless systems. Walsh-Hadamard codes when synchronized at zero time-shift have practically zero MSACC. These codes also possess poor MSAACF characteristics. Therefore, for practical implementation using Walsh codes, the wireless system and user channels must be synchronized e.g. as in downlink.

In this paper, a comprehensive study is carried out to analyze different spreading codes for a DS-CDMA system, and new orthogonal code sets has been presented. It is observed that the new proposed GIG codes have better correlation properties and have improved merit figure. It can be concluded that large Kasami sequence has both good correlation values and high Merit Factor, which make these sequences to have wide flat spectrum that is better suited to be used in the WCDMA uplink transmission. In the downlink of WCDMA, variable data rate is supported by using orthogonal variable spreading factor (OVSF) codes. It has been seen that the length of the code, cross-correlation and auto-correlation properties can help us to determine the best suitable code for any particular communication system. We have tried to find out the code with suitable auto-correlation properties along with low cross-correlation values. GIG codes are orthogonal codes used in DS-CDMA. Auto correlation performance of this GIG codes is good while cross correlation performance is somewhat similar to the Walsh codes.

REFERENCES