Study and Analysis of Duckworth Lewis Method

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Abstract—Designed for one-day cricket, this paper considers the use of the Duckworth-Lewis method as an approach to resetting targets in interrupted cricket matches. The Duckworth-Lewis table is reviewed and an alternative resource table is presented. A desideratum of a resource table is monotonicity in both the rows and columns corresponding to wickets and overs respectively. Consequently, a Gibbs sampling scheme related to isotonic regression is applied to the observed scoring rates to provide a nonparametric resource table. A discussion is provided concerning the use of the Duckworth-Lewis method applied to International Cricket matches. It is designed so that neither team benefits or suffers from the shortening of the game and so is totally fair to both. It is easy to apply, requiring nothing more than a single table of numbers and a pocket calculator, and is capable of dealing with any number of interruptions at any stage of either or both innings. The method is based on a simple model involving a two-factor relationship giving the number of runs which can be scored on average in the remainder of an innings as a function of the number of overs is remaining and the number of wickets fallen. It is shown how the relationship enables the target score in an interrupted match to be recalculated to reflect the relative run scoring resources available to the two teams, that is overs and wickets in combination.

Key words*:* Duckworth-Lewis method, isotonic regression, Gibbs sampling scheme, Two-factor relationship, modelling.

I. Introduction

The use of mathematical modelling in sport in general and cricket in particular has been growing in recent years and many techniques have been used in scheduling cricket fixtures. [1-3]. The Duckworth–Lewis method (D/L method) is a mathematical formulation designed to calculate the target score for the team batting second in a limited overs match interrupted by weather or other circumstances. It is generally accepted to be the most accurate method of setting a target score. The D/L method was devised by two English statisticians, Frank Duckworth and Tony Lewis. [4]. The method was adopted by the International Cricket Council (ICC) in 1999 to address the problem of delayed one–day cricket matches for reasons of rain, poor light and floodlight failures although it has also been used in events that have been shortened due to crowd problem, sandstorms and even snowstorms. It is a mathematical formulation designed to calculate the target score for the team batting second in a limited overs match interrupted by weather or other circumstances. It was first used in international cricket in the second game of the 1996–97 Zimbabwe versus England One Day International series, which Zimbabwe won by seven runs [5] and was formally adopted by the International Cricket Council in 1999 as the standard method of calculating target scores in rain shortened one-day matches.

Development of the method involved all three disciplines of maths, stats and OR, but the main discipline illustrated was that of communication - how to communicate the necessity of the mathematical approach to what was essentially a non-mathematical audience. Thus the D/L method can be regarded as a case study in how a mathematical approach was imposed upon a non-mathematical audience.

It is generally accepted to be the most accurate method of setting a target score. The basic principle is that each team in a limited-overs match has two resources available with which to score runs: wickets remaining, and overs to play. These are not in direct proportion to the number of overs available to be faced, as with the average run rate method of correction. Instead they depend on how many overs are to go and how many wickets are down when the interruptions occur. To calculate the revised targets, you need to know the resources available at the stage of the match when suspensions and resumption of play occur. All possible values of resources have been pre-calculated and these are listed in the accompanying table.

One-day cricket has a major problem. It is intolerant of interruptions due to the weather. In first class cricket a stoppage because of rain or bad light is a natural, though generally unwelcome, part of the game. A one-day match, however, is intended to be finished in a single day and there is usually insufficient spare time when playing conditions are acceptable to make up for the loss of more than a very few overs. Some competitions schedule extra days to cover the eventuality of the game not being able to be completed on the day planned. But in many cases this is not practicable. As a 'draw' is contrary to the whole purpose of limited-over cricket, and knock-out competitions demand a positive result anyway, rules have had to be introduced to cope with the possibility of the match having to be shortened. If there is a delay to the start, then the number of overs per team is simply reduced equally and equitably for both teams. But if there is an interruption after play has commenced, there are problems.

II. Review of Other Methods

The following are methods that have been used so far in one-day cricket together with a brief description. Most of these do not take account of the stage of the innings at which the overs are lost or of the number of wickets that have fallen.

A. Average run rate (ARR)

The winning team is decided by the higher average number of runs per over that each team has had the opportunity to receive. It is a simple calculation but the method's major problem is that it very frequently alters the balance of the match, usually in favor of the team batting second.

B. Most productive overs (MPO)

The target is determined for the overs the team batting second (Team 2) are to receive by totaling the same number of the highest scoring overs of Team 1. The process of determining the target involves substantial bookwork for match officials and the scoring pattern for Team 1 is a criterion in deciding the winner. We believe that it is only Team l's total that should be used in setting the target and not the way by which it was obtained. The method strongly tends to favor Team 1.

C. Discounted most productive overs (DMPO)

The total from the most productive overs is discounted by 0.5% for each over lost. This reduces slightly the advantage MPO gives to Team 1 but it still has the same intrinsic weaknesses of that method.

D. Parabola (PARAB)

This method, by a young South African (do Rego8), calculates a table of 'norms' y, for overs of an innings, x, using the parabola y = 7.46x - O.059x2 to model, rather inappropriately since it has a turning point (at about 63 overs, the 'diminishing returns' nature of the relation- ship between average total runs scored and total number of overs available. The method is an improvement upon ARR but takes no account of the stage of the innings at which the overs are lost or of the number of wickets that have fallen.

E. World Cup 1996 (WC96)

This is an adaptation of the PARAB method. Each of the norms has been converted into a percentage, of 225 as an approximation for the 50 over norm and generally regarded as the mean of first innings scores in one-day international matches.

F. Clark Curves (CLARK)

This method, fully described on the Internet, 9 attempts to correct for the limitations of the PARAB method. It defines six types of stoppage, three for each innings, for stoppages occurring before the innings commences, during the innings, or to terminate the innings. It applies different rules for each type of stoppage some of which, but not all, allow for wickets which have fallen. There are discontinuities between the revised target scores at the meeting points of two adjacent types of stoppage [9] [11].

- All of these methods have flaws that are easily exploitable: The average run-rate method takes no account of how many wickets the team batting second have lost, but simply reflect how quickly they were scoring when the match was interrupted, so if a team felt a rain stoppage was likely they could attempt to force the scoring rate without regard for the corresponding highly likely loss of wickets, skewing the comparison with the first team.
- The most productive overs method also takes no account of how many wickets the team batting second have lost, and also has the further effect of penalizing the team batting second for good bowling, as their best overs are ignored in setting the revised target.

III. THE DUCKWORTH/LEWIS METHOD (D/L)

Objective was to find method that must following criteria given below.

1) It must be equally fair to both sides; that is the relative positions of the two teams should be exactly the same after the interruption as they were before it.
2) It must give sensible results in all conceivable situations.
3) It should be independent of Team l's scoring pattern, as indeed is the target in an uninterrupted game.
4) It should be easy to apply, requiring no more than a table of numbers and a pocket calculator.
5) It should be easy to understand by all involved in the game, players, officials, spectators and reporters.

The essence of the D/L method is 'resources'. Each team is taken to have two 'resources' to use to make as many runs as possible: the number of overs they have to receive; and the number of wickets they have in hand. At any point in any innings, a team's ability to score more runs depends on the combination of these two resources. Looking at historical scores, there is a very close correspondence between the availability of these resources and a team's final score, a correspondence which D/L exploits [7].

Using a published table which gives the percentage of these combined resources remaining for any number of overs (or, more accurately, balls) left and wickets lost, the target score can be adjusted up or down to reflect the loss of resources to one or both teams when a match is shortened one or more times. This percentage is then used to calculate a target (sometimes called a 'par score') that is usually a fractional number of runs. If the second team passes the target, then the second team is taken to have won the match; if the match ends when the second team has exactly met (but not passed) the target (rounded down to the next integer) then the match is taken to be a tie.

Commercial confidentiality prevents the disclosure of the mathematical definitions of these functions. Duckworth and Lewis (1998) have provided only partial information concerning the construction of the resource table. However, they do disclose that the table entries are based on the estimation of the 20 parameters [10].

The average total score Z (u) which is obtained in u overs may be described by the exponential equation

$$Z(u) = Z_0 [1 - \exp(-bu)] \qquad (1)$$

Where Zo is the asymptotic average total score in unlimited overs (but under one-day rules) and b is the exponential decay constant.

The next stage of development of a suitable two-factor relationship is to revise (1) for when w wickets have already been lost but u overs are still left to be received. The asymptote will be lower and the decay constant will be higher and both will be functions of w. The revised relationship is of the form

$$Z(u, w) = Z_0(w) [1 - \exp\{-b(w)u\}] \qquad (2)$$

where Zo(w) is the asymptotic average total score from the last 10-w wickets in unlimited overs and b(w) is the exponential decay constant, both of which depend on the number of wickets already lost hence w= 0..9.

They have been obtained following extensive research and experimentation so that Z (u, w) and its first partial derivative with respect to u behave as expected under various practical situations and give sensible results at the boundaries. Figure 1 shows the family of curves described by (2) using parameters estimated from hundreds of one-day internationals [12].

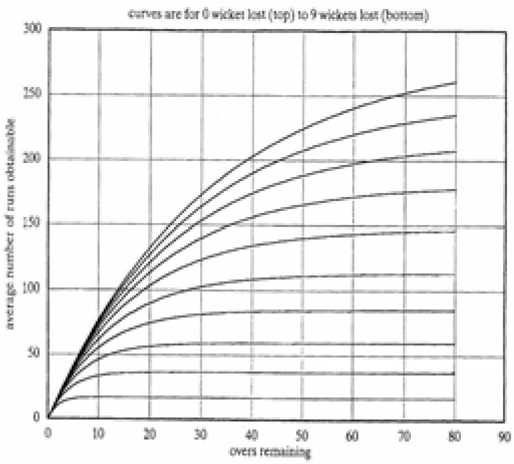

Fig. 1: Average number of runs over remaining with wickets lost.

OVERS LEFT	0	1	2	3	4	5	6	7	8	9	OVERS LEFT
	WICKETS LOST										
50	100.0	93.4	85.1	74.9	62.7	49.0	34.9	22.0	11.9	4.7	50
49	99.1	92.6	84.5	74.4	62.5	48.9	34.9	22.0	11.9	4.7	49
48	98.1	91.7	83.8	74.0	62.2	48.8	34.9	22.0	11.9	4.7	48
47	97.1	90.9	83.2	73.5	61.9	48.6	34.9	22.0	11.9	4.7	47
46	96.1	90.0	82.5	73.0	61.6	48.5	34.8	22.0	11.9	4.7	46
45	95.0	89.1	81.8	72.5	61.3	48.4	34.8	22.0	11.9	4.7	45
44	93.9	88.2	81.0	72.0	61.0	48.3	34.8	22.0	11.9	4.7	44
43	92.8	87.3	80.3	71.4	60.7	48.1	34.7	22.0	11.9	4.7	43
42	91.7	86.3	79.5	70.9	60.3	47.9	34.7	22.0	11.9	4.7	42
41	90.5	85.3	78.7	70.3	59.9	47.8	34.6	22.0	11.9	4.7	41
40	89.3	84.2	77.8	69.6	59.5	47.6	34.6	22.0	11.9	4.7	40
39	88.0	83.1	76.9	69.0	59.1	47.4	34.5	22.0	11.9	4.7	39
38	86.7	82.0	76.0	68.3	58.7	47.1	34.5	21.9	11.9	4.7	38
37	85.4	80.9	75.0	67.6	58.2	46.9	34.4	21.9	11.9	4.7	37
36	84.1	79.7	74.1	66.8	57.7	46.6	34.3	21.9	11.9	4.7	36
35	82.7	78.5	73.0	66.0	57.2	46.4	34.2	21.9	11.9	4.7	35
34	81.3	77.2	72.0	65.2	56.6	46.1	34.1	21.9	11.9	4.7	34
33	79.8	75.9	70.9	64.4	56.0	45.8	34.0	21.9	11.9	4.7	33
32	78.3	74.6	69.7	63.5	55.4	45.4	33.9	21.9	11.9	4.7	32
31	76.7	73.2	68.6	62.5	54.8	45.1	33.7	21.9	11.9	4.7	31
30	75.1	71.8	67.3	61.6	54.1	44.7	33.6	21.8	11.9	4.7	30
29	73.5	70.3	66.1	60.5	53.4	44.2	33.4	21.8	11.9	4.7	29
28	71.8	68.8	64.8	59.5	52.6	43.8	33.2	21.8	11.9	4.7	28
27	70.1	67.2	63.4	58.4	51.8	43.3	33.0	21.7	11.9	4.7	27
26	68.3	65.6	62.0	57.2	50.9	42.8	32.8	21.7	11.9	4.7	26
25	66.5	63.9	60.5	56.0	50.0	42.2	32.6	21.6	11.9	4.7	25
24	64.6	62.2	59.0	54.7	49.0	41.6	32.3	21.6	11.9	4.7	24
23	62.7	60.4	57.4	53.4	48.0	40.9	32.0	21.5	11.9	4.7	23
22	60.7	58.6	55.8	52.0	47.0	40.2	31.6	21.4	11.9	4.7	22
21	58.7	56.7	54.1	50.6	45.8	39.4	31.2	21.3	11.9	4.7	21
20	56.6	54.8	52.4	49.1	44.6	38.6	30.8	21.2	11.9	4.7	20
19	54.4	52.8	50.5	47.5	43.4	37.7	30.3	21.1	11.9	4.7	19
18	52.2	50.7	48.6	45.9	42.0	36.8	29.8	20.9	11.9	4.7	18
17	49.9	48.5	46.7	44.1	40.6	35.8	29.2	20.7	11.9	4.7	17
16	47.6	46.3	44.7	42.3	39.1	34.7	28.5	20.5	11.8	4.7	16
15	45.2	44.1	42.6	40.5	37.6	33.5	27.8	20.2	11.8	4.7	15
14	42.7	41.7	40.4	38.5	35.9	32.2	27.0	19.9	11.8	4.7	14
13	40.2	39.3	38.1	36.5	34.2	30.8	26.1	19.5	11.7	4.7	13
12	37.6	36.8	35.8	34.3	32.3	29.4	25.1	19.0	11.6	4.7	12
11	34.9	34.2	33.4	32.1	30.4	27.8	24.0	18.5	11.5	4.7	11
10	32.1	31.6	30.8	29.8	28.3	26.1	22.8	17.9	11.4	4.7	10
9	29.3	28.9	28.2	27.4	26.1	24.2	21.4	17.1	11.2	4.7	9
8	26.4	26.0	25.5	24.8	23.8	22.3	19.9	16.2	10.9	4.7	8
7	23.4	23.1	22.7	22.2	21.4	20.1	18.2	15.2	10.5	4.7	7
6	20.3	20.1	19.8	19.4	18.8	17.8	16.4	13.9	10.1	4.6	6
5	17.2	17.0	16.8	16.5	16.1	15.4	14.3	12.5	9.4	4.6	5
4	13.9	13.8	13.7	13.5	13.2	12.7	12.0	10.7	8.4	4.5	4
3	10.6	10.5	10.4	10.3	10.2	9.9	9.5	8.7	7.2	4.2	3
2	7.2	7.1	7.1	7.0	7.0	6.8	6.6	6.2	5.5	3.7	2
1	3.6	3.6	3.6	3.6	3.6	3.5	3.5	3.4	3.2	2.5	1
0	0	0	0	0	0	0	0	0	0	0	0

Table. 1: DL Resource Chart

IV. APPLICATIONS AND EXAMPLES

A. *Application*

The Duckworth–Lewis method is fairly simple to apply, requiring a published reference table and some simple mathematical calculations. As with most non-trivial statistical derivations, the D/L method can produce results that are somewhat counter intuitive, and the announcement of the derived target score can provoke a good deal of second-guessing and discussion amongst the crowd at the cricket ground. This can also be seen as one of the method's successes, adding interest to a "slow" rain-affected day of play.

For 50-over matches, each team must face at least 20 overs before D/L can decide the game, unless one or both sides have been bowled out in less than 20 overs and/or the team batting second has reached its target in less than 20 overs. For Twenty20 games, each side must face at least five overs before D/L can decide the game, unless one or both sides have been bowled out in less than five overs and/or the team batting second has reached its target in less than five overs. If these prerequisites are not met, the match is declared a no result.

B. *Examples*

1) *Definitions*

- The team batting first are referred to as 'Team 1' and the team batting second are referred to as 'Team 2'.
- In the table decimal fractions of an over are expressed in standard cricket notation; i.e. 5.4 overs means 5 overs plus 4 balls.
- The terms 'target' and 'revised target' are reserved exclusively for the minimum score Team 2 needs to win.
- As with an uninterrupted match, if Team 2 make a score which is one run short of the target, the match is tied.

The following symbols are used throughout:
N is the number of overs per innings for the match as decided at the moment of delivery of the first ball of the match.
S is Team 1's total score.
R_1 is the resource percentage (relative to a full 50-over innings) available to Team 1.
R_2 is the resource percentage (relative to a full 50-over innings) available to Team 2.
T is Team 2's target score.
For the matches between associate ICC member nations the value of G50 should be 245.
The method of calculating the revised target score T following interruptions to either innings is thus as follows [6] [8]:
For $R_2 < R_1$, T = S (R_2/R1) +1 (rounded down to a whole number, if necessary)
For R_2=R_1, T = S +1
For $R_2 > R_1$, T = S+ G50 (R_2-R_1) +1 (rounded down to a whole number, if necessary)

2) *Example 1 (Suspension during Team 1's innings)*

In a 50 over-per-innings match, Team 1 reaches 79/3 after 20 overs and then there is a suspension in play. It is decided that 20 overs of the match should be lost, 10 of these by each team. Team 1 resumes to reach a final total of 180 in its revised allocation of 40 overs.

Number of overs per innings at the start of match, N = 50

Resource percentage available to Team 1 at start of innings = 100% (5.1)

Resource percentage remaining at suspension (30 overs left, 3 wickets lost = 61.6% (3.1)

Resource percentage remaining at resumption (20 overs left, 3 wickets lost) = 49.1% (3.2)

Resource percentage lost due to suspension = 61.6 – 49.1 = 12.5% (3.3)

Resource percentage available to Team 1, R1 = 100 – 12.5 = 87.5% (5.2)

Number of overs available to Team 2 at the start of its innings = 40

Resource percentage available (40 overs left, 0 wkt lost), R2 = 89.3% (5.4)

R2 is greater than R1, i.e. Team 2 has more resource available than had Team 1, so its target should be increased. S = 180

Team 2's revised target (5.6) is

T = S + G50 x (R2 – R1)/100 + 1 = 180 + 245 x (89.3 – 87.5)/100 + 1 = 185 (rounded down).

3) Example 2 (suspension during Team 2's innings)

In One Day International match (50 overs per innings); Team 1 has scored 250 from its allocation of 50 overs in an uninterrupted innings. Team 2 has received 12 overs and has scored 40/1. Then play is suspended and 10 overs are lost. Number of overs at start of match, N = 50.

Team 1's innings was uninterrupted, so its resource percentage available, R1 = 100% (5.1).

Resource percentage available to Team 2 at start of innings = 100% (5.4).

Resource percentage remaining at suspension (38 overs left, 1 wicket lost) = 82.0% (3.1).

Resource percentage remaining at resumption (28 overs left, 1 wicket lost) = 68.8% (3.2).

Resource percentage lost due to suspension = 82.0 – 68.8 = 13.2% (3.3).

Resource percentage available to Team 2, R2 = 100 – 13.2 = 86.8% (5.5).

R2 is less than R1; S = 250.

Team 2's revised target (5.6) is

T = S x R2/R1 + 1 = 250 x 86.8/100 + 1 = 218, and it needs a further 178 runs from 28 overs.

The most notorious example came in the 1992 Cricket World Cup, where the most productive overs method was used: in the semi-final between England and South Africa, rain stopped play for 12 minutes with South Africa needing 22 runs from 13 balls chasing England's 6/252 off 45 overs. The revised target left South Africa needing 21 runs from one ball, which was a reduction of only one run compared to a reduction of two overs and a preposterous target given that the maximum score from one ball is generally six runs. [13] The D/L method avoids this flaw: in this match, the revised D/L target would have left South Africa four to tie or five to win from the final ball. [14]

The D/L method has been criticized on the grounds that wickets are a much more heavily weighted resource than overs, leading to the suggestion that if teams are chasing big targets, and there is the prospect of rain, a winning strategy could be to not lose wickets and score at what would seem to be a "losing" rate[15].Another criticism is that the D/L method does not account for changes in proportion of the innings for which field restrictions are in place compared to a completed match.[16] More common informal criticism from cricket fans and journalists of the D/L method is that it is overly complex and can be misunderstood.[17]

V. Conclusion

In this paper we have explained the mechanisms of other methods used for resetting target scores in interrupted one-day cricket matches. Each of these methods yields a fair target in some situations. None has proved satisfactory in deriving a fair target under all circumstances. We have explained a DL method which gives a fair revised target score under all circumstances. This is based on the recognition that teams have two resources, overs to be faced and wickets in hand, to enable them to make as many runs as they can or need. We have explained a two-factor relationship which gives the average number of runs which may be scored from any combination of these two resources and hence have derived a table of proportions of an innings for any such combination. This enables the proportion of the resources of the innings of which the batting team are deprived when overs are lost as a result of a stoppage in the play to be calculated simply and hence a fair correction to the target score to be made. Although this method is little bit complex for fans who are not person of mathematical field. But overall this method is quite satisfactory by ignoring some odd cases.

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