Analysis of ZFE and MMSE Schemes for MIMO Communication Systems

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Abstract---MIMO systems are very important technological advancements with respect to the third generation and fourth generation's wireless networks for RF communication. There are multiple equalization schemes for MIMO systems and two very important of them are ZFE and MMSE. In this paper we are analyzing the performance of both the schemes in different conditions.

Keywords: MIMO, SISO, SIMO, MISO, MMSE, ZFE.

I. INTRODUCTION

The concept of MIMO systems comes after the concept of diversity schemes at transmitting scheme as the one given by alamouti in his paper in 1998 and receiving diversity schemes which are very popular because of their good performance of SNR and BER. The traditional digital communication systems involve BPSK modulation or any other modulation scheme and single transmitting and Single Receive antenna systems. The basic block diagram of the wireless communication system has been shown below in the figure 1. The problem in MIMO system arises for the equalization schemes. There are two main equalization schemes Zero forcing error and minimum mean square error equalization schemes [1][2]. This paper aims to analyze both the schemes for better SNR performance under different conditions of successive interference cancellation with and without optimal ordering. Maximal Ratio Combining is a scheme which the strength of the received signals is uses to obtains the corresponding weights and then maximizes the SNR. This scheme is very predominant scheme for the single input multiple out communication systems with CDMA as one very good example. Rake receivers used in CDMA works on this principle only [3] [4] [5].

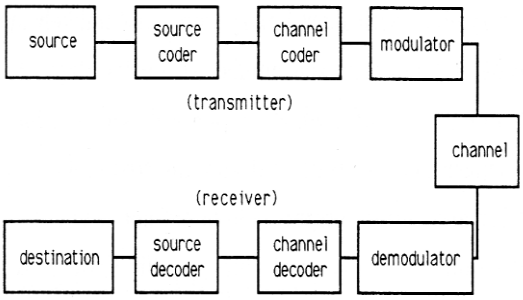

Fig. 1: Wireless Communication System

II. MIMO SYSTEMS

MIMO systems are the multiple input and multiple output systems having multiple transmitting antennas and multi receive antennas at source and destination ends. To achieve this, the total transmitted power is spreaded over to achieve the array gain, and hence throughput of the channel increases linearly which in turn increases the spectral efficiency and link reliability [6] [7].

A. Working of MIMO systems

Working of MIMO communication systems can be defined in three processes [8] [9]:

1) Pre-coding
2) Spatial multiplexing
3) Diversity coding.

1) Precoding

Pre coding is a process of multi-stream beamforming. In more common terms it was supposed to be all spatial processing which occurs at the transmitter end.

2) Spatial multiplexing

It needs MIMO antenna configuration. In this technique a high rate signals is split into several lower rate signals and each signal is transmitted by a different transmit antenna in a common frequency channel.

3) Diversity Coding

MIMO systems uses diversity coding when there is no knowledge about the channel is present at the transmitting end. In diversity methods, a single stream is transmitted, but the signal is coded using techniques called SPACE-TIME CODING.

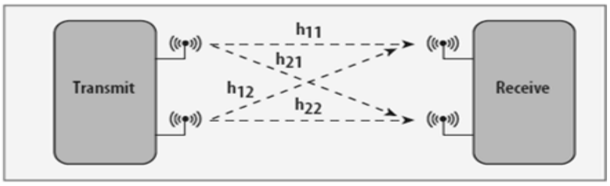

Fig. 2: MIMO Wireless Communication System

B. Equalization schemes for MIMO systems

In MIMO systems, equalization is done at the receiving side or at the destination end. They are Zero Forcing Equalizer and Minimum Mean Square Error. The schemes and the cases are described in further sections [10].

III. ZERO FORCING (ZF) EQUALIZER

Zero Force equalization is a linear equalization process in communication system which works with the principle of frequency inversion at the frequency response of the channel. This technique restores the transmitted signal by applying the channel inverse at the received signal and brings down the inter symbol interference or the ISI. It is a very good scheme to deploy in high ISI conditions w.r.t to channel noise. [11][12].

Let us say two signals are received on antenna 1 and antenna 2, y1 and y2 respectively, h(1,1), h(1,2), h(2,1) and h(2,2) are the channel parameters showing the relationship between transmitting antenna and receiving antenna as shown by the figure 5, x1 and x2 are the transmitted signals from antenna 1 and antenna 2 respectively and n1 and n2 are the noise on receiving antenna 1 and antenna 2respectively such that[12]:

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1} \quad h_{1,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \qquad (1)$$

And

$$y_1 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} \quad h_{2,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \qquad (2)$$

The above quations can be expressed in matrix form as

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \qquad (3)$$

i.e. Y= Hx + n

Now x can be solved by with the help of the matrix Z such that ZH=1, i.e. Z should be the inverse of the channel matrix H. the matrix Z can be expressed mathematically as

$$Z = (H^H H)^{-1} H^H \qquad (4)$$

The term,

$$H^H H = \begin{bmatrix} h_{1,1}{}^* & h_{1,2}{}^* \\ h_{2,1}{}^* & h_{2,2}{}^* \end{bmatrix} \qquad (5)$$

$$= \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2} \\ h_{1,2}^* h_{1,1} + h_{2,2}^* h_{2,1} & |h_{1,2}|^2 + |h_{2,2}|^2 \end{bmatrix} \qquad (6)$$

Fig. 6: The Final matrix of ZFE

For BPSK modulation in Rayleigh fading channel, the bit error rate is derived as,

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{(E_b/N_0)}{(E_b/N_0)+1}}\right) \qquad (7)$$

IV. Minimum Mean Square Error

Minimum mean square error (MMSE) is an estimation scheme which minimizes the mean square error and one very common method used for quality estimation [13]. This does not remove the ISI but however it reduces or minimizes the components of noise and ISI in the output. The MMSE finds a coefficient M which minimizes criteria:

$$E\{[My - x][My - x]^H\} \qquad (8)$$

On solving the above criteria, the mathematical value of M comes out to be:

$$M = [H^H H + N_0 I]^{-1} H^H \qquad (9)$$

If we compare the equation of ZFE with MMSE, both the equation seems similar apart from the term No that means in the absence of noise, MMSE and ZFE works similar to each other[6][13][14][15].

V. SIC and SIC WO

This section will explain the approaches we considered in this paper that is SIC or the successive interference cancellation and the successive interference cancellation with optimal ordering.

A. SIC

The Zero Forcing equalization or the Minimum Mean Square error approaches give an estimate of the transmitted symbols, i.e. x1 and x2. Let us take one of the estimated symbols and subtract its effect from the received vectors and it will result in

$$R = hx1 + n \qquad (10)$$

The above equation is similar to the case of Maximal ratio combining in case of MISO systems [14][15].

The equalized symbol is,

$$x_1 = \frac{h^H r}{h^H h} \qquad (11)$$

This form explains the Successive Interference Cancellation (ZF-SIC) approach.

B. SIC WO

This case is the Zero Force Equalization with Successive Interference Cancellation with optimal ordering. In the previous case, the receiver arbitrarily takes one of the estimated symbols, and subtracts its effect from the received symbol y1 and y2. However, we can implement more intelligence in the system by deciding whether we should subtract the effect of either x1 first or x2 first. In order to make this decision, let us find out the transmit symbol which came at higher power at the receiver [16]. The received power at the both the antennas corresponding to the transmitted symbol x_1 is,

$$Px_1 = |h_{1,1}|^2 + |h_{2,1}|^2 \qquad (12)$$

The received power at the both the antennas corresponding to the transmitted symbol x_2 is,

$$Px_2 = |h_{1,2}|^2 + |h_{2,2}|^2 \qquad (13)$$

If $Px_1 > Px_2$ then the receiver decides to remove the effect of x_1 from the received vector y_1 and y_2 and then re-estimate x_2.

And results in

$$R = hx_2 + n \qquad (14)$$

The equalized symbol is,

$$x_2 = \frac{h^H r}{h^H h} \qquad (15)$$

Else if $Px_1 < Px_2$ the receiver decides to subtract effect of x_2 from the received vector y_1 and y_2, and then re-estimate x_1.

$R = hx_1 + n$

The equalized symbol is,

$$x_1 = \frac{h^H r}{h^H h} \qquad (16)$$

Simulation

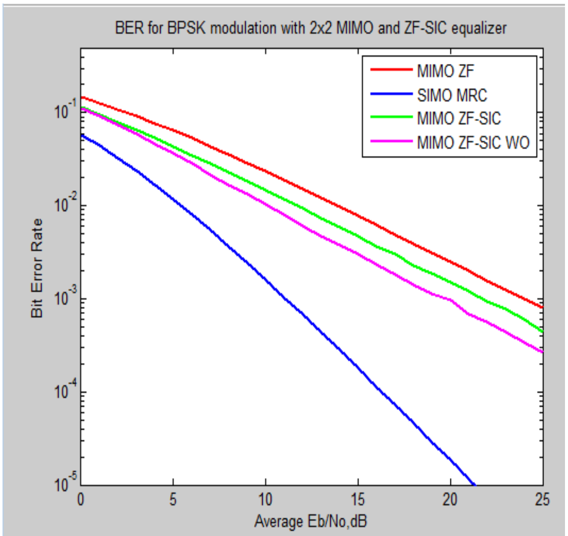

Fig. 3: ZFE, ZFE with SIC and ZFE with SIC WO

Doing successive interference cancellation with optimal ordering ensures that the reliability of the symbol which is decoded first is guaranteed to have a lower error probability than the other symbol. This results in lowering the chances of incorrect decisions resulting in erroneous interference cancellation. Hence gives lower error rate than simple successive interference cancellation [17].

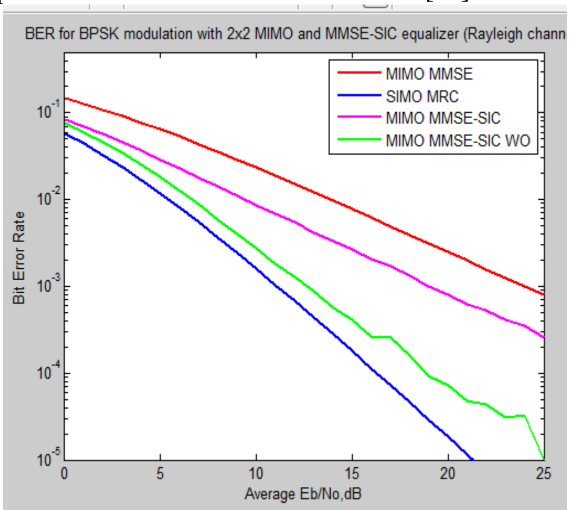

Fig. 4: MMSE, MMSE-SIC and MMSE-SIC WO

VI. CONCLUSION

In this paper, both the equalization schemes have been studied, simulated and analyzed. Compared to Minimum Mean Square Equalization with simple successive interference cancellation case, addition of optimal ordering results in around 5.0dB of improvement for BER of 10-3 and in case of ZFE, Compared to Zero Forcing equalization with successive interference cancellation case, addition of optimal ordering results in around 2.0dB of improvement for BER of 10-3. The overall performance is better n case of MMSE successive interference cancellation and that too with optimal ordering. However still, there is a need to improve the equalization scheme so that the BER performance equivalent to the MRC case can be achieved.

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