Singular Value Decomposition for Image Classification

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Abstract—The purpose of this paper is to study an important application of Singular Value Decomposition (SVD) to image processing. The idea is that by using the smaller number of vectors, one can reconstruct an image that is closer to the original. The clarity of the image depends on how many singular values are used to reconstruct it. In this paper, SVD was applied to the image. Using the MATLAB software the authors have demonstrated how SVD is used to minimize the size needed to store an image.

I. INTRODUCTION

In present scenario, due to internet and availability of large data storage facility, huge numbers of images have been produced and available for active research. Traditionally, retrieval of the images was text based. In this method the images or scene is described by some text annotation and images are searched by using keyword based searching methods. This task is very time consuming and it is very difficult to describe color, texture, shape and object within the image.

Problems with traditional methods of image indexing have led to the rise of interest in techniques for retrieving images on the basis of automatically-derived features such as color, texture and shape – a technology now generally referred to as Content-Based Image Retrieval (CBIR).

Every CBIR system is completely described by answering the two questions: (a) how to mathematically describe an image (computing the feature vector or image signature); and (b) how to assess the similarity (similarity metric) Content-based image retrieval (CBIR), also known as query by image content (QBIC) and content-based visual information retrieval (CBVIR) is the application of computer vision to the image retrieval problem, that is, the problem of searching for digital images in large databases. “Content-based” means that the search will analyze the actual contents of the image. The term ‘content’ in this context might refer to colors, shapes, textures, or any other information that can be derived from the image itself. Without the ability to examine image content, searches must rely on metadata such as captions or keywords, which may be laborious or expensive to produce. The need to find a desired image from a collection is shared by many professional groups, including journalists, design engineers and art historians.

For the given image database, first the features of the database images are extracted. The features can be visual features like color, texture, shape, region or spatial features or some compressed domain features. The extracted features are described by feature vectors. These feature vectors are then stored to form Image feature database.

For a given query image on similar grounds, its features are extracted and feature vector is formed. This feature vector is matched with the already stored vectors in image feature database. The distance between the feature vector of the query image and those of the images in the database are then calculated. Obviously the distance of a query image with itself is zero, if it is in the database. The distances are then stored in increasing order and retrieval is performed with the help of indexing scheme.

In this project a method based on Singular value decomposition is studied and implemented for effective process on image retrieval.

II. RELATED WORKS

Image classification using singular value decomposition, is the problem of searching digital images for large databases. Two main tasks are performed here:

First task is to generate feature vector which represents the content of an image. Second is the similarity measurement where distance between the query image and each image in the database using their feature vectors is used to retrieve the top “closest” images.

III. PROPOSED MODEL

The Singular Value Decomposition (SVD) for square matrix was discovered independently by Beltrami in 1873 and Jordan in 1874 and extended to rectangular matrix by Eckert and Young in 1930.

The singular value decomposition of a rectangular matrix A is decomposed in the form

\[ A = UDV^T \]  

(3.1)

Where A is n matrix. m x n

U, V are the orthogonal matrices.

D is a diagonal matrix comprised of singular value of A.

The singular values appear in the descending order along the main diagonal of D. The singular values are obtained by taking the square root of \( A^T A \) and \( A A^T \)

\[ A^T A = U^T D^2 U \]  

(3.2)

\[ A = UDV^T \]

The relation between SVD and Eigen values are given below

\[ A = UDV^T \]

Now

\[ A^T = UDV^T (UDV^T)^T = UDV^T (VDU^T)^T = UD(V^T)^T \]  

(3.3)

Also

\[ A^T A = (UDV^T)^T (UDV^T) = VDU^T (UDV^T)^T = VDV^T \]  

(3.4)

Properties of SVD are
• The singular values are unique; however the matrix U and V are not unique.
• The matrix U can be computed through the Eigen vector.
• The rank of matrix A is equal to the number of its non-zero singular value.

Application of SVD in image processing is
• SVD approach can be used in the image compression.
• SVD can be used in the face recognition.
• SVD can be used in the texture classification [2].

A. Color Plane Considered For The Proposed Method
As like Gray scale and RGB color planes SVD and block based SVD are applied on the other color planes for computing the feature vector and comparing the performance.

1) YCbCr Color Plane
We applied SVD on YCbCr color planes [3]. Conversion of RGB to YCbCr is given using Equation (3.5).

\[
\begin{bmatrix}
Y' \\
Cb \\
Cr
\end{bmatrix} =
\begin{bmatrix}
0.2989 & 0.5866 & 0.1145 \\
-0.1688 & -0.3312 & 0.5000 \\
0.5000 & -0.1484 & -0.0816
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\] (3.5)

2) YUV Color Plane
In YUV color space Y represent the luminance and U, V represent the chrominance information of given color image. Color conversion of RGB to YUV is given by Equation (3.6).

\[
\begin{bmatrix}
Y \\
U \\
V
\end{bmatrix} =
\begin{bmatrix}
0.299 & 0.587 & 0.144 \\
-0.14713 & -0.22472 & 0.436 \\
0.615 & -0.51498 & 0.1000
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\] (3.6)

3) HSV Color Plane

\[
H = \cos^{-1}\left( \frac{1}{2} \left[ \frac{(R - G) + (R - B)}{\sqrt{(R - G)^2 + (R - B)(G - B)}} \right] \right)
\] (3.7)

\[
S = 1 - \frac{3}{R + G + B} \left[ \min(R, G, B) \right]
\] (3.8)

\[
V = \frac{1}{3} (R + G + B)
\] (3.9)

H (Hue), S (Saturation) and V (Value) is considered as Tint, Shade and Tone by artists. Value represents the intensity of color. The Hue and Saturation components are intimately related to the way human eye perceives colour resulting in image processing algorithms with physiological basis. Conversion formula from RGB to HSV [5] is given in the Equations (3.7), (3.8), (3.9).

4) R’G’I Color Plane
Here we have to used R’G’I color model. This model can be used to separate low and high frequencies in the image without losing any information from the image. This, in turn, allows both distinguishing possible ROIs and retrieving their proper color for further ROI analysis. To get RGI components we need the conversion of RGB to RGI components. The RGB to RGI conversion matrix given in Equations 3.10, 3.11, and 3.12 gives the R’,G’,I components of image for respective R, G, B components.

\[
R' = \frac{R \cdot 256}{(R + G + B)}
\] (3.10)

\[
G' = \frac{G \cdot 256}{(R + G + B)}
\] (3.11)

\[
I = \frac{(R + G + B)}{3}
\] (3.12)

5) CXY Color Plane
Conversion of RGB to CXY color plane is given in the Equation (3.13).

\[
\begin{bmatrix}
C \\
X \\
Y
\end{bmatrix} =
\begin{bmatrix}
0.607 & 0.174 & 0.200 \\
0.299 & 0.587 & 0.14 \\
0.000 & 0.006 & 1.116
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\] (3.13)

B. Feature Extraction

1) SVD of Full Image
When SVD of image having size NXN is computed then we get singular values [13, 37, 36]. These singular values appear in decreasing order. Feature vector size is different because of when we take one, eight, sixteen, thirty two and sixty four singular values for the image reconstruction then the appearance of the image is shown in Fig. 3.1. So in this paper the feature vector size is 8,16,32,64 and 200 for gray scale image and 24,48,93,192 and 600 for color image to find out which SVD values feature suitable for CBIR. Recommended font sizes are shown in Table 3.1[4].

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Category</th>
<th>No. of Images used</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Elephants</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>Flowers</td>
<td>100</td>
</tr>
</tbody>
</table>

Fig. 1: Reconstruction of the images for different value of SVD

From Fig. 3.1 it is clear that as the SVD values goes on increasing the visual quality of the reconstructed image approaches the original image.
### 2) Block Base SVD

Feature vectors have truncated singular values of image. We are testing what is the effect of SVD coefficients on the retrieval accuracy when it goes on increasing. Without truncation when image is divided into sub block and then SVD of each block is computed. In this method the feature vector size goes on increasing as the image block size decreases. In this paper we use 8X8, 16X16, 32X32, 64X64 and 128X128 block size.

#### IV. EXPERIMENTAL RESULT

##### A. Feature Vector Matching

When a query image is submitted by a user, we need to compute the feature vector as before and match it to the precompiled feature vector in the database. This is shown in Fig. 3.2 Block diagram of retrieval process consists of feature extraction process, feature vector storage process and similarity measure process. The feature extraction process is based upon the following. Which the batch feature extraction and storage process as described in the following steps:

1. Images taken one by one from the database.
2. Feature is computed using the feature extraction process.

![Fig. 2: Feature extraction and storage process](image)

After that query image and database image matching is done using similarity measures. In this paper two similarity measures are used Euclidean Distance (ED) and Bray Curtis Distance (BCD) [8] for comparison. Minkowski (Euclidean distance when r=2) distance is computed between each database image and query image on feature vector to find set of images falling in the class of query image.

\[
ED(Q, I) = \left( \sum_{i=0}^{M-1} |H_Q - H_I|^{r} \right)^{1/r} \tag{3.14}
\]

Where Q-Query image
I- Database image
\(H_Q\)-Feature vector query image.
\(H_I\)-Feature vector for database image
\(M\)-Total no of component in feature vector
Bray Curtis Distance is computed between query image and database image using Equation 2.2.16

\[
Bd(Q, I) = \frac{\sum_{i=1}^{n} |H_{Q_i} - H_{I_i}|}{\sum_{i=1}^{n} (H_{Q_i} - H_{I_i})} \tag{3.15}
\]

##### B. Performance of CBIR

Performance of image retrieval system can be analyzed by using two parameters precision and recall. As shown in Fig. 2.3. Testing the effectiveness of the image search engine is about testing how well can the search engine retrieve similar images to the query image and how well the system prevents the return results that are not relevant to the source at all in the user point of view. A sample query image must be selected from one of the image category in the database. When the search engine is run and the result images are returned, the user needs to count how many images are returned and how many of the returned images are similar to the query image. The first measure is called Recall. All the relevant images from the database are recall. The equation for calculating recall is given below:

\[
\text{Recall} = \frac{\text{Number of relevant images retrieved}(A)}{\text{Total number of relevant images in database}(A+D)} \tag{3.16}
\]

The second measure is called Precision. It is accuracy of a retrieval system to present relevant as well as non-relevant images from the database which is mathematically given as:

\[
\text{Precision} = \frac{\text{Number of relevant images retrieved}(A)}{\text{Total number of images retrieved}(A+B)} \tag{3.17}
\]

#### V. CONCLUSION

Thus we have presented a new simple and efficient image retrieval approach using SVD of full image and sub block image. Two similarity measures and seven color planes are used to evaluate the performance of proposed approach. Feature vector is considered as a singular values of each color plane SVD and singular values of sub blocks of each color plane SVD. Bray Curtis Distance similarity outperform Euclidean Distance similarity. Block based SVD performance is better than the full image SVD and truncated SVD.
REFERENCES


