Power System State Estimation-A Review

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Abstract— The review on power system state estimation is presented in this paper. State estimation process is based on the system state variables. The two methodology used for state estimation least square & weighted least square. The treatment of bad data gives the error-free estimate of input data. Bad data detection, identification & suppression is also discussed in this paper.

Key words: Maximum likelihood criterion, Weighted least square estimation criterion, Weighted least square estimation method

I. INTRODUCTION

Selective monitoring of the generation and transmission system has provided the data required for economic dispatch and load frequency control. However, interconnected power system network become more complicated and the process of securely operating the system has become more difficult [3]. The transmission system is under stress, because generation & load are constantly increase but capacity of transmission lines has not increased proportionally. Therefore the transmission system must operate with ever decreasing margin from its maximum capacity [4].

Before any security assessment can be made or control actions taken, a reliable estimate of the existing state of the system must be determined. The input to the conventional power-flow program are confined to the P,Q injections at load buses and P, |V| values at voltage controlled buses. If one of these inputs is not available, the conventional power-flow solution cannot be determined. In practice, other conveniently measured quantities such as P, Q line flows are present, but they cannot be used in conventional power-flow calculations. These drawbacks can be removed by state estimation [3].

The state estimators came in industry of power engineering in 1960s. Since then, state estimators have been installed on a regular basis in a new energy control centers and have proved quite useful. State estimator may be both static & dynamic. Both types of estimators have been developed for power systems [2].

II. STATE ESTIMATION

State estimation is the process of assigning the value to an unknown system state variable based on the measurement from that system according to some criteria. The process involves imperfect or noisy measurements that are redundant and the process of estimating the system states is based on a statistical criterion that estimates the true value of the state variables to minimize or maximize the chosen criterion [3].

In the Power System, The State Variables are the voltage Magnitudes and Relative Phase Angles at the System buses.. The inputs to an estimator are imperfect or noisy power system measurements of voltage magnitude and power, VAR. The Estimator is designed to generate the "best estimate" of the system voltage and phase angles, remembering that there are errors in the measured quantities and these measurements are redundant measurements [3].

We require to produce the best estimate for the state given, the noisy measurement are present. This leads to the problem how to formulate the best estimate of the unknown parameters given the available measurement

Most commonly used traditional methods for obtaining best estimate of system state variable are

A. Maximum likelihood criterion (least square estimation method)
B. Weighted least square estimation criterion [3].

The non-traditional methods are evolutionary optimization techniques like genetic algorithms, differential evolution algorithms etc [4].

III. METHODS OF STATE ESTIMATION.

There are two methods or criterion for the obtaining best estimate of system state variable.

A. Least Square(Maximum Likelihood) Estimation Method:

The objective of least square estimation (maximum likelihood criterion) method is to maximize the probability that the estimate of state variable $\hat{x}$, is the true value of the state vector x, (i.e. maximize P ($\hat{x}$) = x) [1]. Also, minimize the sum of absolute value of difference between measured & calculated values. The objective function 'g' to be minimized is given by

$$g = \sum_{i=1}^{m} wi \times |hi(x) - zi| \quad (1)$$

Subject to constraint zi = hi(x) + ei

Where, m = number of measurements. w = weighting factor of the measurement (reciprocal of variance of measurement) [4]

Error ei = zi – hi(x), i = 1,2,3.....m. (2)

Where, h(x) = measurement function
x = state variables and
z = measured value [5]

If Zmeas be the value of a measurement as received from a measurement device & Ztrue be the true value of the quantity being measured. Finally, let η be the random measurement error. Then, mathematically it is expressed as

$$Zmeas = Z\ true + \eta \quad (3)$$

The random number η serves the inaccuracy in the measurement [1].

If the measurement error is not partial, the probability density function of η is given by

$$PDF\ (\eta) = \frac{1}{\sigma\sqrt{2\pi}} \times e^{\left(-\frac{\eta^2}{2\sigma^2}\right)} \quad (4)$$

Where σ is called the standard deviation and if σ is large, the measurement is relatively inaccurate (i.e. , a poor quality measurement equipment). A small value of σ denotes a small error spread (i.e., a good quality measurement device) [1].

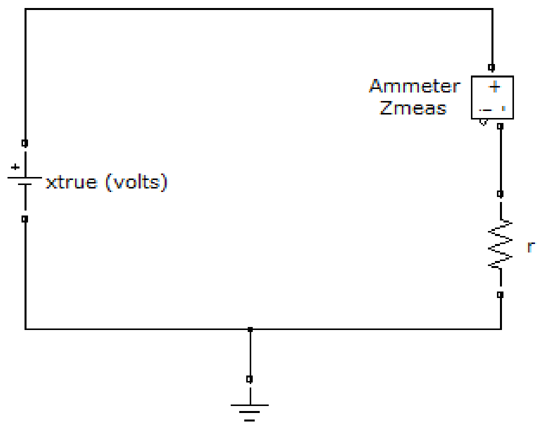

Fig. 1: Simple dc circuit with ammeter.

Fig.1 shows the simple DC circuit with current measurement. Here random error is given by,

η = Zmeas – Ztrue = Zmeas - $\frac{x}{r}$ (5)

Where, r is the resistance in the circuit [1].

By definition of maximum likelihood (least square) criterion Probability of Zmeas is given by

Prob (Zmeas)=$\int_{Zmeas}^{Zmeas+dZmeas} PDF(Zmeas).\,dZmeas$

=PDF(Zmeas)*dZmeas (6)

Now, take maximum of prob (Zmeas) which is the function of x,

Max prob (Zmeas) = max PDF(Zmeas)*dZmeas. (7)

Now, take natural logarithm of PDF(Zmeas) and maximize we get

Max Ln PDF (Zmeas)] (8)

After simplification we get final result of estimated value of x [1].

x^{est} = r. Zmeas (9)

The estimation index J is given by

J = $\tilde{z}$' . $\tilde{z}$ (10)

For minimizing J = f($\hat{x}$) , we must satisfy the following condition, grad ($\hat{x}$) J = 0

The normal equation is given by

H'.H.$\hat{x}$ - H'z = 0 (11)

The LSE of the vector $\hat{x}$ as [2]

$\hat{x}$ = (H'.H)¯1. H'.z (12)

B. *Weighted Least Square Estimation Method:*

The objective of weighted least square estimation is to minimize the weighted sum of squares of the difference between measured & calculated values. In weighted least square method, the objective function 'f' to be minimized is given by

$\sum_{i=1}^{m} \frac{1}{\sigma i^2} \times ei^2$ (13)

The problem of state estimation is to determine the estimate that best fits the measurement model. The state estimation problem can be formulated as a minimization of the weighted least-squares (WLS) function problem.

Min J(x) = $\sum_{i=1}^{m} \frac{(zi-hi(x))^2}{\sigma i^2}$ (14)

This represents the summation of the squares of the measurement residuals weighted by their respective measurement error covariance. Where z is the measurement vector, h(x) is measurement matrix, m is number of measurements & x is vector of unknown variables to be estimated.

In this method nonlinear vector function is linearized using Taylor series expansion

h(x+Δx) ≈ h(x) +H(x).Δx (15)

Where, the jacobian matrix H(x) is defined as:

H(x) = [$\frac{\partial h(x)}{\partial x}$] (16)

Then the linearized least-squares objective function is given by

J(Δx) = $\frac{1}{2}$ (z – h(x) –H(x)Δx)T.R^{-1}.(z – h(x) –H(x).Δx) (17)

Where, R is a weighting matrix whose diagonal elements are often chosen as measurement error variance,i.e.,

$$R = \begin{pmatrix} \sigma 1^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma m^2 \end{pmatrix}$$

J(Δx) = $\frac{1}{2}$ (e(x) – H(x)Δx)T.R^{-1}.(e(x) – H(x).Δx) (18)

Where e = z – h(x) is the residual vector.

At the minimum, first order optimally conditions will have to be satisfied. This can be expressed in compact form as follows:

$\frac{\partial J(\Delta x)}{\partial \Delta x}$ = -H^T. R^{-1}.(e – H.Δx) = 0

H^T.R^{-1}.H..Δx = H^T.R^{-1}.e

G.Δx = H^T.R^{-1}.e (19)

Where G = H^T.R^{-1}.H is called gain matrix. It is sparse, positive definite and symmetric provided that the system is fully observable [3].

IV. TREATMENT OF BAD DATA

One of the essential functions of a state estimator is to detect measurement errors , and to identify and eliminate them if possible. Measurements may contain errors due to Random errors usually exist in measurements due to the finite accuracy of the meters & Telecommunication medium.

Bad data may appear in several different ways depending upon the type , location and number of measurements that are in error. They can be broadly classified as

- Single bad data: Only one of the measurements in the entire system will have a large error.
- Multiple bad data: More than one measurement will be in error.

The treatment of bad data have two levels as :

A. *Bad Data Detection:*

Detection refers to the determination of whether or not the measurement set contains any bad data. Bad data can be detected if removal of the corresponding measurement does not render the system unobservable. One of method used for detecting bad data is the chi-square test. Once bad data are detected, they need to be identified and eliminated or corrected.

Consider the set of N independent random variables X1, X2, X3.....XN.

Where is X is distributed according to the standard normal distribution. Then, a new random variable Y defined by Y = $\sum_{i=1}^{N} Xi^2$

It will have the chi – square distribution of Y~ $\varkappa i^2$

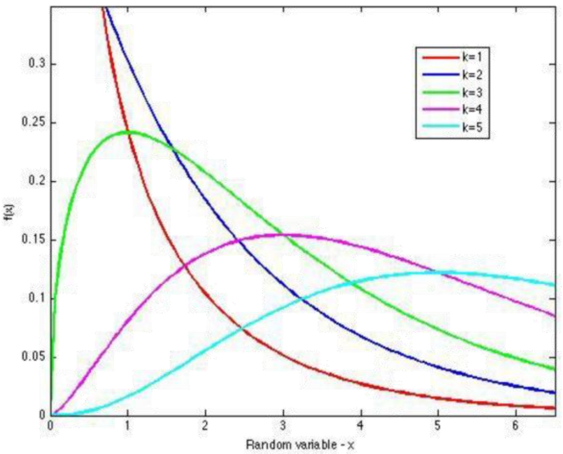

Fig. 2: Chi – square probability density function

The chi – square distribution table is given by

Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of x^2								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09
6	0.872	1.635	2.204	3.455	5.348	7.84	10.64	12.59	16.81
7	1.239	2.167	2.833	4.255	6.346	9.04	12.02	14.07	18.48
8	1.647	2.733	3.490	5.071	7.344	10.22	13.36	15.51	20.09
9	2.088	3.325	4.168	5.899	8.343	11.39	14.68	16.92	21.67
10	2.558	3.940	4.865	6.737	9.342	12.55	15.99	18.31	23.21
11	3.053	4.575	5.578	7.584	10.341	13.70	17.28	19.68	24.72
12	3.571	5.226	6.304	8.438	11.340	14.85	18.55	21.03	26.22
13	4.107	5.892	7.042	9.299	12.340	15.98	19.81	22.36	27.69
14	4.660	6.571	7.790	10.165	13.339	17.12	21.06	23.68	29.14
15	5.229	7.261	8.547	11.037	14.339	18.25	22.31	25.00	30.58
16	5.812	7.962	9.312	11.912	15.338	19.37	23.54	26.30	32.00
17	6.408	8.672	10.085	12.792	16.338	20.49	24.77	27.59	33.41
18	7.015	9.390	10.865	13.675	17.338	21.60	25.99	28.87	34.80
19	7.633	10.117	11.651	14.562	18.338	22.72	27.20	30.14	36.19
20	8.260	10.851	12.443	15.452	19.337	23.83	28.41	31.41	37.57
22	9.542	12.338	14.041	17.240	21.337	26.04	30.81	33.92	40.29
24	10.856	13.848	15.659	19.037	23.337	28.24	33.20	36.42	42.98
26	12.198	15.379	17.292	20.843	25.336	30.43	35.56	38.89	45.64
28	13.565	16.928	18.939	22.657	27.336	32.62	37.92	41.34	48.28
30	14.953	18.493	20.599	24.478	29.336	34.80	40.26	43.77	50.89
40	22.164	26.509	29.051	33.660	39.335	45.62	51.80	55.76	63.69
50	27.707	34.764	37.689	42.942	49.335	56.33	63.17	67.50	76.15
60	37.485	43.188	46.459	52.294	59.335	66.98	74.40	79.08	88.38

Fig. 3: Chi –square distribution table

Step to detect the bad data are:

– Use the raw measurements zi from the system to determine the weighted least-squares estimates x̂i of the system states. Substitute the estimates x̂i in the equation $\hat{z} = H\hat{x}$ to calculate the estimated values ẑi of the measurements and hence the estimated errors êi = zj - ẑi

– Evaluate the weighted sum of squares

$$\hat{f} = \sum_{j=1}^{m} \frac{ei^2}{\sigma i^2} \qquad (20)$$

– If the value of $\hat{f}$ is less than critical value corresponding to ∝ with k degrees of freedom then the measured raw data and the state estimates are accepted as being accurate.

B. Bad Data Identification:

For a non critical (redundant) measurement zi, the normalized residual ei^N can be defined as the ratio of the residual estimate, êi = zj - ẑi , to the standard deviation of the residual , Rii. The vector of normalized residuals is given by

$$(zi - \hat{z}i) / \sqrt{Rii'} \quad , Rii' = (I - HG^{-1} H^T R^{-1}) \qquad (21)$$

Steps to identify the bad data are :

Solve the WLS estimation and obtain the elements of the measurement residual vector ei = zi – hi(x).

Compute the normalized residuals

$$ei^N = \frac{|ei|}{\sqrt{Rii'}} \ , i = 1, 2 ...m \qquad (22)$$

Find p-th measurement such that ep^N is the largest among ei^N.

If $ep^N > c$, then the p-th measurement will suspected as bad data. Else, stop, no bad data will be suspected. Here, c is a chosen identification threshold, for instance 3.0.

Eliminate the p-th measurement from the measurement set and go to step-1.

After identification of bad data in measurement input data they could be removed before it is proceed.

V. Conclusion

This paper presents the very basic concept of state estimation & their conventional topics like maximum likelihood (least square) criterion. And further advanced topics like bad data processing (detection & identification). The state estimation plays very important (key) role in monitoring & control of modern power systems. The least square & weighted least square techniques are important for estimating the system state variables like voltage magnitude, phase angles, power flow & VAR. The bad data detection & identification is responsible for elimination of presence of errors (bad measurement) in measurement data. The bad data detection is based on the chi – square test. After detection & identification of bad data they can be removed from measurement data before it is proceed. It is called as suppression of bad data.

References

[1] Allen J. Wood And Bruce F. Wollenberg, "Power generation, operation and control," 2nd edition, volume no.12, 1996.
[2] D.P. Kothari And I.J. Nagrath , " Modern power system analysis," 3rd edition , volume no. 14, 2003.
[3] John J. Grainger And William D. Stevenson, Jr. , "Power system analysis," 5th edition , volume no. 15 , 2006.
[4] F.C. Schweppe And J.Wildes, "Power system static state estimation, part-1," IEEE trans. Power apparatus and systems, volume no. PAS-89, pp.125, jan 1970.
[5] R.E. Tinney W.F. Tinney, And J.Peschon , "State estimation in power systems, parti :theory and feasibility ," IEEE Trans. Power Apparatus and Systems, vol. PAS-89 , pp. 345-352, March 1970.
[6] F.F. Wu, "Power System State Estimation ," International Journal of Electrical Power and Energy Systems, vol.12, Issue.2, pp.80-87 , April 1990.