Analysis of Adaptive Algorithms

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Abstract—Adaptive Signal Processing has been playing a key role confining itself not just to the field of communications but also had spread into the fields of embedded systems, biological instruments, astronomy, image processing and many other fields. Adaptive filters are slowly replacing traditional filters in many areas. The development of new techniques and trends of adaptation algorithms has provided us with a broader sense of understanding the adaptation phenomena. In this paper some basic algorithms such as Least-Mean-Squares, Leaky-LMS, Normalized-LMS, and Recursive-Least-Squares algorithms have been studied and the convergence of these algorithms has been studied. The study convergence of the algorithms gives us a better picture of how fast the algorithms converge to optimum values. This is an issue of consideration in real-time signal processing as the signal processor implementing these kinds of algorithms has to be converging fast enough to the optimum values to save time and memory.

Key words: Adaptive filters, least mean square, normalized least mean square, leaky least mean square, recursive least square.

I. BACKGROUND

Adaptation has now become a primary characteristic of many systems which involve filters to extract various signals in noisy environments. The frequency bands get congested and crowded due to the increase of wireless systems and various Trans receiver systems; these pose very strict filtering conditions on the systems involving the signal extraction. An adaptive filter can be defined as, "A filter that self-adjusts its transfer function according to an optimization algorithm driven by an error signal."[1]

This congestion of frequency bands not only demands filters to be sensitive to the bands but also simultaneously avoid interference with neighboring bands. These interference issues and noise cancellation processes and extraction of signals with equalization method cannot be achieved by conventional filters. Therefore adaptive filters come into picture to solve these issues.

Weiner filters, proposed by Nobert Wiener in 1940, are a class of filters which solve the problem to a certain extent; Weiner filter is a filter to reduce the noise in the signal by comparison and estimation of the desired noise-less signal [1]. The solution for the optimum weight calculation of the FIR Weiner filter is given by the equation [2]:

$$W_{opt} = R^{-1}{}_{XX} \cdot r_{dx} \quad (1.1)$$

Where the W_{opt} represent the optimum weight vector, which is the product of the inverse of the auto-correlation matrix $R^{-1}{}_{XX}$ of the input vector and the cross-correlation matrix of the desired signal vector with the input vector r_{dx} [2].

This is not an adaptive filter, but it is the base to the adaptive approach as this adaptation-based-filtering is done by a statistical approach rather than the conventional band-pass, band-stop, etc filters which concentrate on the spectrum of input signals [1].

The Weiner filter mentioned above involves a Toeplitz matrix of auto-correlation which is more of a theoretical approach rather than a practical implementation. As the real-time input data statistics are unknown; the auto-correlation of the input is rather impractical to calculate in addition to the inverse of the matrix [2]. Therefore, a different approach has to be taken in-order to settle onto the optimum weights. This approach has been proposed in various ways leading to different adaptive algorithms. Different algorithms have been proposed with trade-offs between the complexity, convergence, input data statistics and various other parameters [1].

II. AIM AND OBJECTIVE

The aim of this paper is to compare and analyse various adaptive algorithms and to point out the trade-offs between the Algorithms tested. The algorithms used here are the least-mean squares algorithm, Leaky LMS algorithm, Normalized LMS algorithm, and Recursive Least Squares algorithm. Two sets of data have been used to verify the algorithms. A set of ECG data and a set of corrupted sinusoidal signal data have been used. The analysis should yield similar results and similar convergence paths.

III. PROBLEM SOLUTION

The algorithms are coded as user-defined functions in the MATLAB software. We have chosen this simulation environment as it is a well-acquainted simulation software and The environment as such is based on matrices, so it has many matrix manipulation functions which decrease the code-size leading to a simpler code, thereby concentrating on the algorithms rather than the coding structures [4].

The adaptation algorithms can be classified into two major groups: statistical adaptation and the deterministic adaptation. The statistical approach concentrates on the statistical quantities of the input vector and tries to minimize the mean square error in the input. Whereas the deterministic approach tries to down the sum of squares of the error [2]. In the latter case the error of the previous iterations are considered in bringing down the error altogether. The algorithms involving the statistical approach are the family of the LMS algorithms. The deterministic approach is taken by the RLS algorithm.

The data used here are the readings of the cardiac and abdominal regions of a pregnant. This data is used to extract the weak foetal heartbeat signal deeply buried under the cardiac and other abdominal signals [10]. This extraction is done using the algorithms mentioned above. The ECG data used here is an 8 channel data sampled at 500Hz for 10 seconds. The data has been filed into a data file with 9 columns each of 5000 samples long. The first column represents the time-steps, and columns 2 to column 6 represent the abdominal signals, columns 7 to column 9 represent the thoracic signals [5]. Second set of data involving a white noise corrupted sinusoidal signal was used to support the analysis observed in the previous data set. The data used here is a Gaussian noise corrupted sinusoidal signal of 10000 samples.

A. *Least Mean Squares*

Firstly the Least-Mean-Squares algorithm is being coded as a function with input parameters as the input corrupted sequence and the desired sequence. These sequences are made into columns with a unit delay and given as input. The weight update equation for the LMS algorithm is [6]:

$$Wn+1 = Wn + \mu E\{x(n) * e(n)\} \qquad (3.1)$$

This equation determines the convergence of the LMS algorithm. Here the expectation parameter can be approximated to [6]:

$$E\{x(n) * e(n)\} \approx x(n) * e(n) \qquad (3.2)$$

B. *Normalized Least Mean Squares*

The normalized least mean squares algorithm has a modified Weight update equation for changing input characteristics; this Weight update equation requires the normalized value of the Input vector so as to take into account the changing input statistics. The weights update equation for the N-LMS algorithm is [7]:

$$Wn+1 = Wn + \left[\frac{\beta}{||x(n)||2}\right] E\{x(n) * e(n)\} \qquad (3.3)$$

Here the L2-Norm value of the input (||x(n)||2) and the β parameters determine the weight updating sequence. Also the restriction on β which is 0< β <2

C. *Leaky Least Mean Squares*

The properties of the input auto-correlation matrix play an important role in determining the stability of the algorithm. If the autocorrelation function of the input signal has zero Eigen values then one or more modes remain undamped which leads to instability of the algorithm [7]. To release the algorithm from the undamped modes and to make it stable a leakage coefficient is attached, this leakage coefficient ensures the stability of the algorithm even when it has zero Eigen values.

The weight update equation of the L-LMS algorithm is given by [7]:

$$Wn+1 = (1 - \mu * \gamma)\, Wn + \mu * E\{x(n) * e(n)\} \qquad (3.4)$$

This weight update involves the leaky coefficient γ. This leaky coefficient determines the amount of leverage that has to be given to the previous weight vectors so as to bring out stability.

D. *Recursive Least Squares*

The recursive least squares is a different approach of sadaptation of the weights. This algorithm takes-up a deterministic approach of converging to the optimum weights [2, 8]. This is the least squares approach rather than the mean square approach. The computation of the gain vectors and the weight update equation is given by [2]:

$$U(n) = Rxx(n-1) * x(n)$$
$$K(n) = U(n)/(\lambda + x'(n) * U(n))$$
$$W(n) = W(n-1) + k(n) * e(n-1)$$
$$R^{-1}xx = tri\{\lambda^{-1}(Rxx^{-1}(n-1) - k(n) * U'(n))\} \qquad (3.5)$$

The above equations form the heart of the looping module in the RLS algorithm.

IV. GRAPHS

The below graphs are the weight-plots of the algorithms discussed above. These weight plots speak of the convergence attained.

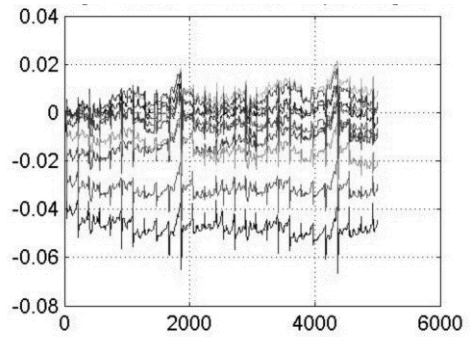

Fig. 1: Weight Plots of Least Mean Square Algorithm of the ECG data-set

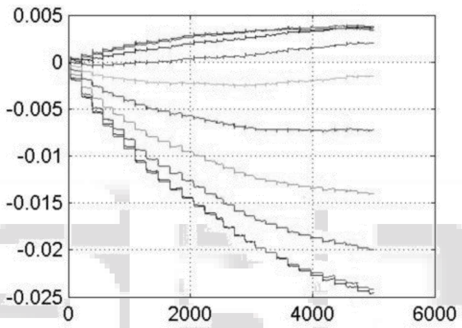

Fig. 2: Weight Plots of Normalized Least Mean Square Algorithm of the ECG data-set

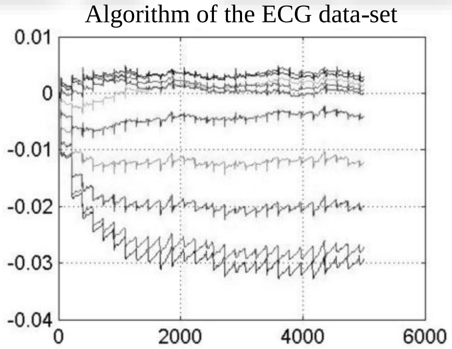

Fig. 3: Weight Plots of Leaky Least Mean Square Algorithm of the ECG data-set

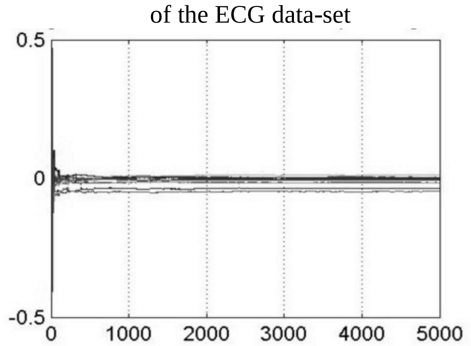

Fig.4: Weight Plots of Recursive Least Squares Algorithm of the ECG data-set

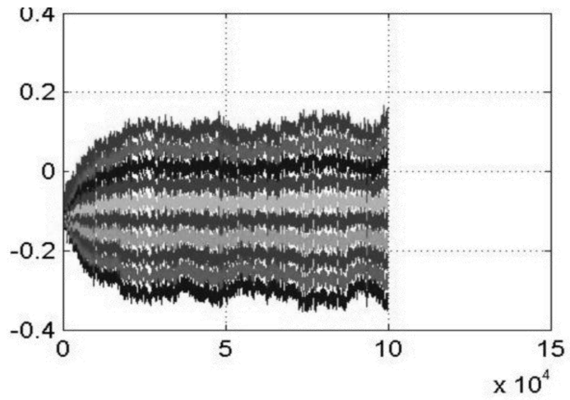

Fig. 5: Weight Plots of Least Mean Square Algorithm of the Corrupted Sinusoidal data-set

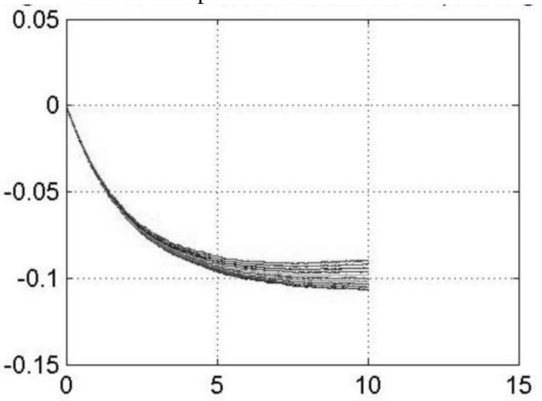

Fig. 6: Weight Plots of Normalized Least Mean Square Algorithm of the Corrupted Sinusoidal data-set

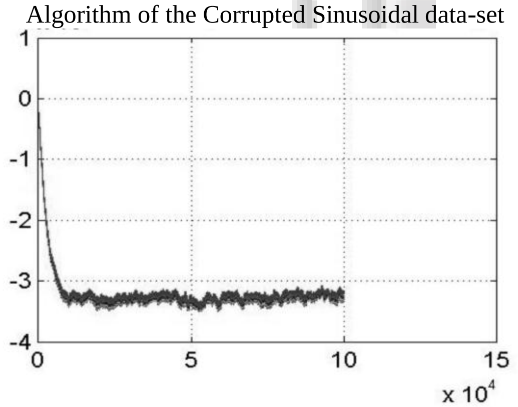

Fig. 7: Weight Plots of Leaky Least Mean Square Algorithm of the Corrupted Sinusoidal Data-set

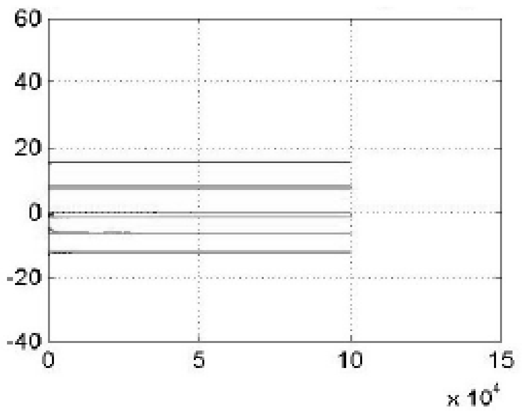

Fig 8 Weight Plots of Recursive Least Squares Algorithm of the Corrupted Sinusoidal Data-set.

The figures are self-explanatory. The figures 1 to 4 represent the convergence graphs of foetal heartbeat data-set. The figures 5 to 8 represent the convergence graphs of the noisy sinusoidal set.

The four figures of each data-set represent the four algorithms that have used to check the convergence. Fig.1 to fig.3 show the different versions of the least-mean-square algorithm which converge slowly i.e. it takes more number of iterations to reach the optimal values. Fig.4 representing the recursive least squares algorithm converges at a faster rate than the other algorithms (in a few hundred iterations).

This convergence is repeated in the case of the noisy sinusoidal signal data-set where the graphs showed a similar convergence of the algorithms. The first three algorithms fig.5 to fig.7 converged slowly. They have taken 10^4 iterations to converge. Whereas the recursive algorithm in this case too has converged with lot lesser iterations (a thousand iterations).

The time of execution of each algorithm has also been noted down and the RLS algorithm has taken roughly 100 times more time than the LMS based algorithms, this proves the computational complexity of the RLS algorithm is high.

The above simulations and the plotting of the weights show that the Recursive Least Squares algorithm in Fig.8 & Fig.4, have converged lot faster than the other Mean Square algorithms at the cost of computational complexity and processing time. On the other hand the LMS based algorithms were of less computational complex but at the cost of slow convergence as like in Fig.6 & Fig.2.

Since the real time signal extraction requires fast processing, the RLS algorithms will have a less demand over the LMS algorithm even though the convergence is fast. This is due to the elongated processing time.

The varying step-size of a standard least-mean square algorithm might provide a better convergence speed, as the weights approach the optimum values the step-size can be decreased to provide accuracy to the optimum weights.

V. CONCLUSION

The results show that the recursive least squares converges slowly but the convergence can be improved by whitening the input signal and modifications in the weight-update equation also contributes for convergence

VI. FUTURE WORK

The comparison of the algorithms, from Fig.1 to Fig.4 and Fig.5 to Fig.8, can be extended to a higher level and can include various other algorithms. This comparison results can give amateurs an idea of the algorithm and can easily choose the best suiting algorithm without much confusion. These comparisons yield in a better algorithms by merging two or more algorithms.

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