

Position Control of Satellite In Geo-Stationary Orbit Using Sliding Mode Control Algorithm

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Abstract— In this paper, sliding mode control, or SMC, in control theory is a form of variable structure control (VSC). It is a nonlinear control method that alters the dynamics of a nonlinear system by application of a high-frequency switching control. It switches from one continuous structure to another based on the current position in the state space. The state feedback control law is not a continuous function of time. Hence, sliding mode control is a variable structure control method. The multiple control structures are designed so that trajectories always move toward a switching condition, and so the ultimate trajectory will not exist entirely within one control structure. Instead, the ultimate trajectory will slide along the boundaries of the control structures. The motion of the system as it slides along these boundaries is called a sliding mode and the geometrical locus consisting of the boundaries is called the sliding (hyper) surface. Using this law we can control the Satellite's position in Geostationary Orbit.

Keywords: Sliding Mode Control, Fast Tracking.

I. INTRODUCTION

In recent years, the sliding mode control methodology has been widely used for robust control of nonlinear systems (Slotine and Li, 1991). Sliding mode control, based on the theory of variable structure systems, has attracted a lot of research on control systems for the last two decades. A comprehensive survey on variable structure control was given in Hung et al. (1993). The salient advantage of sliding mode control is robustness against structured and unstructured uncertainties. In path tracking systems, however, the system invariance properties are observed only during the sliding phase. In the reaching phase, tracking may be hindered by disturbances or parameter variations. The straightforward way to reduce tracking error and reaching time is to increase the control discontinuity gain.

Trajectories from this reduced-order sliding mode have desirable properties (e.g., the system naturally slides along it until it comes to rest at a desired equilibrium). The main strength of sliding mode control is its robustness. Because the control can be as simple as a switching between two states (e.g., "on"/"off" or "forward"/"reverse"), it need not be precise and will not be sensitive to parameter variations that enter into the control channel. Additionally, because the control law is not a continuous function, the sliding mode can be reached in finite time (i.e., better than asymptotic behavior)

Under certain common conditions, optimality requires the use of bang–bang control; hence, sliding mode control describes the optimal controller for a broad set of dynamic systems. Sliding mode control must be applied with more care than other forms of nonlinear control that have more moderate control action.

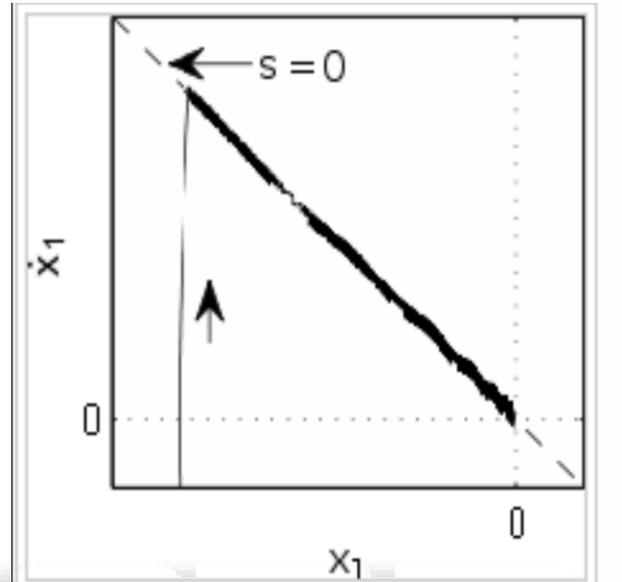


Fig. 1

In particular, because actuators have delays and other imperfections, the hard sliding-mode-control action can lead to chatter, energy loss, plant damage, and excitation of unmodeled dynamics. Continuous control design methods are not as susceptible to these problems and can be made to mimic sliding-mode controllers.

II. SLIDING MODE CONTROL

A. Theory of Sliding Mode control

Consider a nonlinear dynamical system described by $\frac{dx}{dt} = f(X,t)+B(X,t)u(t)$

Where,

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \dots \\ x_{n-1}(t) \\ x_n(t) \end{bmatrix} \quad \text{and} \quad U(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ \dots \\ u_{n-1}(t) \\ u_n(t) \end{bmatrix}$$

A common task is to design a state-feedback control law $u(X(t))$ (i.e., a mapping from current state $X(t)$ at time t to the input u) to stabilize the dynamical system in Equation around the origin $X=[0,0,0,0\dots 0]T$. That is, under the control law, whenever the system is started away from the origin, it will return to it. For example, the component x_1 of the state vector may represent the difference some output is away from a known signal (e.g., a desirable sinusoidal signal); if the control can ensure that x_1 quickly returns to $x_1 = 0$, then the output will track the desired sinusoid. In

sliding-mode control this reduced-order subspace is referred to as a sliding (hyper) surface.

B. The sliding-mode control scheme involves

- 1 Selection of a hyper surface or a manifold (i.e., the sliding surface) such that the system trajectory exhibits desirable behavior when confined to this manifold
- 2 Finding feedback gains so that the system trajectory intersects and stays on the manifold. The sliding-mode designer picks a switching function σ that represents a kind of "distance" that the states X is away from a sliding surface.

A state that is on this sliding surface has $\sigma(X)=0$

The sliding-mode-control law switches from one state to another based on the sign of this distance. So the sliding mode control acts like a stiff pressure always pushing in the direction of the sliding mode

Where $\sigma(X)=0$. Desirable $X(t)$ trajectories will approach the sliding surface, and because the control law is not continuous (i.e., it switches from one state to another as trajectories move across this surface), the surface is reached in finite time. Once a trajectory reaches the surface, it will slide along it and may, for example, move toward the $X=0$ origin. So the switching function is like a topographic map with a contour of constant height along which trajectories are forced to move.

C. To force the system states to satisfy $\sigma(X) = 0$, one must:

- 1 Ensure that the system is capable of reaching $\sigma(X)=0$ from any initial condition.
- 2 Having reached $\sigma(X)=0$, the control action is capable of maintaining the system at $\sigma(X)=0$.

D. Condition for existence of sliding mode

Consider a Lyapunov function candidate

$$V(\sigma(x)) = \frac{1}{2} * \sigma^T(x) * \sigma(x) = \frac{1}{2} * \|\sigma(x)\|^2$$

Where $\|\sigma(x)\|^2$ is a Euclidean norm.

Sufficient condition for the existence of sliding mode that

$$\frac{dV}{dt} < 0, \text{ where } \frac{dV}{dt} = \frac{d\sigma}{dt} * \sigma$$

Roughly speaking

- 1), to achieve $\sigma^T \frac{d\sigma}{dt} < 0$, the feedback control law $u(X)$ is picked so that σ and $\frac{d\sigma}{dt}$ have opposite signs. That is,
 - (1) $u(X)$ makes $\frac{d\sigma}{dt}$ negative when $\sigma(X)$ is positive.
 - (2) $u(X)$ makes $\frac{d\sigma}{dt}$ positive when $\sigma(X)$ is negative.

E. The Sliding Mode Control algorithm

In sliding mode control, the system's representative point is constrained to move along a surface (hyper plane or line) located in the state space. The first property is the fact that application of SMC does not require an accurate model of the plant. Secondly, SMC is robust in the sense that it is insensitive to parameter variations and bounded disturbances. Thirdly, SMC is characterized by accurate and fast responses. Lastly, the algorithm is simple. A typical phase-plane response of a second-order system is shown in

Fig which illustrates the following shortcomings of SMC schemes.

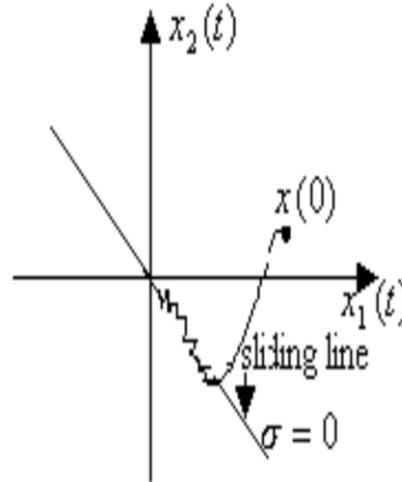


Fig. 2: Phase plane response of an SMC system

First of all, there is a "reaching" phase in which the system's representative point (RP) trajectory starting from a given initial state $x(0)$ away from the sliding line $\sigma = \beta x_1 + x_2$ moves towards the sliding line. Thus the RP in this phase is sensitive to plant parameter variations and disturbances.

F. Sliding Mode Control Law Equation:

Consider a second order plant described in the controllable canonical form

$$\frac{dx}{dt} = Ax + Bu$$

$$y = Cx$$

Where,

$$A = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{and} \quad C = [1 \ 0]$$

Where,

x is the state vector, u is the control signal, y is the output signal, And a_1 and a_2 are constants.

Let the switching surface is defined as

$$\sigma(x) = \beta x_1 + x_2 = 0$$

Where constant is strictly positive Consider the switching control law

$$\frac{d\sigma}{dt} * \sigma = -(\beta^2 + a_1 - \beta a_2 + \psi_0) \quad \sigma y \leq 0$$

$$\Psi_0 = -\hat{\Psi}_0 \text{sgn}(\sigma)$$

Where

$$\hat{\Psi}_0 > |-\beta^2 + a_1 - \beta a_2|$$

Satisfies the condition

$$\frac{d\sigma}{dt} * \sigma \leq 0$$

on the sliding line the system has first order dynamics $X_1 + \beta X_2 = 0$.

Note that the sliding mode can be obtained with only the prior knowledge of the bound Ψ_0 and that a_1 and a_2 could even be time varying. Thus for the plant in equation the sliding mode control law is given by equations.

III. A MATHEMATICAL MODEL FOR GEOSTATIONARY

A. ORBIT

A mathematical model for geostationary orbit can be defined by the Lagrangian function. Which is applied to the sliding mode control algorithm to stabilize the satellite? U_r, U_θ, U_φ are the thrusters are the satellite which is shown in fig.

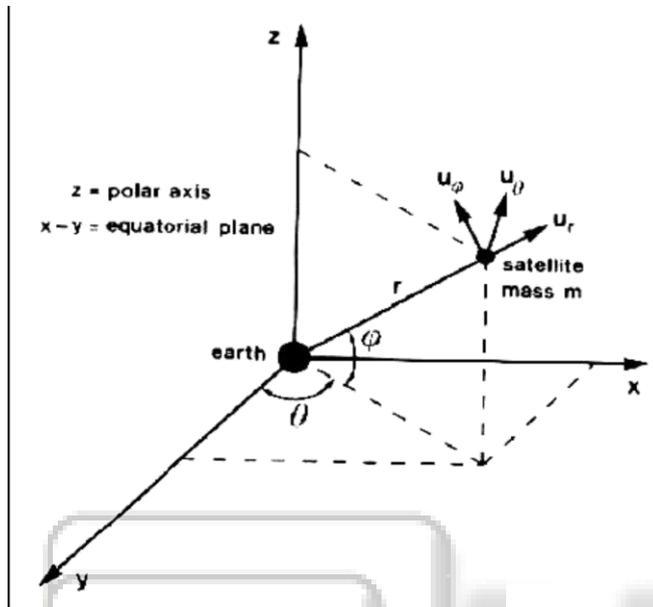


Fig. 3

The Lagrangian function is defined as $L=K-P$ and the dynamic behavior of the system is specified by Lagrange's equations:

The potential energy as:

$$P = -\frac{km}{r}$$

To derive the equations of motion of the system, we express the kinetic energy as:

$$K = \frac{1}{2} m v^2 = \frac{m}{2} \left[\left(\frac{dr}{dt} \right)^2 + \left(r \frac{d\varphi}{dt} \right)^2 + \left(r \frac{d\theta}{dt} \cos\varphi \right)^2 \right]$$

Where K is a known physical constant
N.m² / kg).

$$\begin{aligned} \frac{d}{dt} \left(\frac{dL}{dr} \right) \cdot \frac{dr}{dt} &= U_r \\ \frac{d}{dt} \left(\frac{dL}{d\theta} \right) \cdot \frac{d\theta}{dt} &= r \cos\varphi U_\theta \\ \frac{d}{dt} \left(\frac{dL}{d\varphi} \right) \cdot \frac{d\varphi}{dt} &= r U_\varphi \end{aligned}$$

Define state input and output vectors, $Z(t), V(t)$ and $y(t)$, respectively, as:

$$Z(z,v) = \begin{bmatrix} r(t) \\ \frac{dr}{dt} \\ \theta(t) \\ \frac{d\theta}{dt} \\ \varphi(t) \\ \frac{d\varphi}{dt} \end{bmatrix}, \quad V(t) = \begin{bmatrix} u_r(t) \\ u_\theta(t) \\ u_\varphi(t) \end{bmatrix}, \quad y(t) = \begin{bmatrix} r(t) \\ \theta(t) \\ \varphi(t) \end{bmatrix}$$

$$Y(t) = CZ = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} z$$

Now consider the equation

$$\frac{dX}{dt} = AX + Bu \quad Y = Cx$$

Where,

$$A = \Delta \frac{df}{dz} \quad \text{and} \quad B = \Delta \frac{df}{dv}$$

Thus the linearized and normalized equations of motion in the geostationary orbit are given by

$$\frac{dX}{dt} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 3w^2 & 0 & 0 & 2w & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -2w & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -w^2 & 0 \end{bmatrix} X$$

$$+ \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} x$$

And where the states vector X represents the perturbations about the nominal orbit and u are the forces required to correct the satellite's position. As can be seen, the system is highly nonlinear and multivariable (i.e. it has many inputs and many outputs).

IV. SIMULATION RESULTS AND CONCLUSION

The major aim of the study was to design an optimal sliding mode controller for geostationary communications satellites that have become vital tools of modern global communications. The study begun with a survey of the technical literature on the major parts of communications satellites, the forces that affect satellite orbit and attitude in space as well as the control strategies that have been proposed to maintain correct satellite position and orientation. Started with a brief description of the functions of the following subsystems of communications satellite: attitude and orbit control; telemetry, tracking and command; power supply; and communications electronics and antennas. It was pointed out that geostationary satellites allow use of small and fixed earth antennas in global communications networks. The geostationary orbit is circular, approximately 35,768 km above Earth, and coincides with the equatorial plane. The major factors that cause satellite to change position and attitude in space include: the elliptical shape of Earth around the equatorial plane causes satellites to experience acceleration towards latitudes 75°E and 105°W; variations in the gravitational forces of the moon and sun cause satellites to drift from orbit; solar radiation pressure on the solar panels cause the satellite orbits to be more elliptical than circular. There is increasing congestion of the geostationary arc as more and more countries launch satellites for global and domestic communications. For these reasons there is a growing need for effective and efficient satellite control algorithms. An overview of satellite attitude and orbit control methods available in the technical literature was presented. The essential features of the robust and fast sliding mode control method were presented with an overview of techniques that have been devised to overcome

its major shortcoming of signal chattering in the sliding mode. The Lagrangian method was utilized to derive the orbital dynamic model of a geostationary satellite. The obtained sixth-order state space model comprised highly non-linear and coupled differential equations. The system of equations was normalized such that the nominal mass of the satellite is unity and its nominal orbital radius is unity. The equations were then linearized, using the Taylor series method, about a nominal orbit. Details of the design of sliding mode controllers for such systems were presented. Sliding mode controllers were also presented based on the theories of linear quadratic regulators and sliding mode control. Simulation results revealed that the sliding mode control algorithm employing output feedback alone could not handle the problem of satellite orbit control..

Specifically, the study has achieved the following:

- 1 An up-to-date survey of the technical literature on satellite attitude and orbit control was compiled.
- 2 A state feedback decoupling control law was designed for the system. Sliding mode controllers were designed for the system.
- 3 A structured modular Matlab program was coded for simulation of a satellite system with the designed controllers.

It employs a fourth order Runge-Kutta numerical integration algorithm with fixed step size.

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V. SIMULATION RESULTS

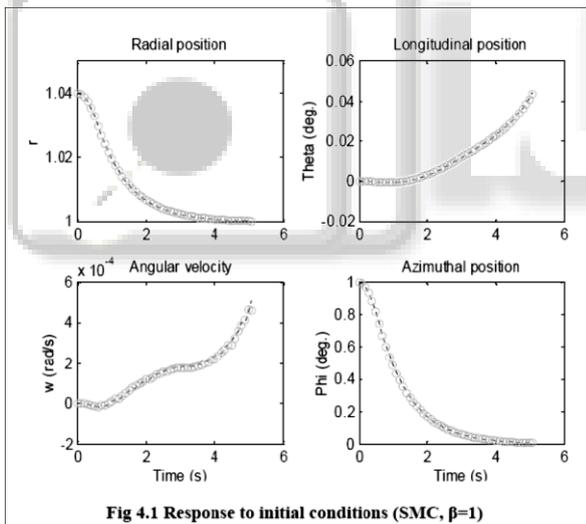


Fig 4.1 Response to initial conditions (SMC, $\beta=1$)

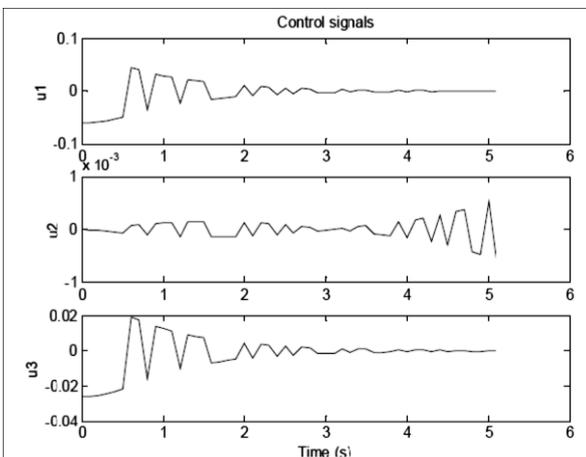


Fig. 4