Efficiency of Hamming Matrix Method as a Test for Isomorphism: A Comparative Study in Generation of EGTs

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Abstract- Any design methodology must ensure identification of all the design alternatives and result in an optimum design. A review and comparison of Characteristic polynomial, Eigen vectors and Eigen value and Hamming number methods is presented. From the adjacency matrix Eigen values are determined for EGTs of given number of links and Degree Of Freedom (DOF). Isomorphism is checked comparing the Eigen values of EGTs. Also characteristic polynomial coefficients are used to detect isomorphism in EGTs that are generated. Keywords: Isomorphism, characteristic coefficients, Eigen values, Hamming number, Rotational graph.

I. INTRODUCTION

Graph theory and Matrix methods are widely used by various kinematicians for synthesis and analysis of EGTs. For analysis and synthesis of EGTs many approaches were introduced by various kinematicians. Identifying the isomorphism for a given number of EGTs generated is the one of challenging problem in structural synthesis to avoid the duplicate graphs.

Different kinematicians [1-24] used adjacency [1], distance [2], flow [3], joint-joint [4] matrices etc to represent graphs of EGTs for analysis and synthesis.

Characteristic polynomial [5], random number technique [6], min-max code [7], identification code [8], link path code [9], Eigen values and Eigen vectors [10-12], genetic algorithm [13], fuzzy logic [14], hamming number technique [15] and loop based hamming [16] etc are the methods used to detect the isomorphism in kinematic chains and EGTs. Each and every method has its own merits and demerits.

A simple EGT with two gear pairs is shown in figure1. Functional [3] representation of a four link EGT with single DOF is shown in fig-1(a). Graphical representation [17] and rotational graph [5,18,19] of the same EGT is shown in Fig-1 (b) and fig-1(c) respectively.

A. *Vertices and Edges:*

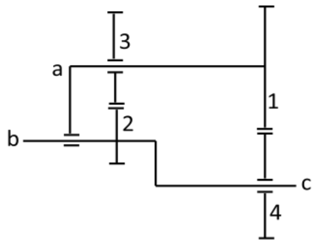

Figure 1(a)

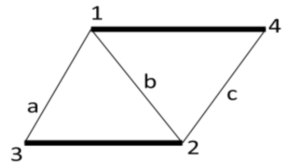

Figure 1(b)

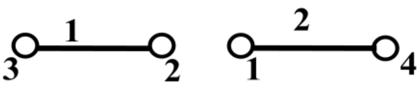

Figure 1(c)

Fig. 1: Representations of EGT

In graph representation vertices indicate the links of a kinematic chain. All pair of joints will become edges connecting the vertices in the graph. All joints are assumed to be binary. There should not be any chance for isolated joint that is all the edges must be connected to at least one vertex.

For mathematical manipulations a graph in turn is represented by a matrix known as linkage adjacency matrix of an EGT, first defined for planar linkages [2] and later modified to suit kinematic chains with gear pairs [5]. Adjacency matrix of a GKC is a Square and symmetric matrix. The adjacency matrix of the EGT shown in figure 1 is given as

$$\text{Adjacency Matrix, A} = \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 2 & 1 \\ 1 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \end{bmatrix}$$

Hamming matrix [15] can be calculated from the adjacency matrix. Hamming matrix for the EGT shown in figure 1 is given as

$$\text{Hamming matrix} = \begin{bmatrix} 0 & 8 & 7 & 5 \\ 8 & 0 & 5 & 7 \\ 7 & 5 & 0 & 6 \\ 5 & 7 & 6 & 0 \end{bmatrix}$$

II. COMPARISON OF METHODS

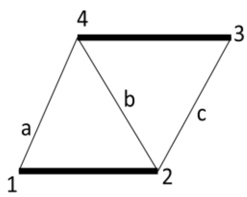

Figure 1(d)

If there exists one to one correspondence between the vertices and edges of any two graphs then the graphs are said to be isomorphic. Depending on the assortment of vertices, two isomorphic graphs may have two different adjacency matrices. .

The adjacency matrix for the EGT shown in figure 1(d) is given as

$$\text{Adjacency matrix} = \begin{bmatrix} 0\,2\,0\,1 \\ 2\,0\,1\,1 \\ 0\,1\,0\,2 \\ 1\,1\,2\,0 \end{bmatrix}$$

The graph shown in figure 1(b) and 1(d) is same but the assortment or naming of the vertices is different. Due to this the adjacency matrix of both the EGTs are different even though they are isomorphic with each other. From this one can say, two isomorphic graphs may not have identical adjacency matrix.

Considering an EGT with seven gear pairs and tested for isomorphism with another two nine link EGTs shown in figures 2, 3 and 4.

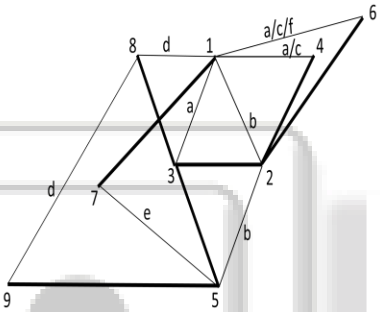

Fig. 2: EGT with 9-links

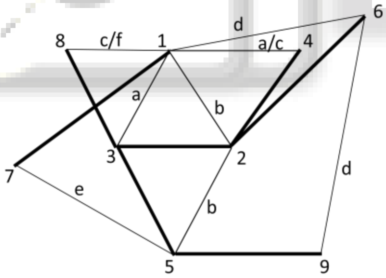

Fig. 3: EGT with 9-links

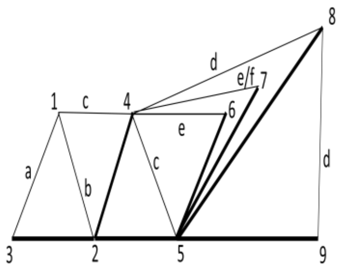

Fig. 4: EGT with seven gear pairs

Characteristic coefficients are used for synthesis of EGTs and results were listed for EGTs of one DOF and with up to six links. However, application of the same method for EGTs with higher number of links gives inaccurate results. Characteristic coefficient of EGT shown in figure 2 is [1.0 0 -36.0 -24.0 303.0 108.0 -779.0 252.0 128.0 0.0] and for the EGT shown in figure 3 is given by [1.0 0.0 -36.0 -24.0 311.0 92.00 -959.0 192.0 588.0 -80.0]. Both the EGTs are not having the identical characteristic coefficient string and accordingly the EGTs are non-isomorphic as per definition [5].

Similarly use of Eigen values and Eigen vectors for the synthesis of EGTs of one DOF and with links not more than six is available in literature. When this method was tested by the authors for EGTs of one DOF and with more than six links the results were found to be not accurate. The Eigen values given for EGT shown in figure 2 and 3 are given and compared as [-4.0048 -3.1759 -2.6256 -0.2740 -0.0000 0.7771 1.3350 2.4391 5.5291] and [-3.8276 -3.3695 -2.5537 -0.7997 0.1340 1.2496 1.6590 1.9762 5.5316] respectively.

The Eigen values of both the EGTs are not identical. And as per the definition [20] if Eigen values of any two EGTs are not identical then the EGTs are said to be non-isomorphic.

If the same two graphs of the EGTs shown in figure 2 & 3 are tested using Hamming strings, hamming string for the EGT in figure 2 is [89 82 76 73 63 59 56 49 49] and the same for the EGT shown in figure 3 is [89 82 76 73 63 59 56 49 49]. Thus the Hamming strings for both of these EGTs are identical. Hence they are isomorphic to each other. This is in contrast to the result obtained above using characteristic coefficients and Eigen values.

Though characteristic coefficients and Eigen values approaches result in two non-isomorphic arrangements, Hamming method asserts that two EGTs are isomorphic. One of these two out comes must be true and not both. Thus to verify the truth of the result, another test must be applied which does not need any further verification. To cross check these results the structural graphs are converted to rotational graphs. If the rotational graphs of two EGTs are identical [5] then the two EGTs are said to be isomorphic. The rotational graphs for the EGTs in figures 2 & 3 are given figures 5 & 6.

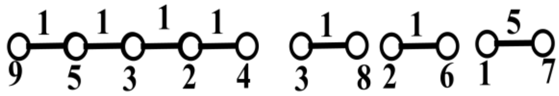

Fig. 5: Rotational graph of an EGT in figure 2.

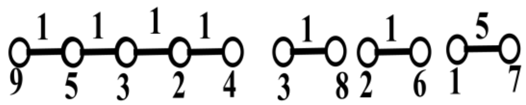

Fig. 6: Rotational graph of an EGT in figure 3

Clearly from the above two figures 5 & 6, rotational diagrams of the two EGTs in figures 2 & 3 are identical. This asserts the fact that these two EGTs are structurally as well as rotationally isomorphic. Hence one may conclude that Hamming string method results in accurate results compared to other methods in test on isomorphism. This asserts that Hamming string method is fool proof and ensures better accuracy in the results. Hamming method is also very simple and easy to adapt with computers.

In this case characteristic coefficients method as well as Eigen value method failed to detect isomorphism in these two geared chains. This is asserted using more number of examples. However it is to be stated here that in case of EGTs with lesser number of links, all the three methods namely Hamming number method, characteristic coefficients and Eigen value methods give results that are in concurrence with the published data, where as with higher number of links the results are not accurate. To give strength to this argument illustrative cases of EGTs with seven and eight links are presented. Figure 7 and figure 8 are the two isomorphic eight link EGTs. Figure 9 and figure 10 are the two seven link EGTs with isomorphism. From the characteristic coefficients and Eigen values methods these EGTs are non isomorphic.

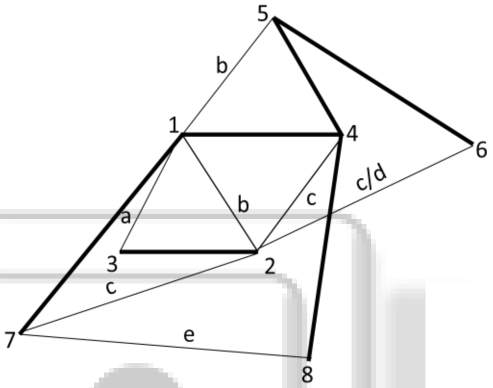

Fig. 7

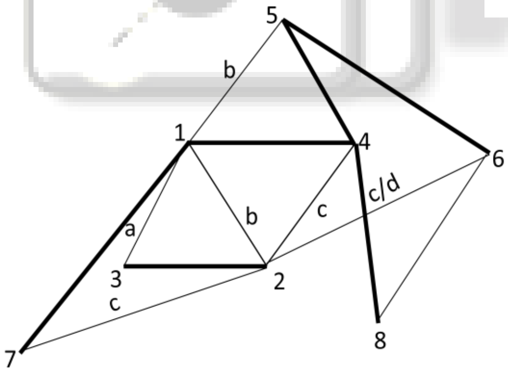

Fig. 8

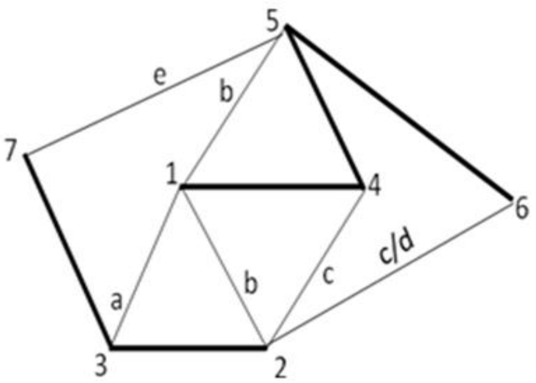

Fig. 9

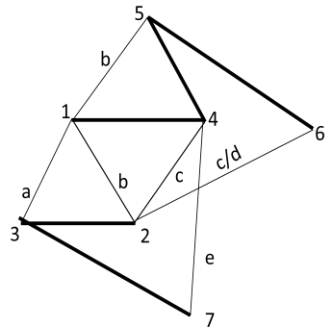

Fig. 10

Fig. 7 – 10 Figures showing Isomorphism

III. CONCLUSION

Graph theory and link adjacency matrix of EGTs are widely used by different researchers for synthesis and analysis of EGTs. A comparison of efficiency of Characteristic polynomial coefficients, Eigen values and Hamming strings is presented. From the adjacency matrix Eigen values are determined for EGTs of given number of links and DOF. Isomorphism is determined by comparing the Eigen values of EGTs. In similar manner characteristic coefficients and Hamming strings are also used to identify isomorphism in the generated EGTs. When Characteristic coefficients and Eigen values are used to detect isomorphism in EGTs, it is observed that in all the cases these two methods may not be give accurate results. However Hamming strings detect isomorphism in EGTs with any number of links and one DOF.

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