PAPR Reduction in DFT- Spread OFDMA using Companding Transform

Minal Jain [1]
[1]Department of Electronics & Communication
[1]U.V.P.C.E.,Ganpat Vidyanagar, Mehsana, Gujarat-384001, India

Abstract— Orthogonal Frequency Division Multiplexing (OFDM) is an efficient transmission technique for high data rate communication systems. The major problem of OFDM system is Peak to Average Power Ratio (PAPR) which reduces the efficiency of the system and increases the system complexity. In this paper, the two different techniques are combined together, so as to give the effective reduction in PAPR for OFDM, namely, DFT spreading and Two Piece-wise Companding (TPWC). DFT spread OFDM (DFTS-OFDM) has been selected as the uplink transmission scheme for long-term 3G evolution (LTE) and TPWC transform can provide significant PAPR reduction with low computational complexity, which is similar to other existing linear transforms. TPWC is proposed so as to compress large signal amplitudes and expand small ones with two different linear functions.

Key words: OFDM, PAPR, DFTS-OFDM, TPWC, LTE.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has gained interest for high-data-rate wireless communications, due to its high spectral efficiency and robustness to frequency selective fading. On the other hand, OFDM suffers from large envelope fluctuation of the transmitted signal, which may occasionally produce a high peak-to-average power ration (PAPR). The high PAPR requires a large dynamic range of both the digital-to-analog (D/A) and analog-to-digital (A/D) converters to cope with the worst-case signal peaks. The high PAPR also requires a wide linear range of the power amplifier (PA) to counteract a nonlinear distortion. Both requirements could cause efficiency problems and would be impractical for most applications. In order to solve these problems, several PAPR reduction techniques have been proposed over the years, such as clipping [5], block coding [6], multiple signal representation [7],[8], tone reservation and injection [10], DFT spreading [1] and companding transform [9]. Here, clipping is the simplest technique. However, it causes significant in-band distortion (IBD) and increases out-of-band radiation (OBR).

DFT-spread OFDM (DFTS-OFDM) is a transmission scheme that can combine the desired properties for uplink transmission i.e.; small variations in the instantaneous power of the transmitted signal (‘single carrier’ property), possibility for low-complexity high-quality equalization in the frequency domain, possibility for FDMA with flexible bandwidth assignment. Due to these properties, DFTS-OFDM has been selected as the uplink transmission scheme for LTE, which is the long-term 3G evolution. Because of the ‘single carrier’ property, it is also known as single carrier FDMA (SC-FDMA) system. As in OFDMA, the transmitters in an SC-FDMA system use different orthogonal frequencies (subcarriers) to transmit information symbols. However, they transmit the subcarriers sequentially, rather than in parallel. Relative to OFDMA, this arrangement reduces considerably the envelope fluctuations in the transmitted waveform. Therefore, SC-FDMA signals have inherently lower PAPR than OFDMA signals.

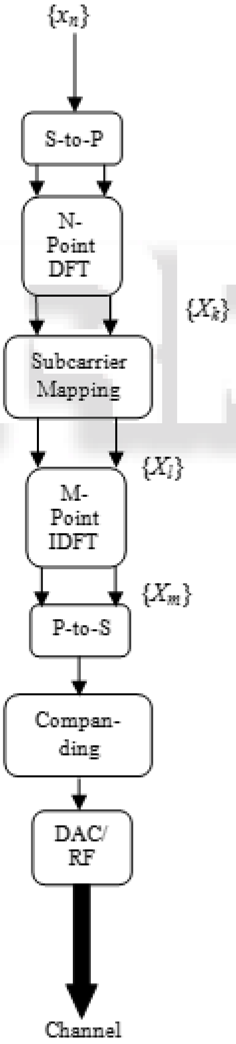

Fig. 1: Transmitter structures of DFTS-OFDMA system with companding transform technique.

Companding transform is also a simple method, which recesses the OFDM signals directly. A μ-law companding transform [11], whose idea came from the use of companding in speech processing. In μ-law transform, the

PAPR was reduced by increasing the average signal power while keeping the peak power unchanged. However, under certain system constraints, the PAPR reduction was limited. In [12], four typical linear and nonlinear companding transforms were investigated, where a linear nonsymmetrical transform (LNST) was the best in terms of both the PAPR reduction and BER performances. In the LNST, small and large signal amplitudes were linearly transformed with different scales, which were achieved by introducing an inflexion point. An important nonlinear companding transform was proposed in [13], named Exponential Companding (EC) transform, where an original Gaussian-distributed OFDM signal was transformed into a uniform-distributed signal. Especially, the input and output of the EC transform were kept at the same average power level. However, compared to the LNST, more complex computation was required. Moreover, the impact of filtering out the OBR in companding transforms, which had been neglected in the earlier reports. In the TPWC transform, similarly to the LNST, small signal amplitudes are linearly transformed only with a scale and, differently to the LNST, large amplitudes are linearly transformed not only with a scale but also with a shift. There exist no inflexion points in the resulting profile. By choosing carefully the two scales and the shift, a good PAPR reduction performance can be achieved with low computational complexity can be realized without changing the average power level.

II. SYSTEM MODEL

At the input to the transmitter, shown in Figure 1, a baseband modulator transforms the binary input to a multilevel sequence of complex numbers x_n in one of several possible modulation formats including binary phase shift keying (BPSK), quaternary PSK (QPSK), 16-level quadrature amplitude modulation (16-QAM) and 64-QAM. The system adapts the modulation format, and thereby the transmission bit rate, to match the current channel conditions of each terminal. The transmitter next groups the modulation symbols, x_n into blocks each containing N symbols. The OFDM samples suffers from large amplitude fluctuation, which can be measured by the PAPR, defined as follows

$$PAPR = \frac{|x|_{peak}{}^2}{x_{rms}{}^2} \qquad (1)$$

Note that the PAPR is a random variable, whose performance can be evaluated by its complement cumulative distribution function (CCDF). The CCDF is defined as the probability of a random variable exceeding a given threshold, Here, it can be written as

$$\text{CCDF} = \Pr\{\text{PAPR} > \text{PAPR0}\} \qquad (2)$$

Moreover, it has been proved in [15] that, for an oversampling rate greater than four, the PAPR of a continuous OFDM signal can be approximated by that of the OFDM samples.

The first step in modulating the SC-FDMA subcarriers is to perform an N-point discrete Fourier transform (DFT), to produce a frequency domain representation X_k of the input symbols. It then maps each of the N DFT outputs to one of the ($M > N$) *orthogonal* subcarriers that can be transmitted. As in OFDMA, a typical value of M is 256 subcarriers and $N = M/Q$ is an integer submultiple of M. Q is the bandwidth expansion factor of the symbol sequence. If all terminals transmit N symbols per block, the system can handle Q simultaneous transmissions without co-channel interference. The result of the subcarrier mapping is the set X_l ($l = 0, 1, 2. . . M - 1$) of complex subcarrier amplitudes, where N of the amplitudes are non-zero. As in OFDMA, an M-point inverse DFT (IDFT) transforms the subcarrier amplitudes to a complex time domain signal Xm. Each Xm then modulates a single frequency carrier and all the modulated symbols are transmitted sequentially.

In order to reduce the PAPR, we introduce a companding transform after the parallel to serial conversion in the system. Let C [·] denotes the companding function that changes only amplitudes of the input signals. After companding, the samples can be denoted as

$$y_n = C\,[|X_m|].sign\,(X_m) \qquad (3)$$

Likewise, C [.] denotes companding transform satisfying the following two conditions [11]:

$|C\,[X_m]| \geq |X_n|$ when $|X_m| \geq m$, and $|C\,[X_m]| < |X_m|$ otherwise;

$E\,\{|X_m|^2\} \approx E\,\{|C\,[X_m]\,|^2\}$

where, m denotes the inflexion value of the transform and ***E* {.}**, the expectation. It is found that, with appropriate selection of companding form and its corresponding inflexion point, significant reduction of PAPR can be achieved, with low complexity. The average power of transmitted signals will be unchanged after the companding transform if the average power is set as the value of the transform inflexion point and the companding form is odd symmetrical with reference to the inflexion point.

Next, the companded samples are converted into analog waveforms by a D/A converter Note that the effect of the D/A converter can be neglected, if its word length is long enough. Finally, the generated OFDM signals are transmitted into the radio channel.

III. DFTS-OFDM

Several approaches to mapping transmission symbols X_k to SC-FDMA subcarriers are currently under consideration. They are divided into two categories; *distributed* and *localized* [1]. In the distributed subcarrier mapping mode, DFT outputs of the input data are allocated over the entire bandwidth with zeros occupying the unused subcarriers resulting in a non-continuous comb-shaped spectrum. *Interleaved* SC-FDMA (IFDMA) is an important special case of distributed SC-FDMA. In contrast with IFDMA, consecutive subcarriers are occupied by the DFT outputs of the input data in the localized subcarrier mapping mode resulting in a continuous spectrum that occupies a fraction of the total available bandwidth.

For IFDMA, time symbols are simply a repetition of the original input symbols with a systematic phase rotation applied to each symbol in the time domain. Therefore, the PAPR of IFDMA signal is the same as in the case of a conventional single carrier signal. In the case of LFDMA, the time signal has exact copies of input time symbols in N conventional single carrier signal. In the case of LFDMA, the time signal has exact copies of input time symbols in N sample positions. The other M-N time samples are weighted sums of all the symbols in the input block.

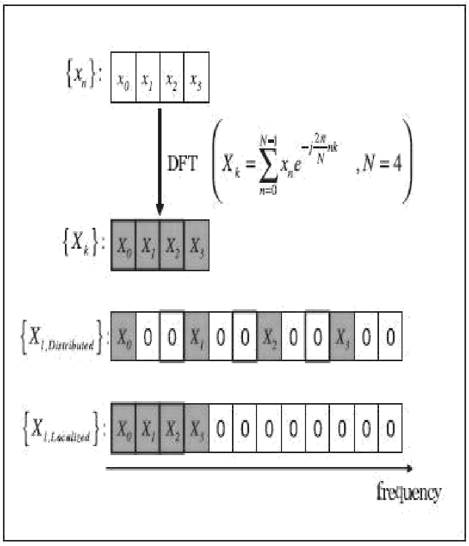

Fig. 2: An example of SC-FDMA transmit symbols in the frequency domain for N=4 subcarrier per user, Q=3 users, and M=12 subcarriers in the system.

$X_{l,Distributed}$ denotes transmit symbols for distributed subcarrier mapping scheme and $X_{l,Localized}$ denotes transmit symbols for localized subcarrier mapping scheme.

IV. TWO PIECE-WISE COMPANDING TRANSFORM

It can be found that small amplitudes occur with high probability, while in contrast large ones occur with low probability. Intuitively, without changing the average power, the expanding of small amplitudes will have great influence on the BER performance, and the compressing of large ones will provide the capability of PAPR reduction.

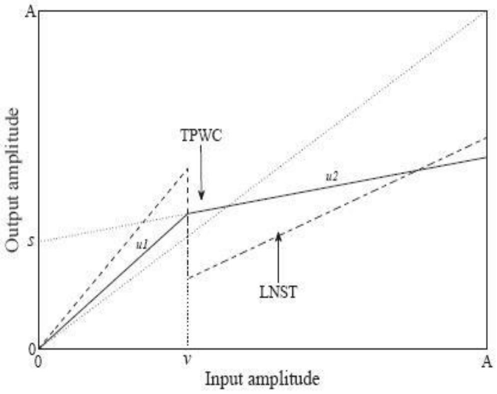

Fig. 3: The proposed Companding transform.

In order to do trade-off between BER and PAPR reduction performances, small and large amplitudes should be treated individually. In the TPWC transform, small amplitudes ($|x_n| \leq v$) are multiplied by a scale factor u_1, while large ones ($|x_n| > v$) are not only multiplied by a scale factor u_2 but also added by a shift s. Moreover, the cut-off point of the amplitude is chosen as $v = s / (u_1 - u_2)$, so that there is no inflexion point. The resulting profile is piecewise-linear and continuous, as shown in Figure 3 where the profile of LNST is also given for comparison. It can be found that both of them have a similar operation, which will result in a similar computational complexity. The proposed companding function is defined as follows, [9]:

$$y_n = f(x) = \begin{cases} u_1|x_n|.sgn(x_n), & |x_n| \leq v \\ (u_2|x_n| + s).sgn(x_n), & |x_n| > v \end{cases} \qquad (4)$$

Where, $u_1 > 1$, $1 > u_2 > 0$ and $s > 0$.

V. PAPR ANALYSIS OF DFTS-OFDM WITH TPWC

Since, the PAPR is reduced already by using DFT spreading technique for OFDM signal. Therefore, overall symbol power is also reduced for the original OFDM. So, the value of m is chosen such that the overall CCDF of the PAPR will reduce for DFT-S-OFDM with TPWC, where (m>0). The value of parameter m which gives the appropriately best results ranges from 0.013 to 0.05, approximately, whereas u_1 & u_2 remain the same and assuming $v = m\sigma$ [9].

This can be seen in Figure 4 and Figure 5 that there is more PAPR reduction occurs when we use TPWC transform after DFT spreading for OFDM system. The CCDFs of the PAPR for LFDMA & IFDMA original and TPWC transformed LFDMA & IFDMA are reduced by 4 dB more when CCDF = $10^{(-2)}$.

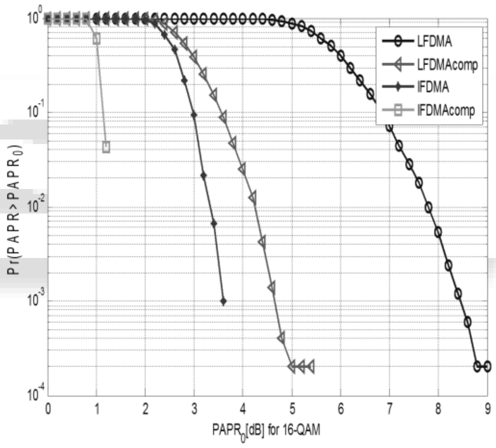

Fig. 4: Comparison of LFDMA & IFDMA with companding and without companding transform technique for 16-QAM

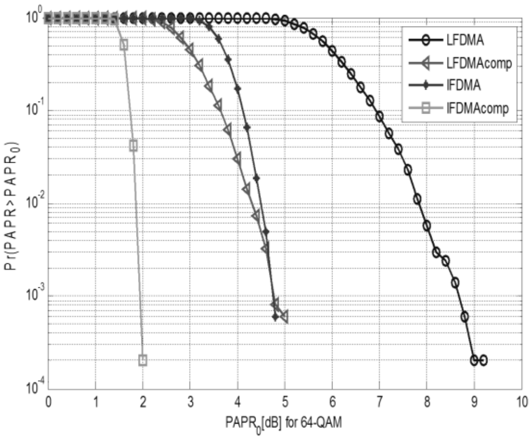

Fig. 5: Comparison of LFDMA & IFDMA with companding and without companding transform technique for 64-QAM.

VI. CONCLUSION

SC-FDMA is a promising technique for high data rate uplink communication in future cellular systems. Within a specific SC-FDMA system configuration, there are many design and operational choices that affect performance in a complex manner.

We have proposed and evaluated the SC-FDMA or DFTS-OFDMA with TPWC transform in this paper. The TPWC transform is an effective technique for PAPR reduction of OFDM signals, due to its low computational complexity and no constraint on system. So, by combining these two techniques we can get overall good PAPR performance. In future we can use an extension of the TPWC transform by modifying the companding function for large amplitudes.

REFERENCES

[1] Hyung G. Myung, Junsung Lim, and David J. Goodman, ”Single Carrier FDMA for Uplink Wireless Transmission”, IEEE Vehicular Technology Magazine, pp. 30-38, Sep-2006.

[2] M. M. Rana, Md. Saiful Islam2 and Abbas Z. Kouzani, “Peak to Average Power Ratio Analysis for LTE Systems”, IEEE 2010 Second International Conference on Communication Software and Networks.

[3] Juraj Gazda, Peter Drot´ar, Pavol Galajda, Duˇsan Kocur, “Comparative evaluation of OFDMA and SC-FDMA based transmission systems”, 8th IEEE International Symposium on Applied Machine Intelligence and Informatics, pp. 28 -30, jan-2010.

[4] S. H. Han and J. H. Lee, “An overview of peak-to-average power ratio reduction techniques for multicarrier transmission,” IEEE Wireless Commun. Mag., vol. 12, no. 2, pp. 56–65, Apr. 2005.

[5] H. Ochiai and H. Imai, “Performance analysis of deliberately clipped ofdm signals,” IEEE Trans. Commun., vol. 50, no. 1, pp. 89–101, Jan. 2002.

[6] K. G. Paterson and V. Tarokh, “On the existence and construction of good codes with low peak-to-average power ratios,” IEEE Trans. Inf. Theory, vol. 46, no. 6, pp. 1974–1987, Sep. 2000.

[7] S. H. Muller and J. B. Huber, “Ofdm with reduced peak-to-average power ratio by optimum combination of partial transmit sequences,” Electron. Lett. vol. 33, no. 5, pp. 368–369, Feb. 1997.

[8] M. Breiling, S. H. Muller, and J. B. Huber, “Slm peak - power reduction without explicit side information,” IEEE Commun. Lett. vol. 5, no. 6, pp. 239–241, Jun. 2001.

[9] Pinlu Yang and Aiqun Hu, “Two-Piecewise Companding Transform for PAPR Reduction of OFDM Signals,” IEEE 2011, pp. 619-623.

[10] B. S. Krongold and D. L. Jones, “An active-set approach for ofdm par reduction via tone reservation,” IEEE Trans. Signal Process, vol. 52, no. 2, pp. 495–509, Feb. 2004.

[11] Xiao huang, Jianhua Lu, Junli Zheng, J. Chuang and Jun Gu , “Reduction of peak-to-average power ratio of ofdm signals using a companding transform,” IEE 2001, Electronics Letter, vol. 37, no. 8, April. 2001.

[12] X. Huang, J. Lu, K. B. Letaief, and J. Gu, “Companding transform for reduction in peak-to-average power ratio of ofdm signals,” IEEE Trans. Wireless Commun., vol. 3, no. 6, pp. 2030–2039, Nov. 2004.

[13] T. Jiang, Y. Yang, and Y. H. Song, “Exponential companding technique for papr reduction in ofdm systems,” IEEE Trans. Broadcast., vol. 51, no. 2, pp. 244–248, Jun. 2005.

[14] N. Chaudhary and L. Cao, “Non-symmetric decompanding for improved performance of companded ofdm systems,” IEEE Trans. Wireless Commun., vol. 6, no. 8, pp. 2803–2806, Aug. 2007.

[15] C. Tellambura, “Computation of the continuous-time par of an ofdm signal with bpsk subcarriers,” IEEE Commun. Lett. vol. 5, no. 5, pp. 185–187, May 2001.