Buckling Analysis of Functionally Graded Material Plates

Lekshmi A J[1] Midhula R Krishna[2]
[1]M Tech Student [2]Assistant Professor
[1,2]Department of Civil Engineering
[1,2]Rajadhani Institute of Engineering and Technology, Nagaroor, Kerala, India

Abstract— High performance, microscopically non - homogeneous functionally graded materials (FGMs) have tailored composition and structure gradients with specified attributes in the preferred orientation. On the macroscopic scale, FGMs are inhomogeneous materials because their properties vary progressively with position within the material. The qualities can be varied by progressively modifying the volume percentage of the constituent materials, utilising reinforcements with different properties, sizes, and forms, continuously interchanging the roles of reinforcement and matrix phases, and tweaking the cell structure. The buckling and post-buckling strength are also affected by the volume fraction exponent. To design various structural sections of aeronautical, mechanical, marine and civil constructions, accurate structural and dynamic analyses are required to identify the behaviour of them structural reaction in real time. The current research focused mostly on the buckling analysis of FGM plates. FEAST^{SMT}, finite element software developed by Vikram Sarabhai Space Centre (VSSC) and Indian Space Research Organization, was used to analyse the geometries under consideration (ISRO).

Keywords: Buckling, Functionally graded material (FGM), Critical load, FEAST^{SMT}.

I. Introduction

Functionally graded materials (FGMs) are composite materials deliberately designed so that they possess expedient properties for specific applications, especially for space vehicles, aircrafts and other engineering structures under high temperature environment. The material properties of FGMs, which are heterogeneous composite materials, vary constantly from top to bottom thanks to gradually changing volume fraction indices of the constituent elements. Young's modulus, shear modulus, Poisson's ratio, and material density are the preferred mechanical characteristics of FGMs. The major advantage of using these materials is that they can survive the high thermal gradient environment, while maintaining their structural integrity. FGMs were first intended for use in fusion reactors and aerospace structural applications as heat barrier materials. FGMs set themselves apart from other conventional composite materials by continuously changing their microstructure. Traditional composite materials typically fail under excessive mechanical and thermal stress conditions through a process known as delamination. The very common available FGMs are ceramic-metal composites, where the ceramic part has good thermal and corrosion resistance capability and metallic part provides superior fracture toughness and weldability. The idea of FGM was first introduced by a

Japanese group of scientists and researchers. The establishment of the concept of FGM originated in various research branches: mechanical plate model applications, vibration and control applications, electrical system analysis, fracture analysis, thermal stress analysis etc. With the progress in manufacturing methods, the FGMs were taken into account in the sandwich structure industries. In general, the abrupt change in the material properties of sandwich structure from one layer to another can result in stress concentrations which often lead to delamination. To overcome this problem, the FG sandwich structure is proposed because of the gradually happening material characteristics at the interfaces between the face layers and the core. The effective properties of macroscopic homogeneous composite materials can be obtained from the microscopic heterogeneous material structures using homogenization techniques. Numerous approaches, such as the mixture rules (Voigt Scheme), Hashin-Shtrikman type bounds, Mori-Tanaka the boundaries of the effective properties can be determined using type models and self-consistent techniques. Voigt scheme and Mori-Tanaka schemes are generally adopted in analysis of FGM plates. FGM processing techniques can be divided into two groups based on mass transfer and constructive preparation. In the first category, the FGM is constructed layer by-layer starting with an apposite distribution in which the gradients are literally constructed in space. The advantage of this technique was to fabricate unrestricted number of gradients. Meanwhile in the second category the gradients within a components are depend on natural transport phenomenon such as the diffusion of atomic species, or heat conduction, flow of fluid. However, developments in automation technology over the past two decades have made technological and financial viability of constitutive gradation processes possible.

The main objectives of the study are given as follows:

1) To understand influence of material composition, plate geometry and twist angle in buckling characteristics.
2) To determine buckling behaviour of cantilever twisted functionally graded material plate using FEAST^{SMT} software.

II. Literature review

B A Samsam Shariat and M R Eslami (2007) developed the thermal and mechanical buckling investigation of thick FG plates. The plate was graded linearly through the thickness direction. The boundary conditions were assumed to be simply supported at ends. Employing an unique method equilibrium equations were reduced from two to five equations. The corresponding stability equations were established which yield to the buckling relations. M Bodaghi and A R Saidi (2010) presented an analytical approach for buckling analysis of thick functionally graded rectangular plates. The equilibrium .and stability equations were derived according to the higher order shear deformation plate theory. 5 coupled partial differential equations of FG plates were converted into 2 uncoupled equations in terms of boundary layer function and transverse displacement. These equations were solved for FG rectangular- plate with two opposite edges simply supported subjected to different types of in-plane loading. By imposing different boundary conditions along two other edges, the critical buckling loads were obtained for FG rectangular plate. The effects of power of FGM, aspect ratio, plate thickness, boundary conditions and loading types on the critical buckling load of the FGM plate were studied. Meisam Mohammadi et al., (2010) studied the buckling analysis of thin FG rectangular plates. Based on the classical plate theory and using the principle of minimum total potential energy the equilibrium equations were obtained. The resulting equations were decoupled and solved for different loading conditions. By imposing different boundary conditions along two opposite edges, the critical buckling loads were obtained for FG rectangular plate. The critical buckling loads for a FG plate with different boundary conditions and loading conditions, aspect ratios and various powers of FGM were studied. The critical buckling mode may change as the boundary conditions or aspect ratio changes, however for a biaxial compressed plate or for a plate which was subjected to uniaxial compressive load in y direction; the critical buckling mode was the first mode.

Huu Tai Thai and Dong Ho Choi (2012): presented an efficient and simple refined theory for buckling analysis of FG plates. The theory which has strong similarity with classical plate theory (CPT) in many aspects. Material properties of FG plates, such as Young's modulus, were assumed to vary through the plate thickness with a power law distribution of the volume fraction of the constituents whereas the Poisson's ratio was constant. Governing equations were developed from the principle of minimum total potential energy. Closed-form solutions of rectangular plates were obtained. Comparison studies were performed to verify the validity of the present results. The effects of loading conditions and modulus ratio, variations of power of FGM, thickness ratio and aspect ratio on the critical buckling load of FGP were investigated and discussed. S.A.M. Ghannadpour et al., (2012) presented a finite strip method for analysing the buckling behavior of rectangular functionally graded plates (FGPs) under thermal loadings. The material properties of FGPs were assumed to vary continuously through the thickness of the plate, according to the simple power law distribution. After convergence studies, numerical results for a variety of FGP with different boundary conditions are given and compared with the available results. Additionally, the effects of geometrical parameters and material properties on the buckling temperature difference of FGM plates were determined and discussed. Derivations of equations were based on the CPT. The fundamental equations for rectangular plates of FGM were obtained by discretizing the plate into some finite strips.

III. Buckling Analysis of FGM Plates

Aluminium/Alumina is selected as the material of the FGM plate. Material properties varies according to power law distribution that is expressed as,

$$E = E_m + (E_c - E_m)\left(\frac{2z+h}{2h}\right)^k \quad (3.1)$$

Here m and c represents metal and ceramic constituents, z is the distance from the centre of layer under consideration to the centre of plate in which $-h/2 \le z \le h/2$ and k is the power law index. The material properties of the FGM plate are shown in table 1.

Aluminium	Alumina
Modulus of elasticity E = 70 GPa	Modulus of elasticity E = 380 GPa
Poisson's ratio ν = 0.3	Poisson's ratio ν = 0.3
Mass density ρ = 2707 kg/m^3	Mass density ρ = 3800 kg/m^3

Table 1: Properties of FGM Plate

It is investigated how different parameters affect the buckling of pre-twisted FGM plate. First step is to build a model of a cantilever FGM plate using FEAST. First a flat cantilever FGM plate will be modelled and buckling behaviour will be analysed then a twisted cantilever FGM plate will be modelled and analysed its behaviour subjected to compressive loads. Table 2 presents the results, and Figure 1 illustrates them graphically for the first buckling mode. It is observed that the buckling load increases with increasing angle of twist. The first and second buckling modes of untwisted and 15^0 twisted plates are shown infigure 2 and figure 3. Non-dimensional buckling load is given by the equation,

$$\lambda = \frac{N_{cr} b^2}{E_m h^3} \quad (3.2)$$

Angle of twist	Buckling load N_{cr} (MN)
0	41.22
10	40.645
15	39.7
20	38.6
30	35.933

Table 2: Variation in Buckling Load with Different Twist Angles (A/B =1, A/H = 10, N=1)

The first buckling mode of untwisted and 15^0 twisted plates is shown infigure 2 and 3.

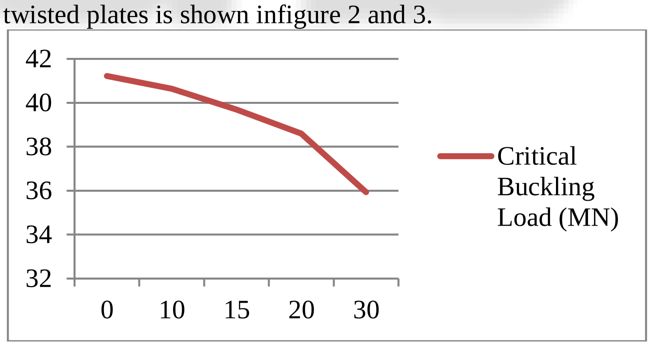
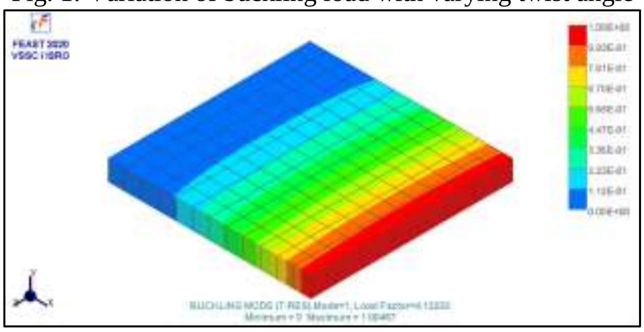


Fig. 1: Variation of buckling load with varying twist angle



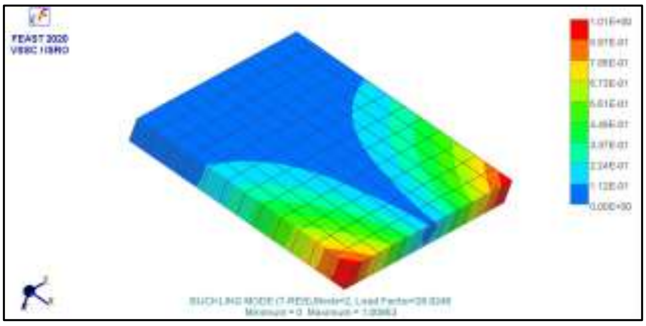


Fig. 2: Buckling modes of untwisted plate

For a 15° pre-twisted plate made of Al/Al2O$_3$ FGM, the variation of non-dimensional buckling load with increasing gradient index is examined. Eleven arbitrary values of the power law index n are considered. Table 4 provides the outcomes that were obtained. According to the findings, the gradient index increases when the non-dimensional buckling load decreases. This is because the critical buckling force is higher when the gradient index is 0 since the plate will be entirely made of ceramic, which is quite stiff. The critical buckling load continues to decrease as it rises along with the metal content of the plate, which results in reduced stiffness. When the gradient index reaches infinity, the plate will be completely metallic which is less stiff than ceramic and thus critical buckling load isless. The variation is shown in the figure 4 and table 4.

In Tables 2-4, the buckling loads for various scenarios are computed and displayed. As can be observed, FGM plates have a higher buckling strength than totally metal aluminium plates but a lower buckling strength than fully ceramic alumina plates. Additionally, it can be seen that the effect of the volume fraction exponent is more pronounced up to a value of 2, beyond which it becomes negligible.

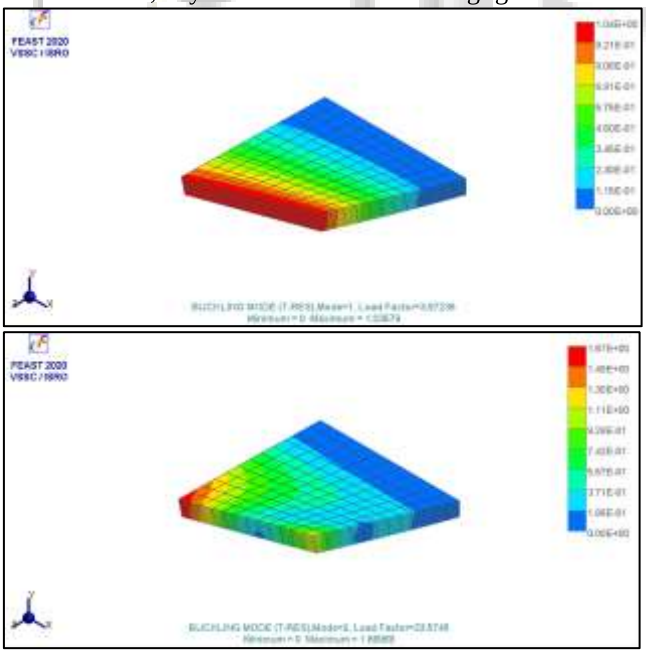


Fig. 3: Buckling modes of twisted plate

Aspect ratio (a/b)	Buckling load (MN)
0.5	77.78
1	39.7
1.5	26.7345
2	20
3	13.26

Table 3: Variation of Buckling Load with Aspect Ratio (a/b =1, a/h = 10)

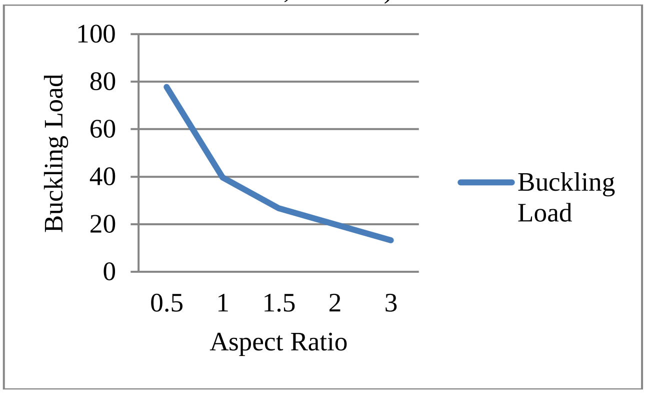


Fig. 4: Variation of buckling load with aspect ratio

It may provide some guidance in selecting the volume fraction exponent when creating FGM plates for desired results.

Gradient Index (n)	Non dimensional Buckling load
0 (Alumina)	1.133
1	0.567
2	0.442
5	0.372
10	0.333
20	0.287
30	0.262
40	0.242
50	0.231
100	0.212
∞ (Aluminium)	0.211

Table 4: Variation of the Non-Dimensional Buckling Load with Different Gradient Index (a/b =1, a/h = 10)

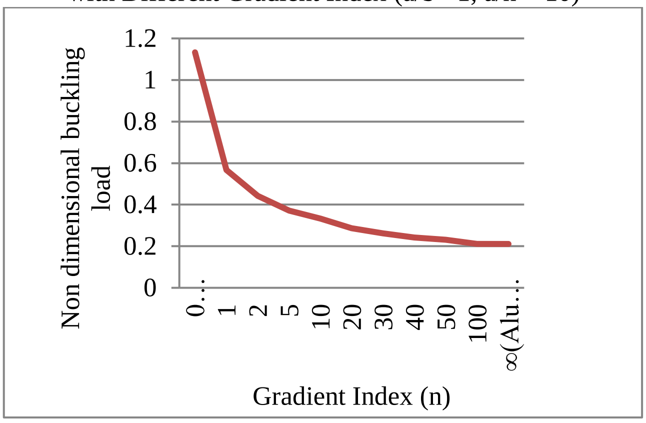


Fig. 5: Variation of non-dimensional buckling load with varying gradient index

IV. CONCLUSION

- The non-dimensional buckling load reduces as the angle of twist increases.
- The non-dimensional buckling load of a twisted FGM plate significantly lowers as the aspect ratio (a/b) rises. This is so that the stiffness decreases as the aspect ratio grows and the length of the plate in the direction of the in-plane compression force similarly increases. As a

result, less critical buckling load is needed to cause critical buckling.

- With an increase in the material index of a pre-twisted FGM plate, the non-dimensional buckling load reduces. This is due to the fact that as the material gradient increases, the metal content of the plate similarly rises, but the ceramic content falls. As a result, the plate becomes less stiff, and the critical buckling force reduces as the gradient index rises.

REFERENCES

[1] A. Hajlaoui, A. Jarraya, K. El Bikri and F. Dammak, “Buckling analysis of functionally graded materials structures with enhanced solid-shell elements and transverse shear correction," *Composite Structures*, Vol. 132, pp. 87-97, 2014.
[2] Ahmed Amine Daikh and Abdelkader Megueni, “Thermal buckling analysis of functionally graded sandwich plates,” *Journal of thermal stresses*, 2017.
[3] A.M. Zenkour and M. Sobhy, “Thermal buckling of various types of FGM sandwich plates,” *Composite structures*, vol. 93, pp. 93-102, 2010.
[4] B.A Samsam Shariat and M. R Eslami, “Buckling of thick functionally graded plates under mechanical and thermal loads,” *Composite Structures*, Vol. 78, pp.433-439, 2007.
[5] Huu Tai Thai and Dong Ho Choi, “An efficient and simple refined theory for buckling analysis of functionally graded plates,” *Applied Mathematical Modelling*, vol. 36, pp. 1008-1022, 2012.
[6] I Ramu and S.C. Mohanty, “Buckling analysis of rectangular functionally graded material plates under uniaxial and biaxial compression load,” *Procedia Engineering*, Vol. 86, pp.748-757, 2014.
[7] M Bodaghi and A.R. Saidi, “Levy type solution for buckling analysis of thick functionally graded rectangular plates based on the higher order shear deformation plate theory,” *Applied Mathematical Modelling*, Vol. 34, pp. 3659-3673, 2010.
[8] Meisam Mohammadi, Ali Reza Saidi and Emad Jomehzadeh, “Levy solution for buckling analysis of functionally graded rectangular plates,” *Applied composite materials*, vol. 17, pp. 81-93, 2009.
[9] S.A.M. Ghannadpour, H.R. Ovesy and M. Nassirnia, “Buckling analysis of functionally graded plates under thermal loadings using the finite strip method,” *Computers and Structures*, pp. 108-109, 2012.
[10] Serge Abrate, “Free vibration, buckling, and static deflections of functionally graded plates,” *Composites Science and Technology*, vol. 66, pp. 2383-2394, 2006.

