Bulk Viscous Bianchi-V Universe with Decaying Vacuum Energy

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Abstract— We present the Bianchi type-V universe with bulk viscosity in general relativity. Exact solutions of the field equations are obtained under the consideration of (i) Hubble parameter of the form $H = \alpha R^{-m}, a > 0, m \geq 0$,,(ii) bulk viscosity of the form $\zeta = \zeta_0 p$, where ζ_0 are positive constant and p is the energy density. (iii) and cosmological term $\Lambda = 3\beta H^2, \beta > 0$. The model obtained evolves with decelerating expansion throughout the evolution. Physical and kinematical behavior of the model are also discussed.

Keywords: Bianchi V Universe, Bulk viscous fluid, Cosmological constant

I. INTRODUCTION

The universe at large scale is homogeneous and isotropic and the accelerating phase of universe [1]. It is well known that the exact solutions of general theory of relativity for homogeneous space times belongs to Bianchi types. Taking into account dissipative process due to viscosity, the nature of cosmological solutions for homogeneous Bianchi type-model has considered by Belinski and Khalatnikov[2] They showed that the viscosity can not remove the cosmological singularity but results in a qualitatively new behaviour of the solutions near singularity. They found the remarkable property that during the time of the big bang matter is created by the gravitational field. Bianchi type-I model with bulk viscosity a power function of energy density when the universe is filled with stiff matter were studied by Banerjee[3]and Huang [4]. The effect of bulk viscosity with a time varying bulk viscous coefficient, on the evolution of isotropic FRW models in the context of open thermodynamics system was studied by Desikan [5].

Exact solutions for homogeneous anisotropic models are not much known in the literature. There are, however, a few [6, 7] which utilize certain simplifying assumptions to get exact solutions at the cost of a physically reasonable equation of state. Belinskii and Khalatnikov [8] assumed an equation of state of the form $\rho \propto p$, but did not give any exact solution. They have, however, investigated some general features of the isotropic and anisotropic homogeneous cosmological models in the presence of bulk as well as shear viscosity in asymptotic limits. We consider in this paper the Bianchi I model with a fluid characterized by both bulk and shear viscosity and having an equation of state $\rho \propto p$. The viscosity coefficients are further assumed to be power functions of the matter density as suggested by Belinskii and Khalatnikov [8]. Exact solutions in several particular cases for stiff fluid $\rho = p$ are worked out.

Bulk viscosity is associated with GUT phase transition and string creation. The model studied by Murphy [9] has an interesting feature that the big bang type of singularity of infinite space-time curvature does not possess a finite past. However, the relationship assumed by Murphy between the viscocity coefficient and the matter density is not acceptable at large density. The effect of bulk viscosity on cosmological evolution has been investigated by number of authors in the frame work of general relativity Johri and Sudershan [10], Maartens [11], Zimdahl [12]. This motivates the study of cosmological bulk viscous fluid model.

In this paper, we have studied Bianchi type-V cosmological model with v bulk viscous and variable cosmological term by taking the condition functional relation on Hubble parameter H. The solution and discussion of field equations are summarized in last section.

A. *Metric and Field Equations:*

We consider the Bianchi type-V space-time given by the line-element

$$ds^2 = -dt^2 + A^2(t)dx^2 + e^{2x}\{B^2(t)dy^2 + C^2(t)dz^2\} \quad ... (1)$$

We assume the cosmic matter consisting of viscous fluid represented by the energy-momentum tensor

$$T_{ij} = (\rho + \bar{p})v_i v_j + \bar{p}\, g_{ij} \quad ... (2)$$

where $\bar{p}$ is the effective pressure given by

$$\bar{p} = p - \zeta\, v^i{}_{;i}\,, \quad ... (3)$$

$$p = w\rho\,, \qquad 0 \leq w \leq 1 \quad ... (4)$$

The Einstein field equation (is gravitational units 8πG = C = 1) with time varying cosmological term Λ (t) are

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} + \Lambda g_{ij} \quad ... (5)$$

For the line-element (1), the field equations (5) in comoving system of coordinates lead to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{1^2}{A^2} = \Lambda - \bar{p} \quad ... (6)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} = \Lambda - \bar{p} \quad ... (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = \Lambda - \bar{p} \quad ... (8)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3}{A^2} = \Lambda + \rho \quad ... (9)$$

$$\frac{2\dot{A}}{A} = \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \quad ... (10)$$

Vanishing divergence of Einstein tensor $R_{ij} - \frac{1}{2}R\, g_{ij}$ gives rise to

$$\dot{\rho} + (\rho + \bar{p})\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \dot{\Lambda} = 0 \quad ... (11)$$

We define average scale factor R for Bianchi V universe as

$$R^3 = ABC \quad \text{... (12)}$$

In analogy with FRW universe, we define generalized Hubble parameter H and deceleration parameter q as

$$H = \frac{\dot{R}}{R} = \frac{1}{3}(H_1 + H_2 + H_3) \quad \text{... (13)}$$

and

$$q = -\frac{\ddot{R}}{RH^2} \quad \text{... (14)}$$

where $H_1 = \dot{A}/A, H_2 = \dot{B}/B, H_3 = \dot{C}/C$ are directional Hubble factors along x, y and z directions respectively.

We introduce volume expansion θ and shear scalar σ for the Bianchi V metric as

$$\theta = v^i_{;i} \quad \text{... (15)}$$

and

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \quad \text{... (16)}$$

For the metric (1), we have

$$\theta = 3\dot{R}/R \quad \text{... (17)}$$

and

$$\sigma = K/R^3 \quad \text{... (18)}$$

where K is an integration constant.

Equations (6) – (9) can be expressed in terms of H, σ and q as

$$p - \zeta\theta - \Lambda = (2q-1)H^2 - \sigma^2 + 1/R^2 \quad \text{... (19)}$$

$$\rho + \Lambda = 3H^2 - \sigma^2 - 3/R^2 \quad \text{... (20)}$$

In this paper, we assume a relation between Hubble parameter H and scale factor R as

$$H = aR^{-m} \quad \text{... (21)}$$

where 'a' and 'm' are positive constant.

For this choice we obtain, scale factor R, Spatial Volume V, expansion scalar θ and deceleration parameter q as

$$R = (maT)^{1/m} \quad \text{... (22)}$$

$$V = (maT)^{3/m} \quad \text{... (23)}$$

$$\theta = \frac{3}{mT} \quad \text{... (24)}$$

$$q = m - 1 \quad \text{... (25)}$$

where $T = t + t_1$ (t_1 is integration constant).

We notice that scale factor R is negligible at T = 0 whereas expansion scalar θ is very larger at initial time, which express that the universe of model starts evolving with zero volume at initial time with a big-bang. It means to say that as T increases, the scale factor R increases unless volume expansion θ decreases. Since q = m – 1 > 0, the model of universe represents decelerating expansion throughout the evaluation of the unvierse. If m = 1, we get H = 1/T and q = 0. Therefore, galaxies moves with constant speed.

II. Solution and Discussion:

To determineρ, p, Λ and ξ explicitly from two equations (19) and (20), we need two more relation. We consider

$$\Lambda = 3\beta H^2, \beta > 0 \quad \text{... (26)}$$

and bulk viscosity is taken as

$$\xi = \xi_0\rho, \ \ \xi_0 > 0 \quad \text{... (27)}$$

Matter energy density ρ, cosmological term Λ, Isotropic pressure p, Bulk viscosity ξ and shear σ for the models are given by

$$\rho = \frac{3(1-\beta)}{m^2T^2} - \frac{k}{(maT)^{6/m}} - \frac{3}{(maT)^{2/m}}$$

$$\Lambda = \frac{3\beta}{m^2T^2}$$

$$p = [(2m-3) + 3\xi_0(1-\beta) + 3\beta]\frac{1}{m^2T^2} - \frac{(1-\xi_0)k}{(maT)^{6/m}} - \frac{(1+3\xi_0)}{(maT)^{2/m}}$$

$$\xi = \xi_0\left[\frac{3(1-\beta)}{m^2T^2} - \frac{k}{(maT)^{6/m}} - \frac{3}{(maT)^{2/m}}\right]$$

$$\sigma = \frac{k}{(maT)^{3/m}}$$

The model starts evolving with a big-bang from its singular state T = 0 with ρ, Λ, p, ξ and σ all infinite.

In the recent time (as T → ∞), matter density ρ, cosmological term Λ, isotropic pressure p, bulk viscosity ξ and shear σ becomes zero. Also for large values of T, ρ + p → 0. Therefore, the model is dominated by vacuum energy. For the model

$$\frac{\sigma}{\theta} = \frac{k}{3(maT)^{\frac{3}{m}-2}}$$

We observe that $\frac{\sigma}{\theta} \to 0$ at T → ∞.

Therefore model of universe represents isotropy at late times.

III. Conclusion:

In this paper, we studied the Bianchi type-V cosmological models with bulk viscosity and decaying vacuum energy density Λ. We adopt condition functional relation on Hubble parameter H, which represents constant value of deceleration parameter q. The cosmological term Λ is decaying function of time.

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